Asset Bubbles and Sudden Stops in a Small Open Economy

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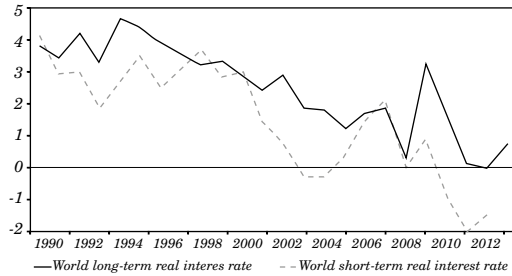*CREI, Universitat Pompeu Fabra and Barcelona GSE*

One of the most striking features of the world economy over the last twenty-five years has been the sharp decline in the real interest rate from approximately 4% in the early 1990s to -1.5% in 2013 (figure 1). During this period, there have been two waves of large capital inflows into emerging economies (figure 2). In the first wave, which began in the early 1990s and ended with the Asian crisis of 1997, net capital flows to these economies went from zero to approximately 3.5% of their combined GDP. The second wave started in the early 2000s and peaked in 2007 as inflows reached approximately 5% of emerging-market GDP. Capital inflows contracted sharply during the financial crisis of 2008-2009, but they have rebounded since.

At first glance, both pieces of evidence appear to be positive news for emerging markets, which have enjoyed increased access to foreign financing at lower interest rates. And yet, there is much debate regarding the desirability of low interest rates and surges in capital inflows.

Figure 1. World interest rates, 1990-2013

Source: OECD. Stat. World Series are real GDP-weighted averages for the short-and long-term real interest rates in the G7 countries. The series for short-term real interest rates excludes, for data availability reasons, Japan from 1990 to 2002, while the long-term real interest rate series excludes Italy from 1990 to 1991.

Figure 2. Net capital inflows to emerging economies
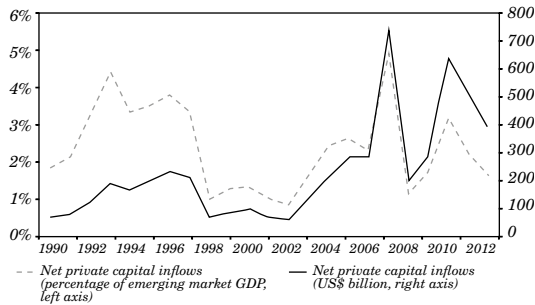
Source: IMF, Balance of Payments Statistics. Emerging economies are defined as in Cardarelli *et al.* (2009).

One of the main reasons is that these episodes may end in so called "sudden stops," sharp reversals in capital flows that are typically accompanied by falling asset prices and deep recessions. This narrative is often invoked to explain the events that transpired across many emerging markets during the 1990s, in particular, the Mexican crisis of 1994 and the East Asian crisis of 1997. As we write this article, it is also invoked to warn of the dangers that lie ahead once the Federal Reserve decides to tighten its monetary policy and increase interest rates in the United States, attracting part of

the capital flows that are now heading to the emerging world. This preoccupation with the downside risks of capital inflows has been accompanied by a growing endorsement of policy intervention in the form of capital controls.

But what is the role of capital controls in a world of financial globalization and low interest rates? How do capital flows behave in such a world, and what are the market failures that should be addressed through intervention? In this paper, we provide a simple framework to think about these questions.

To do so, we first develop a standard model of a small open economy with borrowing constraints. In our model, agents need to borrow from foreigners in order to invest. Due to weak enforcement institutions, however, foreign borrowing must be backed by the value of domestic firms. Our key innovation is to note that, in low interest rate environments, the value of firms has both a fundamental and bubble component. The fundamental component corresponds to the capital that is owned by the firm, i.e., it is the part of the firm's value that is backed by expectations of future production. The bubble component instead corresponds to the part of the firm's value that is positive today only because it is expected to be positive tomorrow, i.e., it is the part of the firm's value that is backed by expectations of the future bubble. The literature on capital flows in the presence of borrowing constraints has focused exclusively on the fundamental component of asset prices. Whenever the interest rate is low enough, however, we show that this view is incomplete. In this case, there is room for investor optimism to sustain bubbles that relax the country's borrowing constraint and fuel capital inflows.

A first contribution of our paper is to characterize the effects of bubbles on capital flows, investment and growth. By definition, bubbles enable domestic borrowers to obtain foreign credit in excess of the fundamental value of their firms. Intuitively, the international financial market is willing to lend in excess of this value because it anticipates that firms will have a bubble component in the future as well. In this sense, bubbles have a crowding-in effect, which raises gross capital inflows, investment and growth. But the bubble component has also fueled "excess" credit in the past, which requires diverting some of today's resources away from investment to pay foreign creditors. This is the crowding-out effect of bubbles, which raises gross capital outflows and reduces investment and growth.

The net effect of bubbles depends on the relative strength of these two effects. In particular, we find that the crowding-in effect dominates during "normal times," when the small open economy faces

an elastic supply of funds from the international financial market. At these times, it is the value of domestic firms that constrains foreign borrowing, and bubbles relax this constraint by raising the value of firms. In contrast, during "sudden stops," it is the supply of funds from the international financial market that constrains foreign borrowing. At these times, bubbles cannot raise gross capital inflows but they do raise gross outflows because foreign creditors must be repaid, and the crowding-out effect of bubbles dominates. Thus, the bubble that attains the optimal level of investment grows during normal times and shrinks during sudden stops.

A second contribution of our paper is to explore the role of capital controls in this environment. An essential feature of bubbles is that they are driven by investor sentiment or market expectations. Their value today depends on market expectations about their value tomorrow, which in turn depends on tomorrow's market expectations about their value on the day after, and so on. Because of this, the bubble provided by the market may be too small—and gross capital inflows insufficient—during normal times, while it may be too large—and gross capital outflows excessive—during sudden stops. We show that a government that can impose taxes and subsidies on gross capital flows can use such controls to replicate the bubble allocation that maximizes output. To do so, it must subsidize gross capital outflows during normal times but tax them during sudden stops.

The view of capital controls that emerges here seems to contradict the recent literature on capital flows and pecuniary externalities, which emphasizes the precautionary nature of controls.[1] This may seem odd because both frameworks are similar, but they differ in two important respects. First, we consider low interest rate environments in which bubbles may arise. These bubbles provide collateral and make it possible for foreign borrowing to expand in normal times. If the bubbles supplied by the market are small, however, the government can complement them by subsidizing gross capital outflows, which amounts to a public guarantee of private

1. See, for instance, Caballero and Krishnamurthy (2001), Mendoza (2010), Bianchi (2011), and Korinek (2011). In most of these models, foreign borrowing is constrained by the value of domestic assets in terms of tradable goods. When a sudden stop occurs, foreign borrowing declines for two reasons: there is the direct effect of the shock that gives rise to the sudden stop (e.g., increased risk aversion of international investors), but there is also an indirect effect because asset prices fall and/or the real exchange rate depreciates. Because domestic agents do not internalize this last effect when they make their borrowing decisions, there is over-borrowing in equilibrium and "prudential" capital controls that reduce net inflows that may be welfare-enhancing.

loans. By doing so, it relaxes the economy's borrowing constraint, and low interest rates ensure that the policy is sustainable. A second difference with the literature is that borrowing constraints in our model depend on expected—as opposed to contemporaneous—asset prices. This means that, in our framework, a fall in asset prices during sudden stops is actually good for economic activity: given expected asset prices, such a fall reduces payments to foreigners and increases domestic resources available for investment.

The rest of the paper is structured as follows. Section 1 develops a stylized model of an emerging market that faces borrowing constraints. Sections 2 and 3 explore bubbly equilibria in low interest rate environments and study the implications of bubbles for capital flows, investment and growth. Section 4 introduces a government and shows how capital controls can be used to maximize investment. Section 5 concludes.

1. A STYLIZED MODEL OF AN EMERGING MARKET

We next describe an economy that is only a very small part of a large world. We refer to the citizens of this economy as domestic residents, and to the citizens of the rest of the world as foreigners. Domestic residents work for domestic firms and manage them. Foreigners cannot do this. Domestic residents and foreigners interact in the credit market, where they exchange consumption goods for promises to deliver consumption goods in the future.

Domestic firms produce consumption goods using capital and labor with a standard Cobb-Douglas technology:

$$y_t = A \cdot k_t^{\alpha} \cdot l_t^{1-\alpha} \tag{1}$$

with $A > 0$ and $\alpha \in (0.1)$; and y_t, k_t and l_t denote output, the capital stock and the labor force, respectively. The capital stock evolves as follows:

$$k_{t+1} = i_t + (1-\delta) \cdot k_t \tag{2}$$

where i_t is investment and $\delta \in (0.1)$ is the depreciation rate. The labor force is constant and equal to one. Workers are paid their marginal product and, as a result, they receive a fraction $1-\alpha$ of output. The remaining output is distributed to firm owners. Domestic residents trade old firms and they can also create new ones at zero cost. Let

v_t be the market value of all firms after output has been distributed and before new investments have been made. Thus, v_t is the market value of firms that contain the undepreciated capital left after production, i.e. $(1-\delta) \cdot k_t$.

There are overlapping generations of domestic residents that live for two periods. All generations have a size of one and contain a fraction μ of patient residents that maximize expected consumption during old age, and a fraction $1-\mu$ of impatient residents that maximize consumption during youth. Patient residents save, own firms and consume when old. Impatient residents consume when young and never own firms. These assumptions imply the following aggregate consumption and investment:

$$c_t = \left(1-s\right) \cdot y_t + v_t - R_t^* \cdot f_{t-1} \tag{3}$$

$$i_t = s \cdot y_t + f_t - v_t \tag{4}$$

where c_t is consumption, f_t foreign borrowing and R_t^* the interest rate paid on foreign borrowing, and $s \equiv \mu \cdot \left(1-\alpha\right)$. In this economy, the impatient young and the patient old are the domestic consumers. Thus, equation (3) says that aggregate consumption equals their combined income which consists of a fraction $1-s$ of the economy's output, plus the price obtained by the old when they sell their firms, minus their foreign interest payments. In this economy, the patient young are the domestic investors. Thus, equation (4) says that aggregate investment equals the income of the patient young, which is a fraction s of the economy's output, plus their foreign borrowing less the price they pay for their firms.

There are two frictions that limit foreign borrowing: one of them originates abroad, and the other at home. The foreign friction is the possibility of sudden stops.[2] In particular, there are two possible states: $z_t \in \{N, S\}$ which we refer to as normal times and sudden stops. In normal times, foreigners provide credit to domestic residents at an expected return $E_t R_{t+1}^* = \rho$, up to a maximum of F. We think of F as being large, which makes this assumption inconsequential for most of the paper. It is nonetheless useful because it ensures that

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2. From the perspective of our economy, this is not really a friction but simply a description of the environment in which it operates. We refer nonetheless to the assumption that the supply of funds is volatile as a foreign friction because we think that this volatility stems from an (unmodeled) imperfection in the international financial market.

the small-open-economy assumption is sensible by guaranteeing that foreign borrowing is always bounded in equilibrium. During sudden stops, foreigners do not lend to domestic residents. Let σ be the probability of a sudden stop starting, i.e. $\sigma = Pr\ (z_{t+1}=S/z_t = N)$; and let η be the probability of a sudden stop ending $\eta = Pr\ (z_{t+1} = N/z_t = S)$. Domestic residents can always lend to the rest of the world at the interest rate ρ.

The domestic friction is insufficient collateral. In particular, domestic courts can seize the price that firm owners obtain when they sell their firms, but not the output that these firms distribute to their owners. As a result, firm owners can only promise a payment of v_{t+1} to their foreign creditors. Define the economy's collateral as the maximum value of payments tomorrow that can be promised today. Since contingent contracts are possible, the economy's collateral equals $E_t v_{t+1}$.[3]

Combining foreign and domestic frictions, we obtain the country's borrowing limit:

$$\overline{f_t} = \begin{cases} \dfrac{E_t v_{t+1}}{\rho} & \text{if } z_t = N \\ 0 & \text{if } z_t = S \end{cases} \tag{5}$$

In normal times, the borrowing limit equals the discounted value of the economy's collateral. During sudden stops, the borrowing limit drops to zero.[4]

Ideally, the country would borrow (or lend) until the return on investment equals the borrowing rate. This is only possible, though, if the borrowing required to achieve this does not exceed the borrowing limit. Otherwise, the country borrows up to the limit. This implies the following dynamics for the capital stock:

$$k_{t+1} = \min\left\{ s \cdot A \cdot k_t^\alpha + \overline{f_t} - v_t + (1-\delta) \cdot k_t, \left(\frac{\alpha \cdot A}{\rho + \delta - 1}\right)^{\frac{1}{1-\alpha}} \right\} \tag{6}$$

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3. If courts could also seize the output that firms distribute to their owners, the economy's collateral would be $\alpha \cdot y_{t+1} + E_t v_{t+1}$. If credit contracts could not be contingent, the economy's collateral would be $\min v_{t+1}$.

4. Equation (5) incorporates our assumption that F is arbitrarily large and that it, therefore, always exceeds the discounted value of the economy's collateral.

For a given borrowing limit, equation (6) shows that high firm prices divert resources away from investment. This is the crowding-out effect of current firm prices, which is always present. But equation (5) shows that high expected firm prices expand the borrowing limit and provide additional resources for investment. This is the crowding-in effect of future firm prices, which operates in normal times when foreign borrowing is constrained by the lack of domestic collateral. To understand the dynamics of capital accumulation and foreign borrowing we must therefore establish how firm prices behave. We turn to this task next.

2. Asset bubbles and their effects

It might surprise some readers that there exist equilibria in which the prices of firms exceed the price of the capital these firms contain. When this is the case, we say that there is a bubble in firm prices. In particular, the appendix shows that there are many equilibria in which firm prices take the following form:

$$v_t = (1-\delta) \cdot k_t + b_t \tag{7}$$

Equation (7) says that the price of firms can be thought of as the sum of two components, which we refer to as the fundamental and the bubble.

The first component of the price of firms is the price of the capital that firms contain, or fundamental: $(1-\delta) \cdot k_t$. Investors are willing to pay this price for the capital contained in the firm since this is exactly what it would cost them to produce such an amount of capital through investment.

The second component of the price of firms is an "overvaluation" or bubble: b_t. At first glance, the existence of this bubble might seem inconsistent with maximization. If firms are bubbly, wouldn't investors prefer to create new firms at zero cost and then obtain the same amount of capital by investing in them? Whether this is a preferable strategy or not depends on how the bubble evolves over time. If the bubble grows fast enough, investors might even prefer purchasing bubbly firms over creating new ones at zero cost.

The question is then: how does the bubble component evolve in equilibrium? The appendix shows that the following bubble dynamics are consistent with maximization and market-clearing:

$$b_{t+1} = \begin{cases} \left(\rho + u_{t+1}\right) \cdot b_t + n_{t+1} & \text{if } z_t = N \\ \left(\alpha \cdot A \cdot k_{t+1}^{\alpha-1} + 1 - \delta + u_{t+1}\right) \cdot b_t + n_{t+1} & \text{if } z_t = S \end{cases} \quad (8)$$

such that $E_t u_{t+1} = 0$ and $n_{t+1} > 0$. Equation (8) says that the bubble has two sources of dynamics. The first one is the growth of pre-existing bubbles, while the second one is the creation of new bubbles. The first term of the right-hand side of equation (8) measures the growth rate of pre-existing bubbles. Since the return to the bubble is its growth, in equilibrium this growth is determined by supply and demand. In normal times, bubbles can be used as collateral to borrow, and their expected growth must equal the world interest rate: ρ. If bubbles grew faster, the demand for bubbly firms would be unlimited, since borrowing to purchase bubbly firms would deliver a net profit. If bubbles grew less, the demand for bubbly firms would be zero, since borrowing to purchase bubbles would produce a loss. During sudden stops, bubbles cannot be used as collateral to borrow and their expected growth must equal the opportunity cost of funds, which is the return to investment: $\left(\alpha \cdot A \cdot k_{t+1}^{\alpha-1} + 1 - \delta\right) \cdot b_t$. The second source of bubble dynamics is the creation of new bubbles: n_{t+1}. The new set of bubbles that are created or initiated by generation t of investors constitute net wealth for them.

The argument above explains why the demand and supply for firms match at the proposed firm prices. But this argument is incomplete because we have implicitly assumed that the foreign borrowing associated with the bubble dynamics in equation (8) never exceeds the maximum F. For this to be the case, the bubble process must not explode and this requires that ρ be low enough. This is a key observation: bubbly equilibria are only possible in low interest rate environments. If ρ is high enough, there is a unique equilibria with $b_t = 0$ always.

Using equations (7) and (8), we can re-write equations (5) and (6) as follows:

$$k_{t+1} = \min\left\{ s \cdot A \cdot k_t^\alpha + \bar{f}_t - b_t, \left(\frac{\alpha \cdot A}{\rho + \delta - 1}\right)^{\frac{1}{1-\alpha}} \right\} \quad (9)$$

$$\bar{f}_t = \begin{cases} \dfrac{(1-\delta) \cdot s \cdot A \cdot k_t^\alpha + E_t n_{t+1}}{\rho + \delta - 1} + b_t & \text{if } z_t = N \\ 0 & \text{if } z_t = S \end{cases} \quad (10)$$

Figure 3. The law of motion for capital
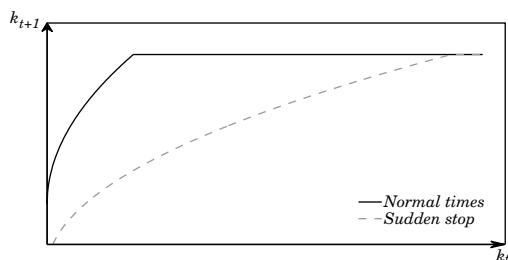
Equations (9) and (10) jointly describe the law of motion of the capital stock. Figure 3 shows that this law of motion contains three differentiated regions. For high levels of capital stock, the law of motion is flat and independent of whether there is a sudden stop or not. In this range of capital stocks, the borrowing limit is never binding. The economy is capital-abundant and it exports savings until the return on investment equals the world interest rate. Sudden stops are irrelevant because the economy does not borrow abroad.

For intermediate levels of the capital stock, the law of motion is flat in normal times but upward-sloping during sudden stops. In this range of capital stocks, the borrowing limit is binding during sudden stops only. The economy is no longer capital-abundant but it has enough collateral to import savings until the return on investment equals the world interest rate in normal times. During sudden stops, however, domestic investors cannot borrow abroad and investment equals domestic savings. As a result, the capital stock drops and the return on investment increases above the world interest rate.

For low levels of the capital stock, the law of motion is upward-sloping both in normal times and during sudden stops. In this range of capital stocks, the borrowing limit is always binding. The economy does not have enough collateral to import savings until the return on investment equals the world interest rate. The law of motion in normal times is above that of sudden stops because domestic investors can supplement domestic savings with foreign savings in normal times, but this is not possible during sudden stops.

Equations (9) and (10) show that asset bubbles only affect capital accumulation if the borrowing limit is binding. Interestingly, the effects of bubbles on capital accumulation differ in normal times and during sudden stops. In normal times, bubble creation provides collateral that raises foreign credit and investment. This raises capital accumulation. During sudden stops, pre-existing bubbles divert part of the economy's savings away from investment. This lowers capital accumulation. To determine the relative importance of these two effects, we must look at specific equilibria.

To find equilibria of this economy, we take an initial condition $\{k_0, b_0, z_0\}$ and a joint stochastic process for $\{u_t, n_t, z_t\}$ such that $Pr(z_{t+1}=S/z_t = N) = \sigma$, $Pr(z_{t+1} = N/z_t = S) = \eta$, $E_t u_{t+1} = 0$ and $n_{t+1} \geq 0$. If all possible sequences for $\{k_t, b_t, z_t\}$ generated in this way are such that $b_t \geq 0$ and $k_t \geq 0$, then we have found an equilibrium of the model. It turns out that this simple model can give rise to a large set of equilibria. A full analysis of this set is beyond the scope of this paper, and we refer the reader to our earlier work. Here, we just show some examples to build intuition.

3. EXAMPLES

We now construct a set of examples to illustrate the effect of bubbles. First, we simplify by assuming that $\eta=1$. That is, sudden stops only last one period. Second, we assume a very simple bubble process. During normal times the bubble is constant and equal to b. Bubble creation adjusts to ensure this. During sudden stops the bubble drops by a fraction d and there is no bubble creation.[5]

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5. In particular, we assume the economy starts in $k_t = k_0$, $b_t = b$ and $z_t = N$ and $\{u_t, n_t\}$ follows this stochastic process:

$$n_{t+1} = \begin{cases} \left(1 - \rho - d \cdot \dfrac{\sigma}{1-\sigma}\right) \cdot b & \text{if } z_t = N \text{ and } z_{t+1} = N \\ 0 & \text{if } z_t = N \text{ and } z_{t+1} = S \\ \left[1 - \left(\alpha \cdot A \cdot k_{t+1}^{\alpha-1} + 1 - \delta\right) \cdot (\rho - d)\right] \cdot b & \text{if } z_t = S \end{cases}$$

$$u_{t+1} = \begin{cases} d \cdot \dfrac{\sigma}{1-\sigma} & \text{if } z_t = N \text{ and } z_{t+1} = N \\ -d & \text{if } z_t = N \text{ and } z_{t+1} = S \\ 0 & \text{if } z_t = S \end{cases}$$

There are thus two key parameters that drive the example: the size of the bubble b, and its reaction to a sudden stop d. A higher value of b implies more bubble creation and thus greater borrowing and investment during normal times, but it also diverts more resources away from investment during sudden stops. A higher value of d means that the bubble becomes smaller during sudden stops and reduces its negative impact on investment at these times.

Based on these observations, we consider five different equilibria that are defined as follows:

Size \ Reaction to Sudden Stop	$d=0$	$d=\rho$
$b=0$	1	1
$b=b_L$	2	3
$b=b_H$	4	5

We analyze each equilibrium under the assumption that $\rho < 1 - \sigma$ (i.e., that the probability of experiencing a sudden stop is sufficiently low). In equilibria 2 and 4 the size of the bubble does not change during sudden stops, whereas in equilibria 3 and 5 the bubble disappears during sudden stops.

We simulate each of these equilibria by assuming that the economy starts in the steady state that corresponds to normal times and then simulating it forward for 100,000 periods. Table 1 reports the parameter values used in these simulations. Table 2 reports the mean and the standard deviation of k_t, y_t, c_t, f_t, and v_t for each simulation. Figure 4 plots the simulated time series of k_t, c_t and f_t during a window of 20 periods. These simulations offer an excellent summary of the main effects of bubbles in our economy.

Table 1. Parameter values

Parameter	Value	Parameter	Value
A	1	δ	0.8
α	0.47	b_L	$1.25k_A^*$
μ	0.9	b_H	$2.5k_A^*$
ρ	0.6	σ	0.15

Notes: k_A^* stands for the steady-state capital stock that would prevail in autarky (that is, $k_A^* = (sA)^{\frac{1}{1-\alpha}}$).

Table 2. Summary statistics

Equilibrium	1	2	3	4	5
k_t					
Mean	0.485	0.813	0.768	1.097	1.020
Standard deviation	0.063	0.238	0.144	0.406	0.219
y_t					
Mean	0.710	0.894	0.879	1.011	1.002
Standard deviation	0.046	0.150	0.086	0.248	0.116
c_t					
Mean	0.372	0.467	0.460	0.529	0.524
Standard deviation	0.024	0.079	0.045	0.130	0.061
f_t					
Mean	0.147	0.680	0.617	1.200	1.079
Standard deviation	0.058	0.267	0.241	0.471	0.421
v_t					
Mean	0.097	0.456	0.422	0.805	0.741
Standard deviation	0.013	0.059	0.105	0.108	0.208

First, bubbles improve the workings of the economy in normal times but worsen them during sudden stops. In normal times, the collateral that bubbles provide raises foreign borrowing and investment; during sudden stops, these same bubbles divert resources away from investment in what could be termed a "bubble overhang" effect. Thus, bubbles create additional volatility. Table 2 indeed shows that the mean values and standard deviations of k_t, y_t, c_t, f_t, and v_t are highest in equilibria 4 and 5, followed by equilibria 2 and 3 (in which $b=b_L$) and, finally, by equilibrium 1 (in which $b=0$). Figure 4 also illustrates this point for a given subsample of the simulation. In each panel, the solid line depicts the simulated time series of the specific equilibrium being analyzed, whereas the dashed line depicts the simulated time series for equilibrium 1. The figure shows how bubbles raise volatility by increasing economic activity in normal times and, in equilibria, by lowering it during sudden stops.

Figure 4. Simulated time series
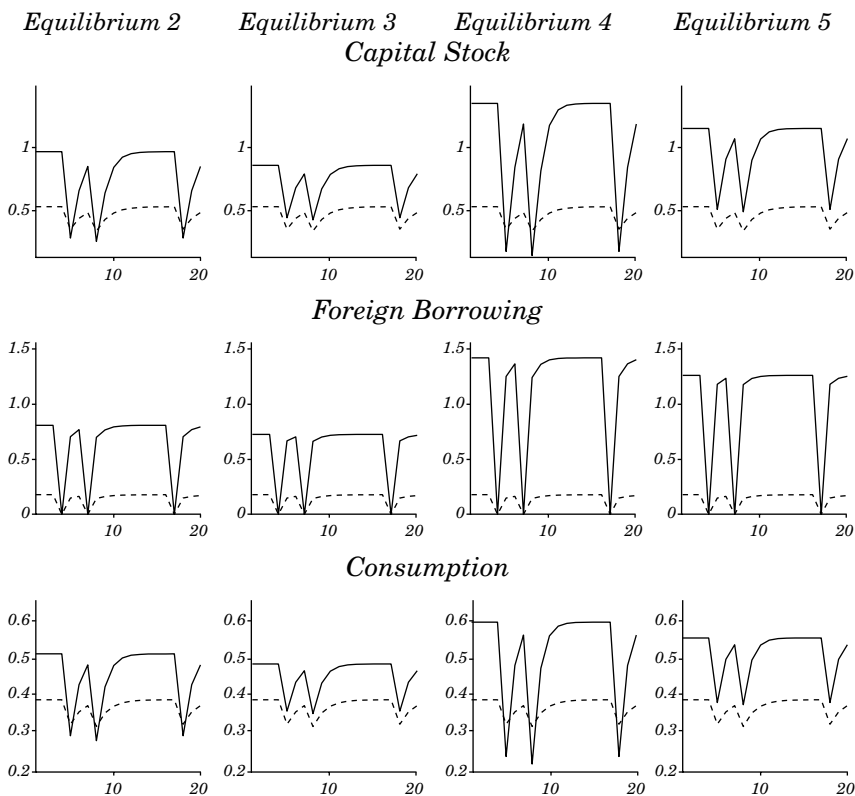
Note: Dashed lines show the time series for Equilibrium 1 (no bubble).

Second, all bubbles are not alike. In terms of economic activity, the best bubbles are those that maximize investment in normal times while hurting it as little as possible during sudden stops. Intuitively, these are bubbles that are large on average but small during sudden stops. In our examples, we have already mentioned that the average values of k_t, c_t and y_t are in fact maximized in equilibria 4 and 5, in which $b=b_H$. Among these, equilibrium 4 displays both a higher average and volatility of these variables. In particular, even though the values of k_t, c_t and y_t are substantially higher in equilibrium 4 than in 5 during normal times, the opposite is true during sudden stops. The reason is that $d > 0$ in economy 5. This reduces volatility

by making the bubble smaller during sudden stops, but it also reduces investment and output by reducing bubble creation during normal times.

This second result is important to contrast our conclusions with those obtained from related models in the literature. In our model, a fall in asset prices during sudden stops is actually good for economic activity: given expected asset prices, such a fall reduces payments to foreigners and increases domestic resources available for investment. This follows in our setup because credit contracts are contingent and borrowing constraints depend on expected asset prices. This is contrary to most other models in the literature, which assume that borrowing constraints depend on contemporaneous asset prices. Thus, a fall in asset prices during sudden stops tightens borrowing constraints even further, and leads to an even more severe drop in investment.

A third important point illustrated by our example is that, even though all bubbles are different, it is not possible to determine whether the market will select any specific one over the rest. Thus, sometimes the bubble sustained by market expectations may be too small, preventing the country from taking advantage of foreign borrowing; at other times the bubble may be too large, generating large recessions during sudden stops. This raises the question of whether policy can be used to improve upon the market equilibrium. We now turn to his question, focusing on the specific role of capital controls.

4. MANAGING CAPITAL FLOWS

We introduce a government that manages capital flows by introducing capital controls, i.e., taxes and/or subsidies to foreign borrowing and/or repayment. We then use this modified model to ask two key questions: Can capital controls be used to replicate a desired equilibrium bubble? If so, what should they look like?

To address these questions, let us revisit the role played by the bubble in our economy. The bubble creates a series of transfers between individuals. Current borrowers use b_t of the funds borrowed today to buy the bubble from previous borrowers, which in turn use this income to make payments to foreigners. But the bubble also allows current borrowers to increase their foreign borrowing by $\frac{E_t b_{t+1}}{\rho}$, since they will sell the bubble to future borrowers for a price of b_{t+1}, and they will use this income to make additional payments to

foreigners. Of course, this last effect is absent during a sudden stop. Thus, from the perspective of the country as a whole these transfers generate the following funds for investment:

$$v_t^b = \begin{cases} \dfrac{1}{\rho + \delta - 1} \cdot \left(\dfrac{E_t b_{t+1}}{\rho} - b_t \right) & \text{if} \quad z_t = N \\ -b_t & \text{if} \quad z_t = S \end{cases} \tag{11}$$

Equation (11) summarizes the effect of a bubble on investment. In normal times, the country as a whole is paying foreigners with a fraction of the funds that it's borrowing from them. The bubble therefore raises both gross inflows and outflows (i.e., payments from foreigners to domestic residents and vice-versa). The bubble also raises the difference between the two, i.e., the increase in net inflows provides resources that can be leveraged to boost investment. During sudden stops, the presence of the bubble does not increase foreign borrowing, but it does mean that the patient young must divert part of their resources towards purchasing the bubble from the old. Thus, equation (11) restates the main result of our examples: the bubble raises investment in normal times but detracts from it during sudden stops. In terms of maximizing investment, therefore, the ideal bubble is one that entails the most bubble creation during normal times while being as small as possible during sudden stops. Of course, nothing guarantees that the market will deliver this bubble, and this is where capital controls may be of help.

Assume that the government imposes a lump-sum tax on foreign borrowing (or gross capital inflows) equal to p_t, where p_t can be state contingent and $p_t < 0$ indicates a subsidy on borrowing. We assume initially that these tax revenues are transferred to previous borrowers so that they can make payments to foreigners, i.e., taxes on gross capital inflows go hand-in-hand with subsidies on gross capital outflows. In order to establish an analogy with the dynamics of the bubble, we can express the evolution of p_t as:

$$p_{t+1} = \begin{cases} \left(\rho + e_{t+1} \right) \cdot p_t + m_{t+1} & \text{if } z_t = N \\ \left(\alpha \cdot A \cdot k_{t+1}^{\alpha - 1} + 1 - \delta + e_{t+1} \right) \cdot p_t + m_{t+1} & \text{if } z_t = S \end{cases}, \tag{12}$$

such that $E_t e_{t+1}$ and $m_{t+1} \lessgtr 0$ reflects the net wealth that the policy transfers to patient individuals of generation t. To see this, consider

an individual of generation t that pays a tax p_t when borrowing during youth but expects to receive a subsidy p_{t+1}, to be used for the repayment of creditors, during old age. For such an individual, the policy represents a transfer of present-value wealth equal to,

$$
\begin{aligned}
\frac{E_t m_{t+1}}{\rho} &= \frac{E_t p_{t+1}}{\rho} - p_t && \text{if } z_t = N \\
\frac{E_t m_{t+1}}{\alpha \cdot A \cdot k_{t+1}^{\alpha-1} + 1 - \delta} &= \frac{E_t p_{t+1}}{\alpha \cdot A \cdot k_{t+1}^{\alpha-1} + 1 - \delta} - p_t && \text{if } z_t = S
\end{aligned}
$$

Although equation (12) has been deliberately written so that the evolution of p_t mimics the evolution of b_t in equation (8), both processes are not subject to the same constraints. In particular, whereas b_t and n_t must be non-negative, p_t and m_t can be either positive or negative: $p_t < 0$ means that the policy prescribes a subsidy on gross capital inflows at time t, and $E_t m_{t+1} < 0$ means that—in expectation—the policy extracts wealth from generation t.

We are now ready to analyze the main effects of capital controls on our economy. First, the expectation of subsidies on gross capital outflows tomorrow enables patient individuals to expand their borrowing today, i.e., current borrowers can pledge both the expected value of their firms and the expected value of subsidies. Taking this into account, we can re-write equations (5) and (6) as follows:

$$
k_{t+1} = \min\left\{ s \cdot A \cdot k_t^\alpha + \overline{f}_t - b_t - p_t, \left(\frac{\alpha \cdot A}{\rho + \delta - 1} \right)^{\frac{1}{1-\alpha}} \right\} \tag{13}
$$

$$
\overline{f}_t = \begin{cases} \dfrac{E_t v_{t+1} + E_t p_{t+1}}{\rho} & \text{if } z_t = N \\ 0 & \text{if } z_t = S \end{cases} \tag{14}
$$

Equations (13) and (14) are natural generalizations of equations (9) and (10) and they illustrate the conflicting effects of policy on capital accumulation. Every period, the policy imposes taxes on current borrowers and gives it to previous borrowers so that they can make payments to foreigners. This is the crowding-out effect of the policy. During normal times, though, the policy also has a crowding-in effect because it allows current borrowers to expand their foreign

borrowing against the expected subsidies that they will receive at the time of repayment $\frac{E_t p_{t+1}}{\rho}$, which will be funded with the taxes of future borrowers. In the aggregate, this policy generates the following funds for investment:

$$
v_t^p = \begin{cases} \dfrac{1}{\rho + \delta - 1} \cdot \left(\dfrac{E_t p_{t+1}}{\rho} - p_t \right) & \text{if } z_t = N \\ -p_t & \text{if } z_t = S \end{cases}
\tag{15}
$$

To find equilibria in this economy, we take an initial condition $\{k_0, b_0, z_0, p_0\}$ and a stochastic process for $\{u_t, n_t, e_t, m_t\}$ such that $E_t u_{t+1} = 0$, , and $n_{t+1} \geq 0$ for all t. If all possible sequences for $\{k_t, b_t, z_t, p_t\}$ generated in this way are such that $b_t \geq 0$ and $k_t \geq 0$, then we have found an equilibrium of the model.

What can a government achieve by adopting capital controls? A crucial result, which follows directly from comparing equations (9) and (10) with equations (13) and (14), is that taxes and subsidies on gross capital flows can be used to complement or counteract fluctuations in the bubble. If the bubble is not high enough to take full advantage of foreign funds during normal times, a policy that sets $E_t m_{t+1} > 0$ by taxing gross capital inflows and subsidizing gross capital outflows helps raise foreign borrowing. Such a policy amounts to a government guarantee on foreign payments and, as long as the rate of economic growth is higher than the interest rate, it can be designed to raise the wealth of borrowers at each point in time. During sudden stops, if the bubble is positive, thereby diverting funds from investment, a policy that sets $p_t < 0$ and taxes gross capital outflows is also useful. By reducing foreign payments and transferring resources to the patient young, such a policy raises the availability of domestic resources for investment and fuels growth. Thus, although capital controls do not directly affect the equilibrium value of the bubble, they do enable the government to "select" an equilibrium allocation among those that are feasible.

We return to our example with $\eta = 1$ to see how this can be done. In particular, we choose a policy rule that maximizes output. In normal times, this policy injects sufficient collateral to allow the economy to borrow until the return on investment equals the world interest rate. During sudden stops, borrowing is not possible and the goal of

policy is to offset the bubble and ensure that all domestic savings are used for capital accumulation.[6]

The effects of this policy rule can be analyzed by simulating the economy for each of the equilibria considered in section 3. Table 3 reports the mean and the standard deviation of k_t, y_t, c_t, f_t, and v_t for these simulations, where as figure 5 and 6 plots the simulated series for k_t, c_t and f_t, v_t and p_t over a window of 20 periods. The main result here is that, as figure 5 shows, the evolution of k_t, c_t and f_t is the same for all equilibria.

Figure 6 shows that, to attain the policy goals, taxes on gross inflows and outflows are contingent on the evolution of the bubble. In particular, taxes on gross capital inflows are used to complement bubble creation during normal times, raising collateral and thus foreign borrowing. Taxes on gross capital outflows are used instead during sudden stops to reduce payments to foreigners in an amount equal to the size of the bubble. By doing so, they redirect resources away from foreign creditors and towards young borrowers, enabling them to purchase the bubble without reducing investment.

This example shows how bubbles provide a new rationale for capital controls. When foreign borrowing is limited by the demand for funds, the government can complement the existing bubble with subsidies to capital outflows. When foreign borrowing is instead limited by the external supply of funds, the government can reduce the bubble's impact on investment by taxing capital outflows. Note that this policy can be loosely interpreted as an insurance or "counter-cyclical" fund, to which patient individuals contribute during youth in the expectation of receiving a transfer during old age if the bubble turns out to be low. But this insurance is not actuarially fair, and this is a crucial aspect of the policy. If $E_t m_{t+1} > 0$, as our proposed policy is during normal times, it provides net resources to the patient individuals of generation , thereby enabling them to expand their foreign borrowing. If $E_t m_{t+1} < 0$, it extracts net resources from the patient individuals of generation t.

This policy might seem at odds with the type of capital controls usually stressed in the literature, which are used in a "prudential" fashion to reduce capital inflows during normal times. In those models, as we have mentioned, capital inflows are inefficiently high during normal times and controls can be useful to reduce them. In our model, instead, capital inflows are inefficiently low during normal times because the economy suffers from a constant lack of collateral.

6. In particular, we assume that the economy starts in $k_t = k_0, b_t = b$ and $z_t = N$, and for a given stochastic process $\{u_t, n_t\}$, we define the following policy rule:

Figure 5. Simulated time series with capital controls

Capital stock

Foreign borrowing

Consumption
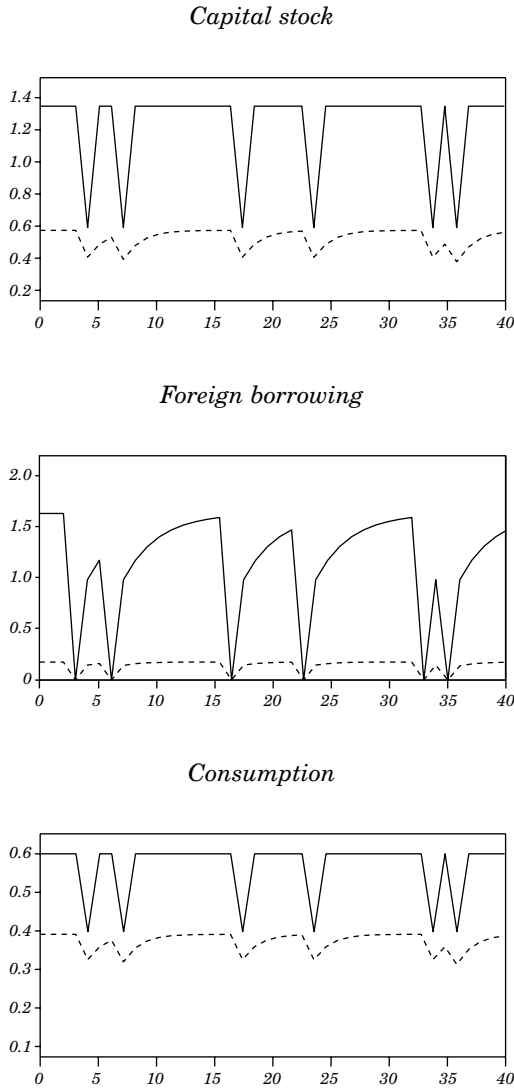
Notes: Dashed lines show the time series for Equilibrium 1 (no bubble) without capital controls.

Figure 6. Tax on capital inflows and firm values

Equilibrium 2   Equilibrium 3   Equilibrium 4   Equilibrium 5

Tax on capital inflows

Firm value
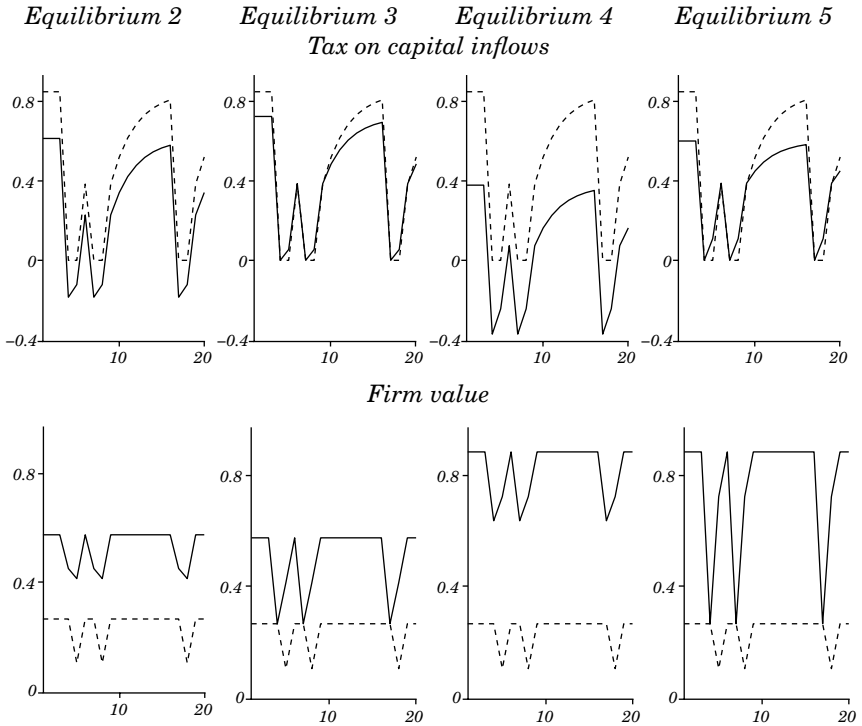
Note: Dashed lines show the time series for Equilibrium 1 (no bubble).

The same low interest rate environment that gives rise to bubbles, however, makes it possible for the government to "inject" collateral during normal times by taxing gross capital inflows and subsidizing gross capital outflows. If ρ was higher than one, the tax dynamics in equation (12) would require p_t to explode during normal times, and this would violate our implicit assumption that foreign borrowing cannot exceed the maximum F.

A final consideration refers to the robustness of our results. We have assumed throughout that the government runs a balanced budget in the sense that subsidies on capital outflows are fully financed by levying taxes on capital inflows. Nothing depends on this assumption, however, and it is possible to show that all of the results of this section go through if the government finances subsidies by issuing debt.

Table 3. Summary statistics

Equilibrium	1	2	3	4	5
k_t					
Mean	1.250	1.250	1.250	1.250	1.250
Standard deviation	0.272	0.272	0.272	0.272	0.272
y_t					
Mean	1.101	1.101	1.101	1.101	1.101
Standard deviation	0.135	0.135	0.135	0.135	0.135
c_t					
Mean	0.576	0.576	0.576	0.576	0.576
Standard deviation	0.071	0.071	0.071	0.071	0.071
f_t					
Mean	1.220	1.321	1.440	1.423	1.659
Standard deviation					
v_t					
Mean	0.521	0.544	0.582	0.572	0.654
Standard deviation	0.055	0.064	0.110	0.093	0.208

Although we refer the reader to our earlier work for a thorough treatment of this issue, the intuition behind this result is that debt has the same effects on investment as the bubble does.[7] During normal

7. For a full treatment of the effects of debt in this kind of model, see Martin and Ventura (2014a).

$$
m_{t+1} = \begin{cases} \dfrac{\rho}{1-\sigma} \cdot \max\left\{ \left(\dfrac{\alpha \cdot A}{\rho + \delta - 1} \right)^{\frac{1}{1-\alpha}} \cdot \dfrac{\rho + \delta - 1}{\rho} - s \cdot A \cdot k_t^\alpha - \dfrac{E_t n_{t+1}}{\rho}, 0 \right\} & \text{if } z_t = N \text{ and } z_{t+1} = N \\ 0 & \text{if } z_t = S \text{ or } z_{t+1} = S \end{cases}
$$

$$
e_{t+1} = \begin{cases} \dfrac{\sigma}{1-\sigma} \cdot \left(\dfrac{b_{t+1}}{p_t} + \rho \right) & \text{if } z_{t+1} = N \\ -\dfrac{b_{t+1}}{p_t} - \rho & \text{if } z_{t+1} = S \end{cases}
$$

times, a high expected value of debt allows borrowers to obtain more financing today, because—all else being equal—higher expected debt means that they will receive higher subsidies during old age. This is the crowding-in effect of debt. During sudden stops, however, debt has also a crowding-out effect: although foreign borrowing collapses, the existing stock of debt must still be purchased by the patient young and it therefore diverts resources away from investment.

5. CONCLUDING REMARKS

We live in a world of financial globalization and historically low interest rates. These interest rates create the conditions for bubbles to exist, which in turn affect the size and direction of capital flows. In this paper, we have developed an analytical framework to think about this interplay, with three main results. First, the effects of bubbles on economic activity depend on the circumstances at hand. During "normal times," when the supply of funds from the international financial market is high, bubbles raise net capital inflows, investment and growth. During sudden stops, when foreign funding dries up, bubbles instead have a negative effect on net capital inflows and economic activity. Second, the bubble that attains the optimal level of investment should therefore be large during normal times and small during sudden stops. But bubbles are driven by market expectations, and nothing guarantees that the equilibrium bubble will behave in the way that it is desired. This leads to our third result, which says that the government can replicate the desired bubble allocation through the appropriate use of capital controls. In particular, we show that it can maximize investment by subsidizing gross capital outflows and taxing gross capital inflows during normal times while adopting the opposite policy during sudden stops.

The framework developed here is closely related to models of financial frictions that have been recently used to advocate the usefulness of capital controls.[8] And yet, its implications for policy are quite different. The literature stresses the use of capital controls in a "prudential" fashion to reduce capital inflows during normal times. Our model instead suggests that policy interventions should be used to boost inflows even further during normal times. The main reason behind this difference is that we focus on a low interest rate

8. For a survey of this literature, see Korinek (2011).

environment, in which bubbles are possible. This same assumption guarantees that the government can boost net capital inflows, and thus foreign liabilities, beyond their equilibrium level. Basically, it can do so by using subsidies on gross capital outflows to roll over foreign liabilities during normal times. Once the sudden stop comes, the policy is reverted and gross capital outflows are taxed to raise domestic resources for investment.

These results provide a rich view of the relationship between bubbles and capital flows, and their implications for the design of policy. The framework has an important limitation, however: it considers only the case of a small open economy. Relaxing this assumption has crucial implications, both from the perspective of the economy that we are considering and of the world as a whole. From the perspective of our economy, abandoning this assumption means that it no longer faces a fully elastic supply of funds during normal times. This means that bubbles are unable to replicate the optimal level of investment even during normal times, because they raise the interest rate faced by the country so that there is always both a crowding-in and a crowding-out effect. From the perspective of the world as a whole, abandoning the small-open economy assumption means that capital controls have effects on third countries. When an economy adopts the policy analyzed here, it boosts capital inflows for instance, thereby reducing the availability of funds in the rest of the world and raising the international interest rate. This lowers investment and growth in other countries and, in a low-interest rate environment like the one that we consider, it may generate severe crises in the rest of the world by bursting existing bubbles. These global or systemic issues are fundamental. In Martin and Ventura (2014b), we develop a general equilibrium model of the world economy in order to address them appropriately.

APPENDIX

Let $j = 1,...,J$ be the patient young, and let $l = 1,...,L$ be the patient old. The old sell their firms to the young. The equilibrium prices of these firms is v_t^l and they contain undepreciated capital $(1-\delta) \cdot k_t^l$. Since there is one firm per old, they have the same index: $l = 1,..., L$. The young purchase these firms and make additional investments to build their own firms. Since there is one firm per young, they also have the same index: $j = 1,...,J$. L_j now defined as the set of firms that young purchases. He/she chooses i_t^j, f_t^j and L_j so as to maximize:

$$E_t c_{t+1}^j = R_{t+1}^K \cdot k_{t+1}^j + E_t v_{t+1}^j - \rho \cdot f_t^j \tag{16}$$

where $R_{(t+1)}^K = \alpha \cdot A \cdot k_{t+1}^{\alpha-1}$. Maximization is subject to these constraints:

$$w_t = i_t^j + \sum_{l \in L_j} v_t^l - f_t^j \tag{17}$$

$$k_{t+1}^j = i_t^j + \sum_{l \in L_j} (1-\delta) \cdot k_t^l \tag{18}$$

$$f_t^j \leq \begin{cases} \dfrac{E_t v_{t+1}^j}{\rho} & \text{if } z_t = N \\ 0 & \text{if } z_t = S \end{cases} \tag{19}$$

These constraints are self-explanatory.

We need to find a price process such that, given this price process, maximization leads to market-clearing. We propose the following one:

$$v_t^j = (1-\delta) \cdot k_t^j + b_t^j \tag{20}$$

$$b_{t+1}^j = \begin{cases} (\rho + u_{t+1}^j) \cdot b_t^j + n_{t+1}^j & \text{if } z_t = N \\ (R_{t+1}^K + 1 - \delta + u_{t+1}^j) \cdot b_t^j + n_{t+1}^j & \text{if } z_t = S \end{cases} \tag{21}$$

with $E_t u_{t+1}^j = 0$ and $n_{t+1}^j > 1$. Everybody expects this stochastic process to drive prices. Let us see if markets clear when individuals maximize.

The old are willing to sell at any price. What about the young? We show next that they are also willing to buy since they are indifferent about L_j.

Assume first that $R_{t+1}^K > \rho$ and the collateral constraint is not binding. Then, using this price process and substituting constraints (17)-(19) into the objective function (16), we find that:

$$E_t c_{t+1}^j = \rho \cdot \left(i_t^j + \Sigma_{l \in L_j} \left(1 - \delta\right) \cdot k_t^l \right) + \rho \cdot \Sigma_{l \in L_j} b_t^l + E_t n_{t+1}^j$$
$$- \rho \cdot \left(i_t^j + \Sigma_{l \in L_j} \left(\left(1 - \delta\right) \cdot k_t^l + b_t^l \right) - w_t \right) = \rho \cdot w_t + E_t n_{t+1}^j$$

With these prices, young j is indifferent about L_j. The old are happy to sell.

Assume next that $R_{t+1}^K > \rho$ and the collateral constraint is binding. There are two subcases here. First, assume we are in a sudden stop and $f_t = 0$. Using the price process and substituting constraints (17), (18) and (19) into the objective function (16), we find that:

$$E_t c_{t+1}^j = R_{t+1}^K \cdot \left(w_t - \Sigma_{l \in L_j} b_t^l \right) + \left(1 - \delta\right) \cdot \left(w_t - \Sigma_{l \in L_j} b_t^l \right) +$$
$$\left(R_{t+1}^K + 1 - \delta \right) \cdot \Sigma_{l \in L_j} b_t^l + E_t n_{t+1}^j = \rho \cdot w_t + E_t n_{t+1}^j$$

Again, with these prices young j is indifferent about L_j.

Second, assume that we are in normal times and $f_t = \dfrac{E_t v_{t+1}^j}{\rho}$.

Using the price process and substituting constraints (17), (18) and (19) into the objective function (16), we find:

$$E_t c_{t+1}^j = R_{t+1}^K \cdot \left(w_t - \Sigma_{l \in L_j} b_t^l + \Sigma_{l \in L_j} b_t^l + \frac{E_t n_{t+1}^j}{\rho} \right) = R_{t+1}^K \cdot \left(w_t + \frac{E_t n_{t+1}^j}{\rho} \right)$$

Again, with these prices, young j is indifferent about L_j. Thus, we have reached the conclusion that this price process is consistent with maximization.

Is this price process also consistent with market-clearing? It follows from the previous discussion that demand and supply for firms match with this price process, but there is a loose end in the discussion. In particular, we have implicitly assumed that the upper limit on foreign borrowing F is never binding. To ensure this, firm prices cannot grow too much. And this, in turn, requires that ρ be sufficiently low. We assume throughout that this is the case.

Having proved now that this price process is consistent with both maximization and market-clearing, we can aggregate over j and l, obtaining equations (7) and (8) in the text.

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