

THE FINANCIAL ACCELERATOR UNDER LEARNING AND THE ROLE OF MONETARY POLICY

Rodrigo Caputo
Central Bank of Chile

Juan Pablo Medina
Central Bank of Chile

Claudio Soto
Central Bank of Chile

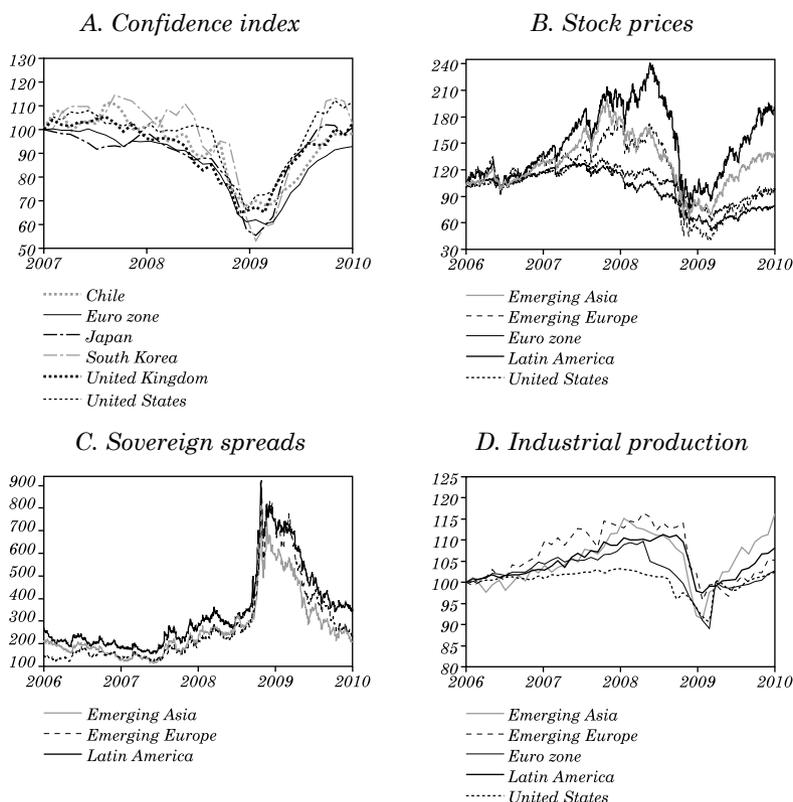
The financial crisis that unraveled after the Lehman Brothers collapse affected in different degrees almost all countries around the world, independently of the direct exposure of their financial institutions to toxic assets. Most countries saw a sharp drop in demand, together with sudden increases in financial spreads and a dramatic fall in stock markets. Developed countries and many emerging market economies responded with a considerable monetary easing accompanied by unconventional central bank policies and fiscal stimulus packages to moderate the downturn in the economy.

The sharp fall in assets prices was followed by a striking recovery, which has gone hand-in-hand with a reduction in financial market stress and an initial recovery in activity.¹ In fact, financial conditions in many countries quickly returned to their pre-Lehman levels at the end of 2009 (figure 1). While this may, in part, reflect the strong policy responses around the world, it is also consistent with the view that market participants overreacted to the initial shock and have been adjusting their expectations upward as the crisis turned out to be milder than initially thought.

We acknowledge comments and suggestions by Brian Doyle, Sofia Bauducco, Alejandro Justiniano and an anonymous referee. The research assistance of Ursula Schwarzhaupt is greatly appreciated.

1. This striking recovery took place, mainly, in emerging economies.

Monetary Policy under Financial Turbulence, edited by Luis Felipe Céspedes, Roberto Chang, and Diego Saravia, Santiago, Chile. © 2011 Central Bank of Chile.

Figure 1. Some Recent Stylized Facts

Sources: Bloomberg and Central Bank of Chile.

In this paper, we show that imperfections in financial markets, coupled with small departures from the standard rational expectations assumption used in most macroeconomic models, may lead to a significant amplification of the effects of shocks. We develop a dynamic stochastic general equilibrium (DSGE) model with nominal frictions and a financial accelerator mechanism as in Bernanke, Gertler, and Gilchrist (1999), under the assumption that agents form their expectations based on adaptive learning (Evans and Honkapohja, 2001). With a rather standard parametrization, our model is able to generate the dramatic fall in output and a relatively rapid recovery, similar to what we have observed in the recent crisis.

While both the financial accelerator mechanism and the learning assumption alone are able to amplify the effects of detrimental shocks on output, it is the combination of the two elements that turns out to be key for generating a sizable drop in output in response to negative shocks and producing a relatively fast recovery in asset prices. In this sense, the presence of a learning process, different from rational expectations, tends to exacerbate the impact of negative productivity shocks.

The model is a standard New Keynesian model with sticky prices and sticky wages. We introduce a financial accelerator mechanism as in Bernanke, Gertler, and Gilchrist (1999) and Gilchrist and Saito (2008), where firms are subject to idiosyncratic shocks and need to borrow to finance investment. Under the assumptions that the realization of idiosyncratic shocks is private information and that there is a costly state verification, the optimal contract between lender and borrower is a standard debt contract. The interest rate in this debt contract exhibits a premium above the risk-free interest rate, which is a positive function of the leverage of the borrower (the firm). Bernanke, Gertler, and Gilchrist (1999) show that the existence of this finance premium generates an accelerator effect that amplifies the impact of shocks on activity.² The adaptive learning approach that we use, in turn, follows from the idea that private agents and policymakers in the economy behave like applied economists and econometricians. In practice, econometricians base their forecasts on estimated models that are adapted and reestimated quite often. In our model, agents form their expectations of macroeconomic variables precisely by using the statistical forecasting models that applied economists use. The learning mechanism in our model is a constant-gain learning process, which does not guarantee convergence to a rational expectations (RE) equilibrium after a shock.³

This adaptive learning approach generates important propagation and amplification mechanisms that are not present under rational expectations equilibrium models. This is shown, for example, by Marcet and Nicolini (2003) in a standard monetary model with a quasi-rational learning process that is able to match the recurrent

2. Other contributions in this literature include Carlstrom and Fuerst (1997) and Kiyotaki and Moore (1997).

3. We do not analyze E-stability in our model, but the simulations discussed in sections 2 and 3 suggest that we do not obtain E-stability. For a deeper discussion on this topic, see Evans and Honkapohja (2001).

hyperinflations experienced by several countries in the 1980s. Milani (2005, 2007) also shows that when learning replaces rational expectations in a New Keynesian model, the estimated degrees of habits and indexation—which are usually important in RE models to explain inertia—are close to zero. This finding suggests that the propagation of shocks arises in the model economy mainly from expectations and learning. Similar conclusions are obtained by Slobodyan and Wouters (2009), who find that a DSGE model under adaptive learning can fit business cycle fluctuations much better than a model under rational expectations.

In our framework, the adaptive learning assumption, combined with the financial accelerator mechanism, leads to a large amplification of the effects of shocks on activity, demand, inflation, and asset prices. A detrimental shock that reduces output—modeled as a persistent fall in productivity—leads to a fall in the asset prices observed by agents and in the net worth of the firms, feeding back into expectations formation. If shocks are sequential, the expectations formation mechanism can endogenously generate a significant deviation of asset prices from their fundamental values, considerably amplifying the financial accelerator effect of detrimental shocks. These asset price fluctuations interact with the financial accelerator mechanism, reinforcing movements in real variables that, in turn, affect expectations and asset prices. Adam, Marcet, and Nicolini (2008) refer to this phenomenon under learning as momentum in asset prices. Eventually, the response of the monetary authority lowering the interest rate reverses the evolution of asset prices, reducing the risk premium and generating a recovery that feeds back into an improvement in asset prices. Thus, assets prices recover rapidly and activity approaches its equilibrium path under rational expectations relatively quickly.

We consider a model with nominal rigidities because it induces nontrivial policy trade-offs in the face of negative productivity shocks. This allows us to analyze the implication of alternative monetary policy regimes in the context of learning.⁴

In the baseline specification of the model, we assume that monetary policy is conducted by a simple Taylor rule. However, we are also interested in analyzing alternative specifications for the design of the monetary policy. In particular, we are interested in analyzing

4. The amplification of shocks due to the interaction of the financial accelerator mechanism and learning also holds in a simple real business cycle model that does not incorporate the nominal frictions. Results are available on request.

the case of a central bank that responds not only to fluctuations in output or inflation, but also to asset price movements. This question has been extensively debated in recent years.⁵ One prominent view is that a monetary policy that directly targets asset prices appears to have undesirable side effects. Bernanke and Gertler (1999, 2001) show that even in a model with a financial accelerator mechanism, like the one considered in this paper, asset prices become relevant only if they signal potential inflationary or deflationary forces. In other words, it is desirable for the central bank to focus exclusively on underlying inflationary pressures.⁶

An alternative view favors a more active role of monetary policy in the detection and prevention of asset market misalignments. For instance, Cecchetti and others (2000) argue that asset price bubbles create distortions in investment and consumption, leading to excessive fluctuations in activity and inflation. Hence, a monetary policy rate that responds modestly to deviations of asset prices from fundamentals would enhance overall macroeconomic and financial stability. Moreover, they suggest that a systematic policy of “leaning against the bubble” might reduce the probability of bubbles arising in the first place. Borio and Lowe (2002) also support the view that a monetary response to credit and asset markets may be appropriate to preserve both financial and monetary stability.⁷

As Bernanke and Gertler (2001) note, how monetary policy should behave in the face of endogenous panic-driven financial distress is an open question. One limitation of models that claim that central banks should not react directly to asset price fluctuations when

5. See IMF (2009, chapter 3) for a recent overview on this issue.

6. Other recent studies reach similar conclusions based on alternative frameworks in which financial frictions amplify the propagation of economic disturbances. For example, Gilchrist and Leahy (2002) do not find a strong case for including asset prices in monetary policy rules, despite the fact that asset prices exhibit large fluctuations that affect the real economy. Iacoviello (2005) develops a theoretical model in which collateral constraints are tied to housing values; he finds that responding to asset prices yields negligible gains in terms of output and inflation stabilization. In an economy with credit market imperfections, Faia and Monacelli (2007) find that monetary policy should respond to increases in asset prices, but the marginal welfare gain of responding to the asset price flattens out when monetary policy responds more aggressively to inflation. Finally, a recent empirical analysis by Ahearne and others (2005) tends to support the view that in practice, central banks have not reacted to episodes of rising asset prices, beyond taking into account their implications for inflation and output growth.

7. Tetlow and von zur Muehlen (2002) argue that a nonlinear monetary feedback rule that responds to bubbles might improve welfare, but only when the bubbles become large enough and, especially, when their size leaves little doubt that fundamentals cannot be their sole driving factor.

they deviate from their fundamentals is that their nonfundamental movements are generated exogenously. More precisely, asset prices deviate from fundamentals because agents have incomplete information about the driving forces in the economy. As time goes by, agents learn about these forces and asset prices converge back to their fundamentals. Thus, in these models, there is no feedback from the nonfundamental component of asset prices into the expectation formation. Our model with adaptive learning and financial frictions is able to tackle this issue since it endogenously generates asset price bubbles through the interaction of movements in different variables and the learning mechanism. This raises the question of whether responding aggressively only to inflationary pressures is still efficient in this environment. Our preliminary results indicate that this is the case. Even in the presence of endogenous bubbles, responding aggressively to inflation reduces output and inflation volatility. If the central bank adjusts its policy instrument in response to asset price fluctuations, it may reduce output volatility and even inflation volatility in the short run. However, this monetary policy leads to a surge in inflation some periods after the shocks. On the other hand, a policy that aggressively responds to *changes* in asset prices may marginally reduce output volatility with respect to a policy that reacts aggressively to inflation, but at the cost of generating inflationary pressures.

The rest of the paper is organized as follows. Section 1 presents a linearized version of a closed economy model with a financial accelerator. Section 2 discusses the adaptive learning mechanism that governs agents' expectations. In section 3, we use a standard calibration to analyze the effects of a sequence of bad productivity shocks on the economy. Section 4 analyzes alternative monetary policy rules and their stabilizing properties. Finally, section 5 concludes.

1. THE MODEL ECONOMY

This section sketches a closed economy New Keynesian DSGE model that features sticky prices and wages and costly adjustments in the capital stock. The model also incorporates an external finance premium as in Bernanke, Gertler, and Gilchrist (1999), which amplifies the responses of the endogenous variables to different shocks. Fluctuations in the economy are triggered by trend productivity shocks that are persistent over time. We present a linearized version of the model that is obtained by taking first-order

expansions of the decision rules and equilibrium conditions around the flexible-price steady state. In what follows, a lower case variable represents the log deviation of the respective variable from its trend. Details of the model derivation can be found in Bernanke, Gertler, and Gilchrist (1999) and Gilchrist and Saito (2008).

Households maximize their intertemporal expected utility subject to their budget constraint. The log-linearized version of the Euler equation for consumption, c_t , is given by

$$c_t = - (i_t - E_t \pi_{t+1}) + E_t c_{t+1} + E_t z_{t+1}, \quad (1)$$

where z_t corresponds to productivity growth at time t . The expectations operator, E_t , encompasses the standard rational expectations (RE) operator and also the expectations obtained under the adaptive learning mechanism that we discuss in detail below. The interest rate, i_t , corresponds to the market interest rate at which households are able to borrow, and π_t is the inflation rate at time t . For simplicity, we assume that i_t is the risk-free interest rate determined each period by the monetary policy authority.

Households are assumed to supply differentiated labor services, whose elasticity of substitution in the production technology is ε_L . Each household optimally sets its wage rate only infrequently and then supplies all labor demanded at its current wage rate. When not optimally adjusted, wages are updated according to past inflation. Let ϕ_L be the fraction of households that do not optimally adjust wages in a particular period, and χ_L the weight given to past inflation in their indexation scheme. The evolution of real wages, wr_t , is thus given by

$$\begin{aligned} [\kappa_L + (1 + \beta)]wr_t &= \kappa_L mrs_t + wr_{t-1} + \beta E_t wr_{t+1} \\ &\quad - (1 + \beta \chi_L)\pi_t + \chi_L \pi_{t-1} + \beta E_t \pi_{t+1}, \end{aligned} \quad (2)$$

where $mrs_t = \sigma_L l_t + c_t$ is the marginal rate of substitution between labor, l_t , and consumption; σ_L corresponds to the inverse labor supply elasticity; and β is the intertemporal discount factor. The parameter $\kappa_L = [(1 - \beta \phi_L)(1 - \phi_L) / \phi_L (1 + \sigma_L \varepsilon_L)]$ defines the sensitivity of real wages to fluctuations in mrs_t .

We assume that a large set of retail firms rebrand intermediate varieties, which they sell to assemblers who pack them into final goods. Those goods are consumed by households and are accumulated as new capital. Retailers have monopoly power over a particular

variety of the intermediate goods and optimally set their prices only infrequently, as in Calvo (1983). A fraction ϕ of the firms are not able to reoptimize in a particular period. When prices are not reoptimized, firms adjust them according to past inflation. From these assumptions, we derive the following extended New Keynesian Phillips curve, which links inflation, π_t , with marginal costs, mc_t :

$$\pi_t = \frac{(1-\beta\phi)(1-\phi)}{\phi(1+\beta\chi)} mc_t + \frac{\beta}{1+\beta\chi} E_t \pi_{t+1} + \frac{\chi}{1+\beta\chi} \pi_{t-1}, \quad (3)$$

where $mc_t = (wr_t + l_t) - y_t$ corresponds to the real marginal costs relevant to firms producing intermediate varieties; y_t is output; and χ is the weight given to past inflation in the indexation mechanism.

Firms producing intermediate varieties hire labor from households and rent capital from entrepreneurs to produce new intermediate varieties of goods. They use a Cobb-Douglas production technology that features a stationary productivity trend growth rate, g , which is subject to shocks, z . Cost minimization by these intermediate firms determines the optimal composition of factors of production:

$$k_{t-1} - z_t - l_t = wr_t - r_{k,t}, \quad (4)$$

where k_{t-1} is the capital stock at the beginning of period t , and $r_{k,t}$ is the net rental rate of capital. A large set of entrepreneurs accumulate capital and rent it to the producer of intermediate varieties. Assuming that there are quadratic adjustment costs to adding new units to the capital stock, we obtain the following expression relating investment, inv_t , to the real price of installed capital, qr_t , and the capital stock at the beginning of period t :

$$inv_t = -\frac{1}{\zeta_I} qr_t + (k_{t-1} - z_t), \quad (5)$$

where ζ_I is the inverse of the elasticity of the adjustment cost of capital. The equilibrium condition in the financial market determines the real price of capital, qr_t . By a no-arbitrage condition, this price is a function of the expected rental price of capital to intermediate producers, $E_t r_{k,t+1}$, and the relevant discount factor for capital producer firms, $E_t(i_{k,t} - \pi_{t+1})$, which, in turn, depends on the interest rate charged on loans, $i_{k,t}$:

$$qr_t = -E_t(i_{k,t} - \pi_{t+1}) + \frac{r_k}{R_k} E_t r_{k,t+1} + \frac{1-\delta}{R_k} E_t qr_{t+1}, \quad (6)$$

where δ is the depreciation rate of capital, and R_k corresponds to the gross return of capital in steady state (see the appendix). The interest rate charged on loans to entrepreneurs investing in new capital goods corresponds to the risk-free interest rate plus an external financial risk premium, which arises from an incomplete information approach to the financial intermediation process. Entrepreneurs finance part of their investment with internal resources and the rest by borrowing from financial intermediaries. These financial intermediaries charge a premium over the risk-free interest rate, which stems from a costly state-verification problem. Entrepreneurs are subject to idiosyncratic shocks and may default on their debt. Given that information is incomplete and that verifying the realization of the idiosyncratic shock is costly, the optimal contract takes the form of a standard debt contract, in which the interest rate available to entrepreneurs has a premium above the risk-free interest rate. As Bernanke, Gertler, and Gilchrist (1999) and Calstrom and Fuerst (1997) show, this premium is a function of the entrepreneur's leverage:

$$i_{k,t} = i_t + \chi_k (qr_t + k_{t-1} - n_t). \quad (7)$$

The second term on the right-hand side of equation (7) corresponds to the external risk premium, which, as mentioned above, is a function of the leverage of the firm, $qr_t + k_{t-1} - n_t$. The parameter χ_k defines the sensitivity of the external risk premium to the evolution of leverage in the economy. As in Bernanke, Gertler, and Gilchrist (1999), we limit the amount of equity entrepreneurs are able to accumulate over time. For this reason, we assume that entrepreneurs do not live infinitely long and die with a certain probability each period. Equity from entrepreneurs that die is split among all members of the society. The evolution of the aggregate net worth, n_t , is thus given by⁸

$$n_t = n_{t-1} - z_t + \frac{K}{N} \left(\frac{r_k}{R_k} r_{k,t} + \frac{1-\delta}{R_k} qr_t - qr_{t-1} \right) - \left(\frac{K}{N} - 1 \right) (i_{k,t-1} - \pi_t). \quad (8)$$

8. We are assuming that the share of entrepreneur's labor in aggregate production is close to zero. See Gilchrist and Saito (2008).

The steady-state leverage, $(K/N) - 1$, is a measure of the degree of financial fragility in the economy. It determines the sensitivity of a firm's equity to changes in the interest rate charged on loans and thus to asset price fluctuations. Installed capital evolves over time according to the following expression:

$$k_t = \frac{1-\delta}{1+g}(k_{t-1} - z_t) + \left(1 - \frac{1-\delta}{1+g}\right) inv_t. \quad (9)$$

The equilibrium condition in the goods market is given by

$$y_t = \frac{C}{Y}c_t + \frac{I}{Y}inv_t, \quad (10)$$

where C/Y is the ratio of steady-state consumption to GDP and I/Y is the ratio of investment to GDP. The total supply of final goods, in turn, is given by

$$y_t = (1-\alpha)l_t + \alpha(k_{t-1} - z_t), \quad (11)$$

where α is the share of capital in the production function. Finally, we assume in the first part of our analysis that the central bank follows a simple rule to conduct its monetary policy:

$$i_t = \varphi_i i_{t-1} + (1-\varphi_i)\varphi_\pi \pi_t + (1-\varphi_i)\varphi_y (y_t - y_{t-1} + z_t), \quad (12)$$

where φ_i is the smoothing coefficient, φ_π is the inflation feedback coefficient, and φ_y is the output feedback coefficient. Below, in section 4, we depart from this simple policy rule to analyze the effects of productivity shocks when some of the coefficients of this rule are chosen so as to minimize a particular loss function. The only exogenous process considered is trend productivity, z_t . This variable is subject to independent and identically distributed shocks, $\varepsilon_{z,t}$, and evolves according to

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t},$$

where $\varepsilon_{z,t} \sim N(0, \sigma_z^2)$, and ρ_z is a persistence parameter.

2. INTRODUCING ADAPTIVE LEARNING

We depart from rational expectations slightly by assuming that agents form their expectations based on a learning mechanism. We follow the approach discussed in Adam (2005) and Evans and Honkapohja (2001). Let us consider the structural form representation of our model:

$$\mathbf{F}E_t\mathbf{x}_{t+1} + \mathbf{G}\mathbf{x}_t + \mathbf{H}\mathbf{x}_{t-1} + \mathbf{D}\varepsilon_{z,t} = 0, \quad (13)$$

where $\mathbf{x}_t = [y_t, c_t, inv_t, l_t, k_t, n_t, \pi_t, qr_t, wr_t, r_{k,t}, i_{k,t}, i_t, z_t]'$ corresponds to a vector of endogenous and exogenous variables, and \mathbf{F} , \mathbf{G} , \mathbf{H} , and \mathbf{D} are matrices containing structural coefficients. E_t is an operator that measures agents' expectations based on their information up to period t . The rational expectations solution of equation (13) is given by the following expression:

$$\mathbf{x}_t = \mathbf{\Omega}\mathbf{x}_{t-1} + \mathbf{\Lambda}\varepsilon_{z,t}, \quad (14)$$

where $\mathbf{\Omega}$ and $\mathbf{\Lambda}$ are invariant matrices whose elements are only functions of the structural parameters of the model.

We deviate from the rational expectations assumption and follow the approach by Marcet and Sargent (1989) and Evans and Honkapohja (2001). In particular, we assume that agents have the following perceived law of motion (PLM) for the endogenous variables:

$$\mathbf{x}_{t+1} = \tilde{\mathbf{\Omega}}_{t-1}\mathbf{x}_t + \tilde{\mathbf{\Xi}}_{t-1}\varepsilon_{z,t} + \mathbf{e}_{t+1}, \quad (15)$$

where \mathbf{e}_{t+1} is orthogonal to $\tilde{\mathbf{\Omega}}_{t-1}\mathbf{x}_t$ and $\tilde{\mathbf{\Xi}}_{t-1}\varepsilon_{z,t}$. This PLM nests the rational expectations solution given by equation (14).⁹ Agents forecast future values of \mathbf{x}_t using their PLM with $E_t\mathbf{e}_{t+1} = 0$:

$$E_t\mathbf{x}_{t+1} = \tilde{\mathbf{\Omega}}_{t-1}\mathbf{x}_t + \tilde{\mathbf{\Xi}}_{t-1}\varepsilon_{z,t}. \quad (16)$$

The model information assumptions are as follows: in period t , agents observe \mathbf{x}_{t-1} and $\varepsilon_{z,t}$, and then they use $\tilde{\mathbf{\Omega}}_{t-1}$ and $\tilde{\mathbf{\Xi}}_{t-1}$ to form expectations about \mathbf{x}_{t+1} . Substituting equation (16) into the structural

9. In particular, the RE solution has a PLM with $\tilde{\mathbf{\Omega}}_{t-1} = \mathbf{\Omega}$, $\tilde{\mathbf{\Xi}}_{t-1} \equiv \mathbf{0}$ and $\mathbf{e}_{t+1} = \mathbf{\Lambda}\varepsilon_{z,t+1}$.

representation of the model (equation 13), we obtain the actual law of motion (ALM) under the previous PLM:

$$\mathbf{x}_t = -(\mathbf{F}\tilde{\Omega}_{t-1} + \mathbf{G})^{-1}[\mathbf{H}\mathbf{x}_{t-1} + (\mathbf{D} + \mathbf{F}\tilde{\Xi}_{t-1})\varepsilon_{z,t}]. \quad (17)$$

As in Evans and Honkapohja (2001) and Orphanides and Williams (2005), we assume agents use recursive least squares with perpetual learning to update their belief regarding the system's law of motion. Under this assumption, we have

$$\begin{aligned} [\tilde{\Omega}_t, \tilde{\Xi}_t] &= [\tilde{\Omega}_{t-1}, \tilde{\Xi}_{t-1}] \\ &+ \gamma \left\{ \mathbf{R}_{t-1}^{-1}(\mathbf{x}'_{t-1}, \varepsilon_{z,t-1})' [\mathbf{x}'_t - (\tilde{\Omega}_{t-1}\mathbf{x}_{t-1} + \tilde{\Xi}_{t-1}\varepsilon_{z,t-1})'] \right\}' \end{aligned} \quad (18)$$

and

$$\mathbf{R}_t = \mathbf{R}_{t-1} + \gamma \left[(\mathbf{x}'_{t-1}, \varepsilon_{z,t-1})' (\mathbf{x}'_{t-1}, \varepsilon_{z,t-1}) - \mathbf{R}_{t-1} \right], \quad (19)$$

where the gain parameter γ is constant. At time $t = 0$, we start from the RE equilibrium solution (equation 14): $\tilde{\Omega}_0 = \Omega$ and $\tilde{\Xi}_0 = \bar{0}$, together with $\mathbf{R}_0 = \mathbf{I}$.¹⁰ After a shock hits the economy, the system evolves by iterating over equations (18) and (19). Under the constant-gain assumption, past data is discounted when agents update their expectations. This is equivalent to using weighted least squares with the weights declining geometrically as we move back in time.

By setting a small number for γ , the solution under learning remains close to the starting RE solution. Therefore, the equilibrium path under learning does not significantly differ from the RE equilibrium. If we set γ to a large number, the initial data have a big effect on the estimated matrices $\tilde{\Omega}_t$ and $\tilde{\Xi}_t$, and agents adjust their expectations away from what is implied by the RE equilibrium.

This constant-gain learning mechanism does not guarantee convergence toward the RE equilibrium path after a shock hits the economy as long as $\gamma > 0$. Milani (2007) argues that the asymptotic distribution of these learning beliefs (for $t \rightarrow \infty$) approaches the RE beliefs as $\gamma \rightarrow 0$. We do not further discuss the E-stability properties of this type of learning mechanism. Evans and Honkapohja (2001, 2009) discuss at length the implications of the constant-gain assumption for the convergence properties of this learning scheme.

10. This assumption constrains our degree of freedom. Other papers that analyze propagation consider an initial equilibrium that is deviated from RE, for example, Milani (2007).

3. PRODUCTIVITY SHOCKS, FINANCIAL FRICTIONS, AND LEARNING

The parametrization of the model turns out to be an important element for the results discussed below. In calibrating the model, we closely follow the parameter values chosen by Christiano and others (2008) and Gilchrist and Saito (2008). The steady-state real interest rate is set to 2.5 percent, whereas the steady-state labor productivity growth rate is assumed to be 1.5 percent, both on an annual basis. The probability of adjusting nominal wages is set to 0.80, while that of prices is fixed at 0.60. These values imply that wages are optimally adjusted every five quarters, and prices are optimally adjusted every two and a half quarters. The weights of past inflation for wage and price indexation are set to 0.1 and 0.8, respectively.

The smoothing coefficient in the Taylor rule is 0.85, and the feedback coefficients for inflation and output are set to 1.75 and 0.25, respectively. These parameters are in line with several empirical studies of policy rules for advanced economies (see Clarida, Galí, and Gertler, 1998; Christiano and others, 2008).

The steady-state external finance premium is fixed at 3.0 percent (annual basis), in line with previous studies (Calstrom and Fuerst, 1997; Gilchrist and Saito, 2008; Bernanke, Gertler, and Gilchrist, 1999). The steady-state leverage ratio is set to 0.90, which falls between the values used by Gilchrist (2004) to characterize high-leverage economies (1.5) and by Gilchrist and Saito (2008) (0.8). Below we discuss the implication of assuming different values for this ratio. The elasticity of the external premium to the leverage ratio is assumed to be 0.065, which is consistent with the range of values used by Bernanke, Gertler, and Gilchrist (1999): 0.065–0.040. Finally, the persistence coefficient of productivity growth is 0.8, which is smaller than the value used by Gilchrist and Saito in their analysis. Table A1 in the appendix provides a detailed description of the values used for all different parameters in the model.

As mentioned above, the constant-gain parameter, γ , governing the weights given to current forecast errors when updating expectations formation is crucial for the dynamics of the system. We set this parameter to 0.025, which represents a minor departure from RE (see Orphanides and Williams, 2005). Slobodyan and Wouters (2009), in their simulation of a DSGE model with learning, assume different values for this gain parameter (0.01, 0.02, and 0.05)

corresponding roughly to a regression with a forgetting half-length of 69, 34, and 14 periods. In our case, it corresponds to a regression with a forgetting half-length of 23 periods. Milani (2007) estimates this parameter together with other structural parameters for the United States. He finds values in the range of 0.005 and 0.035, depending on the specification of his model.

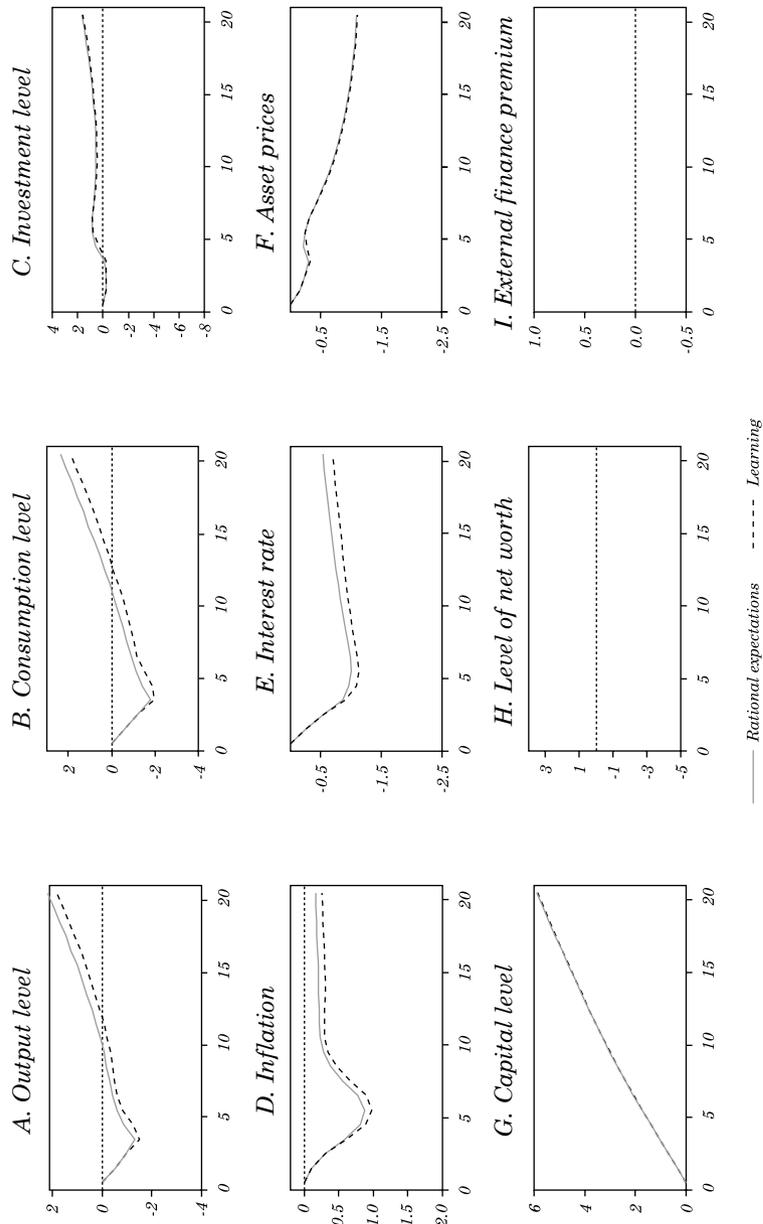
We now turn to the analysis of the effects of a detrimental shock to the productivity trend, comparing the results under RE and under adaptive learning. Before we present the results for the model featuring financial frictions, it is useful to discuss the result obtained under both RE and learning in the standard version of the model. Figure 2 presents the response of different variables to a sequence of three negative productivity trend shocks, each of size 0.5. Having a sequence of shocks is important to generate a differentiated response under learning. If the economy were hit by only one or two consecutive shocks, the adaptive learning approach we are using would imply a very fast convergence toward the RE equilibrium path.

As the figure illustrates, a sequence of negative productivity shocks leads to a transitory fall in output, consumption, and investment.¹¹ The fall in investment is more muted than that of output and consumption. Inflation decreases for several quarters, and there is a slow decline in asset prices. The monetary policy rate decreases in line with inflation and the slowdown in activity. When agents form their expectations based on adaptive learning, the response in activity is a bit more intense than under RE and the fall in inflation slightly more severe. Moreover, these variables remain below their equilibrium path under RE for several periods. These results are in line with Adam (2005), who finds that output and inflation are persistent under adaptive learning but not under RE in his model. In our case, however, the responses under learning and under RE seem to be quite similar from a quantitative point of view.

We now turn to the case in which financial frictions are present in the economy. Figure 3 displays the responses to the same sequence of negative productivity shocks discussed above, assuming that there is an external finance premium that is a function of the leverage of the firms. Now we observe a sharp difference between the responses under RE and learning. Under RE the fall in activity is larger than in

11. Impulse response functions are expressed in levels rather than deviations from the steady-state balanced growth path.

Figure 2. Response of Variables without the Financial Accelerator (Baseline Calibration)



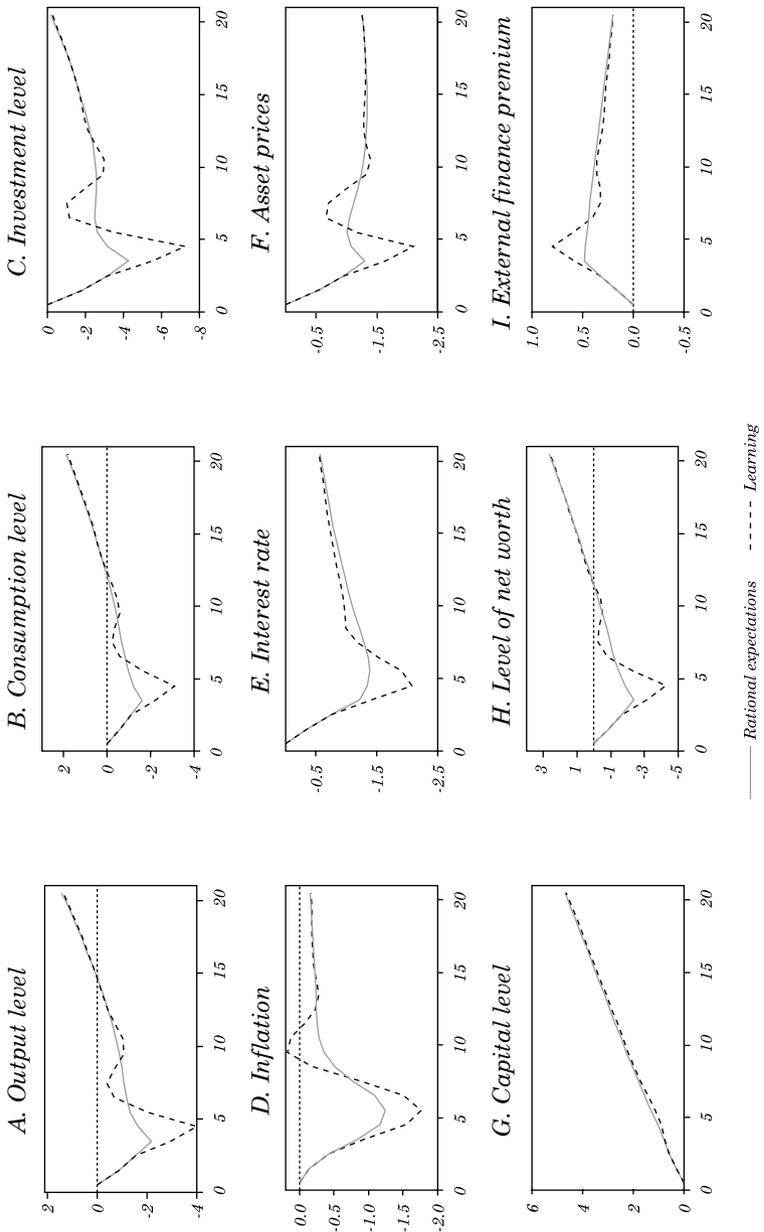
Source: Authors' calculations.

the case without financial frictions, as shown by Bernanke, Gertler, and Gilchrist (1999). However, under learning the fall in activity is exacerbated when financial frictions are present. Inflation also decreases more, and there is an abrupt fall in asset prices. Recovery is much faster following the sharp fall in activity and inflation, so after a few quarters output and inflation are above the levels they would have had under RE. In sum, when financial frictions are present, a small departure from the RE assumption leads to an amplified response of several variables to productivity shocks and an increase in their volatility.

In the above scenario, output falls more under learning and then increases substantially. In particular, it overshoots when compared to the rational expectations scenario (see figure 3). This overshooting is also present in the case of inflation. This result is similar to the findings obtained by Bloom (2009) using a different framework: uncertainty shocks generate short, sharp recessions and fast recoveries. The reason behind this behavior is related to the way in which expectations are determined under this scenario with learning and financial frictions. Given the policy rule in place, agents do not perceive important changes in the policy rate. As a result, inflation and output drop substantially when the external finance premium is increasing. Eventually, there is a turning point at which the interest rate declines and agents modify their expectations. In this later stage, inflation and output overshoot when the real interest rate and the external finance premium decline.

This overshooting is, in part, explained by the way in which monetary policy is conducted. If the central bank reacts more aggressively toward output and inflation (by reducing the degree of policy inertia, φ_i), then the real interest rate will decline more, thus attenuating the decline in output and inflation. In this case, the overshooting will not be present, but output and inflation will still decline more under learning than under rational expectations. Overall, the way in which monetary policy is designed determines how expectations are formed (as we show in the next section). In the presence of learning, a more aggressive monetary policy will induce a faster convergence to the rational expectations equilibrium (see Orphanides and Williams, 2008, 2009). Why do small departures from RE generate such large downturns in activity and inflation in response to productivity shocks in the presence of a financial friction? Figure 4 presents the path of the expected variables four steps ahead

Figure 3. Response of Variables with the Financial Accelerator (Baseline Calibration)



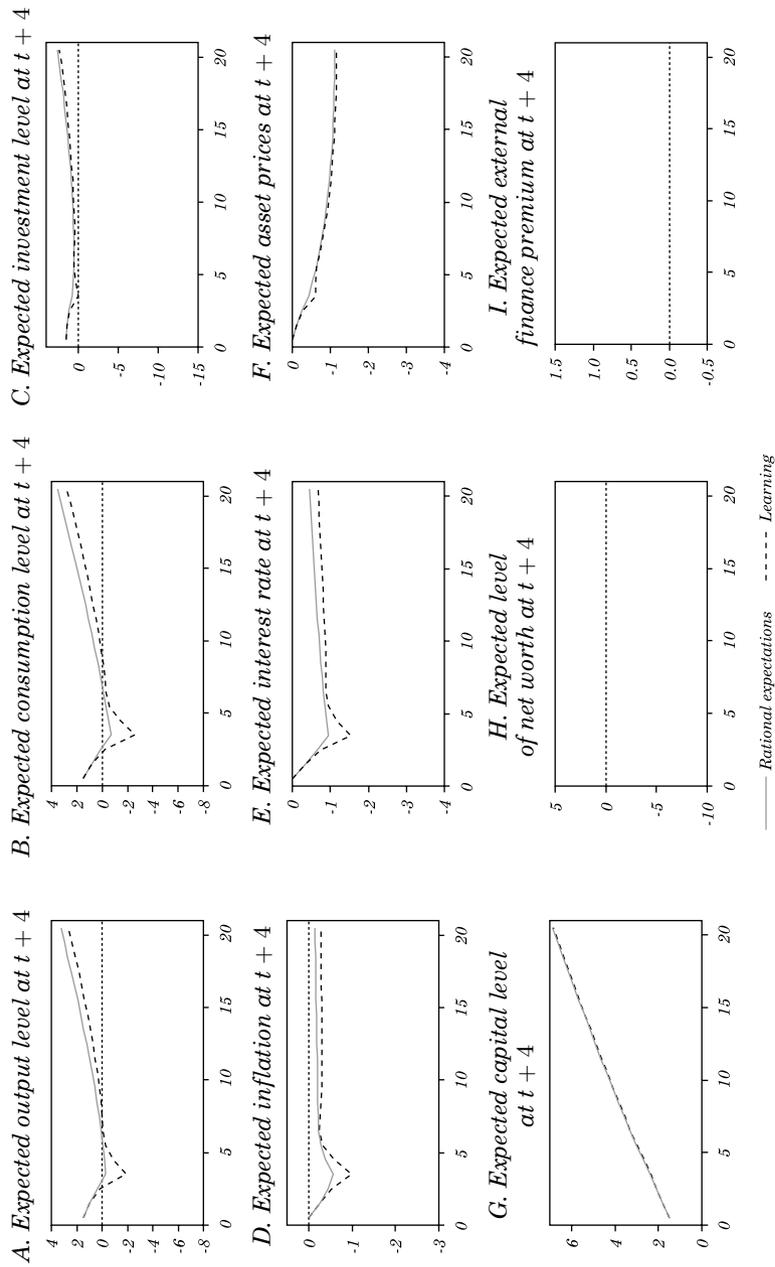
Source: Authors' calculations.

in the model without financial frictions. As the figure shows, the expected drop in activity and inflation is more intense when agents forecast using adaptive learning. However, when we include the financial accelerator mechanism, the expected fall in variables is much more intense, as the fall in net worth leads to an important increase in the expected risk premium (figure 5). This dramatic fall in activity and the increase in the expected risk premium feed back into asset prices, which decrease even further. This amplification mechanism through expectations does not work in the model with RE. Adam, Marcet, and Nicolini (2008) present similar results in a different context. They show that the reinforcing effect between beliefs and stock prices can produce large and persistent deviations of the price-dividend ratio from fundamentals. Thus, if expectations about stock price growth increase in a given period, the actual growth rate of prices has a tendency to increase beyond the fundamental growth rate, amplifying the initial belief of higher stock price growth. They further show that the model under adaptive learning exhibits mean reversion, so that even if expectations are very high or very low at some point, they will eventually return to fundamentals.

We identify two elements that are crucial for the amplified responses under adaptive learning: the size of the shocks and the degree of financial fragility, measured by the steady-state leverage of firms. Figure 6 depicts the difference in the responses to the shocks under RE and under learning for different shock sizes. For relatively small shocks (namely, 0.25) the response of the variables under learning is indistinguishable from that obtained under RE. As the size of the shocks increases, the contraction in activity and the fall in inflation under learning become relatively more intense. The jump in the external finance premium also rises considerably under learning when the shocks are sizable.

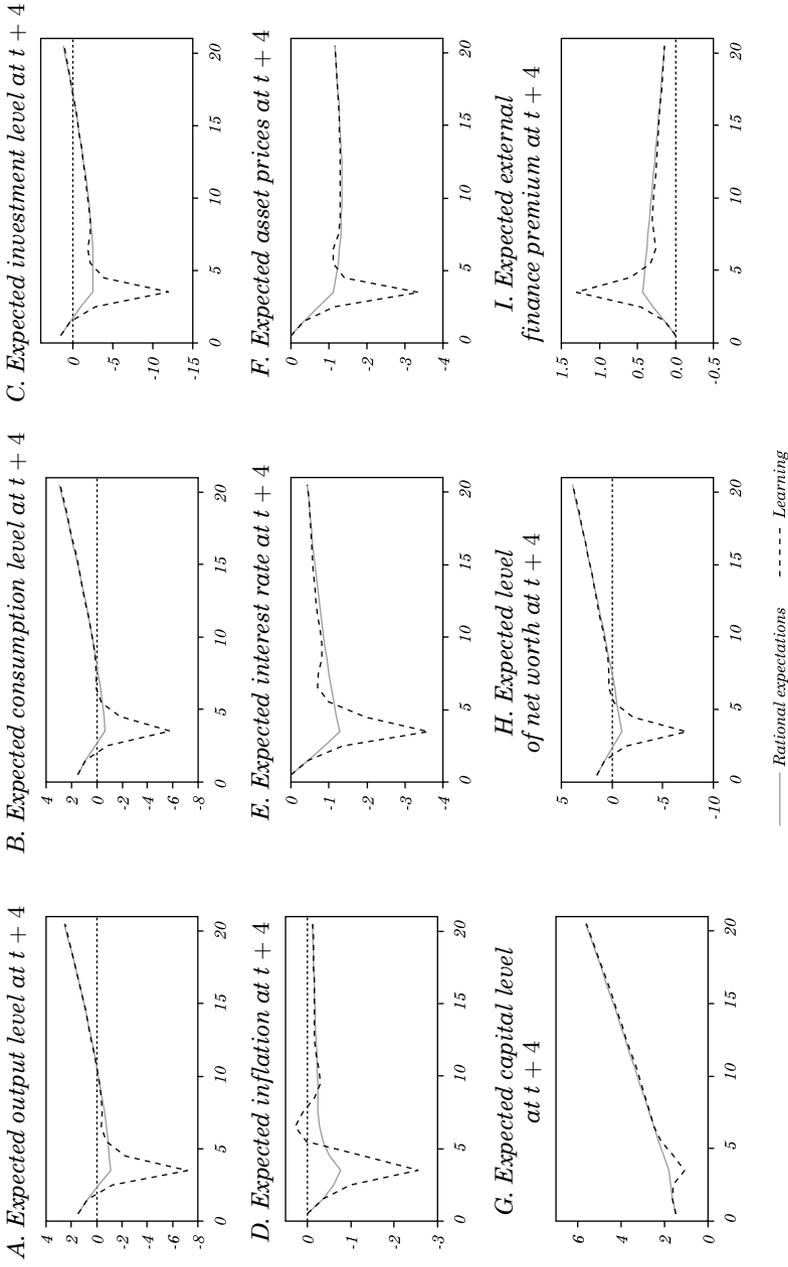
A second amplification mechanism for the transmission of shocks is the degree of financial fragility, measured by the steady-state leverage of the firms. Figure 7 compares the responses under RE and learning for different degrees of leverage for the firms. When the leverage increases, the difference between the response of the external premium under learning and the response under RE also increases. That, in turn, implies that the contraction of output and the fall in inflation that result from the shock are amplified when the economy is financially fragile and agents form their expectations based on a learning mechanism.

Figure 4. Expected Response of Variables Four Periods ahead without the Financial Accelerator (Baseline Calibration)



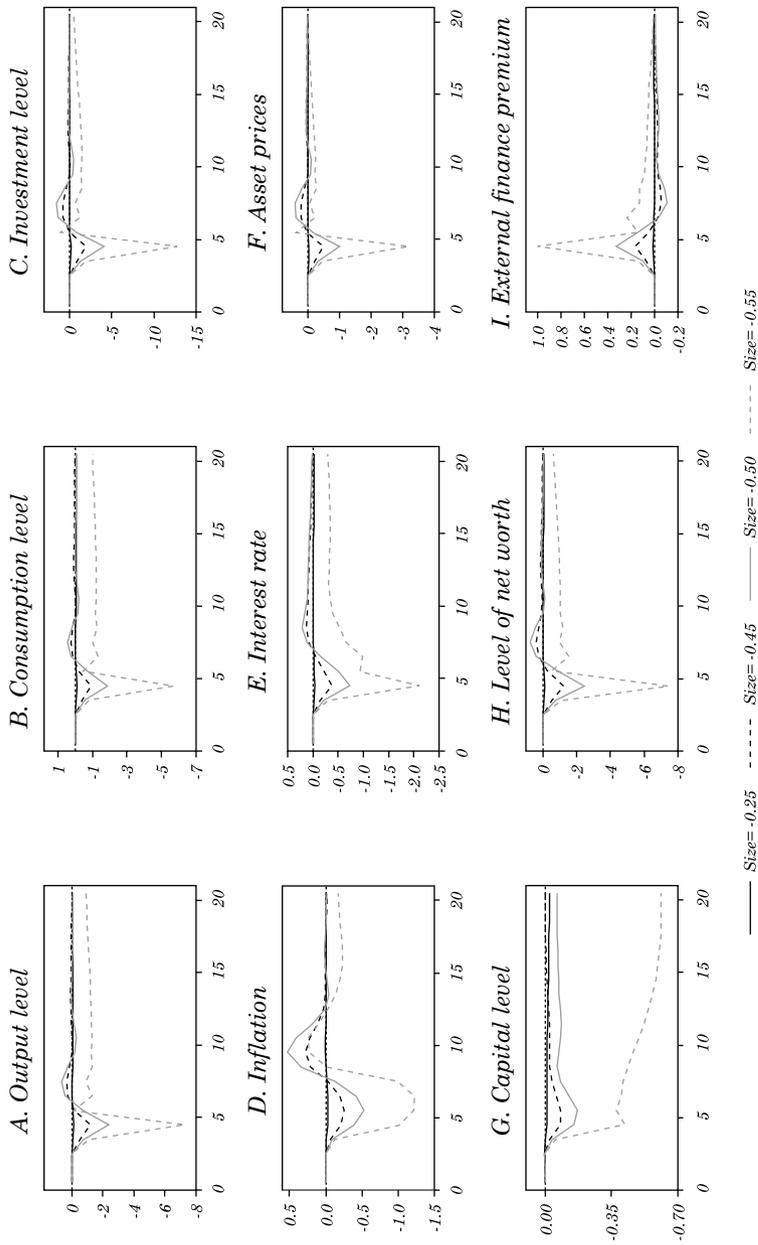
Source: Authors' calculations.

Figure 5. Expected Response of Variables Four Periods ahead with the Financial Accelerator (Baseline Calibration)



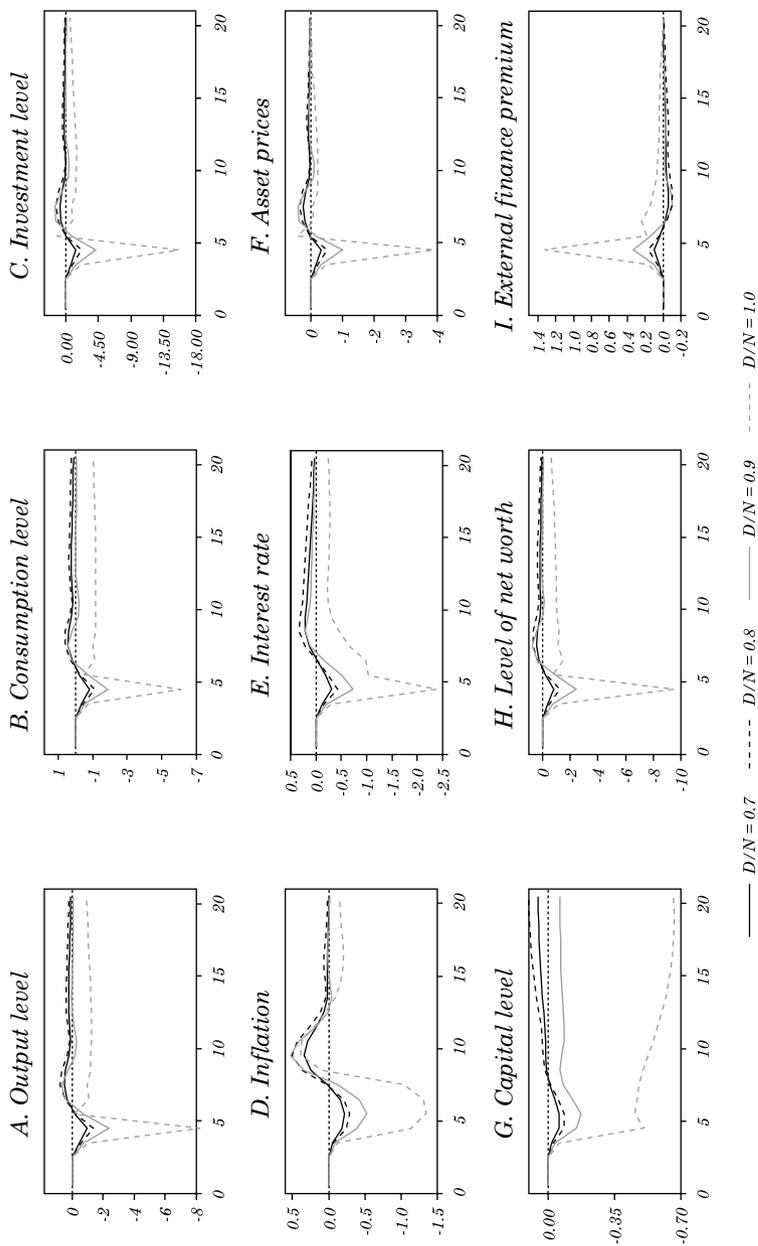
Source: Authors' calculations.

Figure 6. Difference in Response of Variables between Learning and Rational Expectations with the Financial Accelerator under Alternative Sizes of Shocks



Source: Authors' calculations.

Figure 7. Difference in Response of Variables between Learning and RE with the Financial Accelerator under Alternative Leverage Ratios



Source: Authors' calculations.

4. MONETARY POLICY RESPONSE TO ASSET PRICE FLUCTUATIONS

The housing market bubble in the United States and Europe that generated the conditions for the current crises led to a significant amount of research into whether the interest rate should respond to asset price fluctuations. Most of the theoretical papers found that the basic prescriptions of the inflation targeting approach to conducting monetary policy could deliver optimal outcomes even in the presence of asset price bubbles. Bernanke and Gertler (2001), for example, show that it is desirable for central banks to focus on underlying inflationary pressure and that asset prices become relevant only to the extent they signal potential inflationary or deflationary forces. They also find that rules that directly target asset prices appear to have undesirable side effects. Gilchrist and Saito (2008) extend the analysis of Bernanke and Gertler to discuss the implications of incomplete information on the fundamentals behind asset prices. They find that the gains from responding to the asset price gap (that is, the difference between observed asset prices and the potential level of asset prices in a flexible-price economy without financial market imperfections) are greater when the private sector is uninformed about asset price fundamentals, while the monetary authority is well informed. When monetary policy is less informed about fundamentals than market participants, responding to the wrong asset price gap may be detrimental. Dupor (2005) obtains similar conclusions. He finds that when the central bank has limited information about the nature of asset price movements, it should respond less aggressively to nonfundamental shocks.

Here we perform a preliminary analysis of the implications of endogenously generated asset price bubbles for the conduct of the monetary policy. Rather than looking for a fully optimal policy rule, we compare the economy's responses to the sequence of shocks described above under alternative policy rules. For this, we modify one by one each of the feedback coefficients in the monetary policy equation (12), keeping the degree of persistence, φ_i , constant. The modified coefficient is chosen so as to minimize the following loss criterion:

$$L(T) = \min_{\varphi} \frac{1}{T} \sum_{t=0}^T (\hat{y}_t^2 + \pi_t^2), \quad (20)$$

where $\hat{y}_t = \Delta y_t + z_t$. The relative weight of inflation fluctuations is set to one, which is consistent with Orphanides and Williams (2008).¹²

12. In the computations below, we use $T = 21$.

This loss function is not derived from first principles, although it is a standard function used in evaluating the implications of alternative monetary policies rules. Examples of this loss criterion are found in Orphanides and Williams (2008), Adolfson and others (2008a, 2008b), and Justiniano and Preston (2009), among others. The reason for using this criterion is that it reflects the policymaker's preferences for stabilizing an average between output and inflation volatility.

Consider first the case of a policy rule that responds not only to output and inflation, but also to fluctuations in the level of asset prices, qr_t . The value of the corresponding feedback coefficient, φ_q , in the policy rule that minimizes $L(T)$ is 0.14 (rule q in table 1). Following a sequence of negative shocks, the monetary policy response is such that it moderates the fall in asset prices. This, in turn, avoids the decline in net worth and attenuates the increase in the external finance premium (figure 8). As a result, output and inflation are more stable than in the baseline case. Under this rule, however, the monetary authority attempts to sustain the real asset price at its initial level, under circumstances in which the equilibrium price should fall. In this attempt, the monetary policy incubates inflationary pressures that lead to an increase in inflation after some periods.

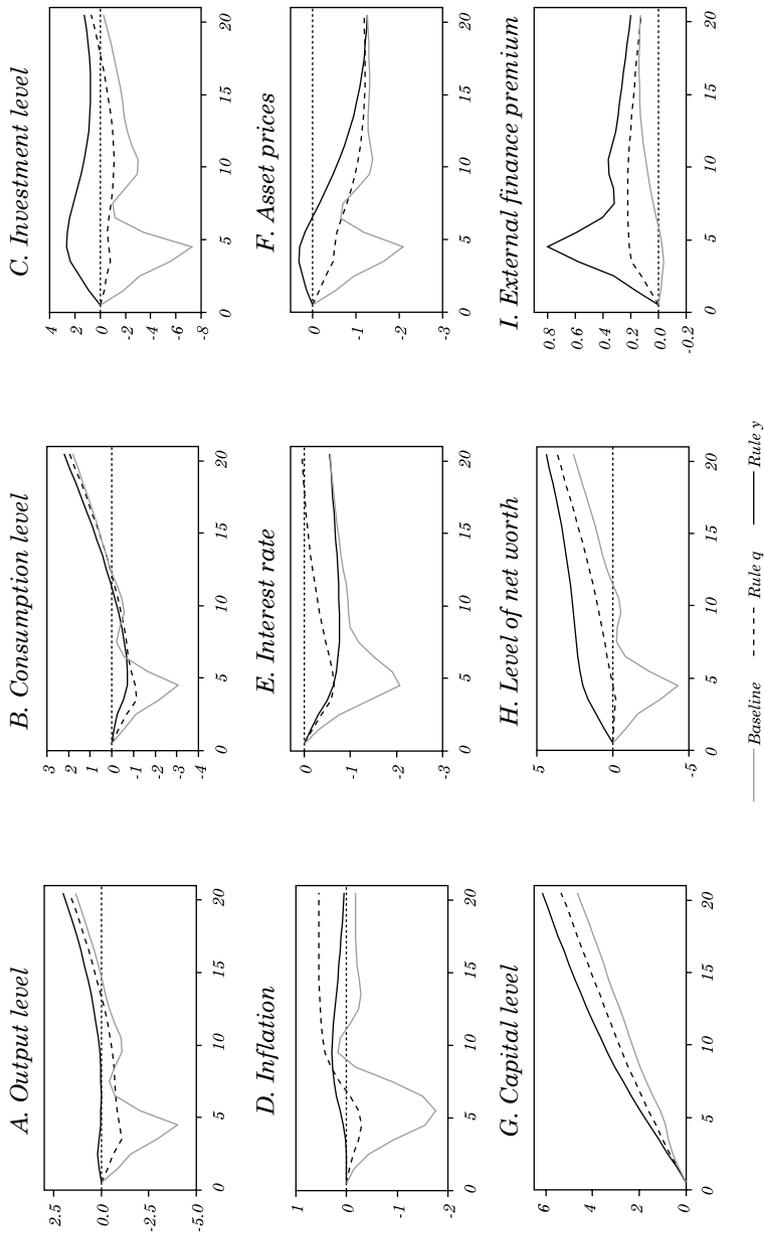
Table 1. Alternative Policy Rules^a

<i>Long-run coeff.</i>	<i>Baseline rule</i>	<i>Rule q</i>	<i>Rule y</i>	<i>Rule Δq</i>	<i>Rule π</i>	<i>UOSR</i>
φ_i	0.85	0.85	0.85	0.85	0.85	0.85
φ_π	1.75	1.75	1.75	1.75	21.17	1.62
φ_y	0.25	0.25	0.96	0.25	0.25	-0.05
φ_q	—	0.14	—	—	—	—
$\varphi_{\Delta q}$	—	—	—	4.05	—	5.63
$\hat{\sigma}_\pi^2$	0.0486	0.0144	0.0021	0.0066	0.0002	0.0068
$\hat{\sigma}_y^2$	0.6627	0.1287	0.0883	0.0632	0.0845	0.0560
$L(T)$	0.7113	0.1431	0.0904	0.0698	0.0847	0.0627

Source: Authors' calculations.

a. $\hat{\sigma}_\pi^2$ and $\hat{\sigma}_y^2$ are computed for $T = 21$.

Figure 8. Comparing Alternative Policy Rules



Source: Authors' calculations.

In the second exercise, we consider a policy that responds to fluctuations in output growth (rule y in table 1). The optimal feedback coefficient in this case is $\varphi_y = 0.96$. This policy manages to reduce the real interest rate fast enough to initially raise asset prices and reduce the external finance premium. Output is stabilized in the short run, as is inflation. After some quarters, this expansive policy generates a mild increase in inflation, which converges back to its long-run equilibrium level slowly. Output remains somewhat below trend for several quarters, but it is more stable than in the previous cases (figure 8).

Consider now a policy that reacts to changes in asset prices (rule Δq in table 1). The feedback coefficient that minimizes the loss criterion is $\varphi_{\Delta q} = 4.05$. Under this policy rule, asset prices remain nearly constant after the sequence of negative shocks, and the external finance premium increases slightly (figure 9). Output and inflation remain virtually unaltered in the first quarters. This policy turns out to be more expansive over a medium- or long-term horizon than the previous rules. In particular, it is able to reduce output volatility, although it marginally increases inflation volatility when compared to rule y (see table 1 and figure 9). Also, this rule induces a smooth decline in asset prices.

Finally, we consider a rule that responds aggressively to inflation deviations (rule π in table 1). The feedback coefficient on inflation that minimizes the loss criterion is $\varphi_\pi = 21.17$. This policy leads to an aggressive reduction of the interest rate in response to the shocks. This reduction in the policy rate avoids a decline in asset prices and the external finance premium declines marginally in the first quarters after the shock. Inflation is almost completely stabilized under this rule (figure 9). This policy, however, generates a higher volatility in output growth than the rule that reacts to changes in the asset price level.

All these alternative policy rules have in common a more aggressive response to output and inflation than the baseline policy. All of them induce a more stable path for asset prices and attenuate the increase in the external finance premium. Real variables therefore tend to be more stable than in the baseline case, and the sharp decline in inflation is avoided. In addition, by avoiding the dramatic fall in inflation, these alternative policy rules succeed at effectively lowering the real interest rate in response to the sequence of shocks.

To check the robustness of our results, we perform an alternative exercise where instead of optimizing just in one dimension (one

feedback coefficient at a time), we look for the joint combination of coefficients that minimizes the welfare loss.¹³ We follow Justiniano and Preston (2009) by choosing the feedback coefficients starting from several initial points to find the minimum of equation (20). Our results indicate that this unconstrained optimal simple rule (UOSR) considers an aggressive response to the change in asset prices, a feedback coefficient to inflation that is somehow lower than in the baseline case, and a response to output that is nearly zero (table 1). Nevertheless, the UOSR is able to reduce output volatility, mainly because asset prices turn out to be less volatile (see figure 9). This UOSR, like rule Δq , generates more inflation volatility than rule π .

These exercises are far from an optimal monetary policy analysis. First, the loss function is rather ad hoc, and variances are conditional to a particular sequence of shocks. Second, we restrict our analysis to consider only simple rules. Nevertheless, these exercises suggest that a policy rule that responds to changes in asset prices may improve on traditional policy rules that do not consider an endogenous response to financial variables. In any case, all the rules considered here induce a more stable path for the asset price than the one obtained under a simple Taylor rule.

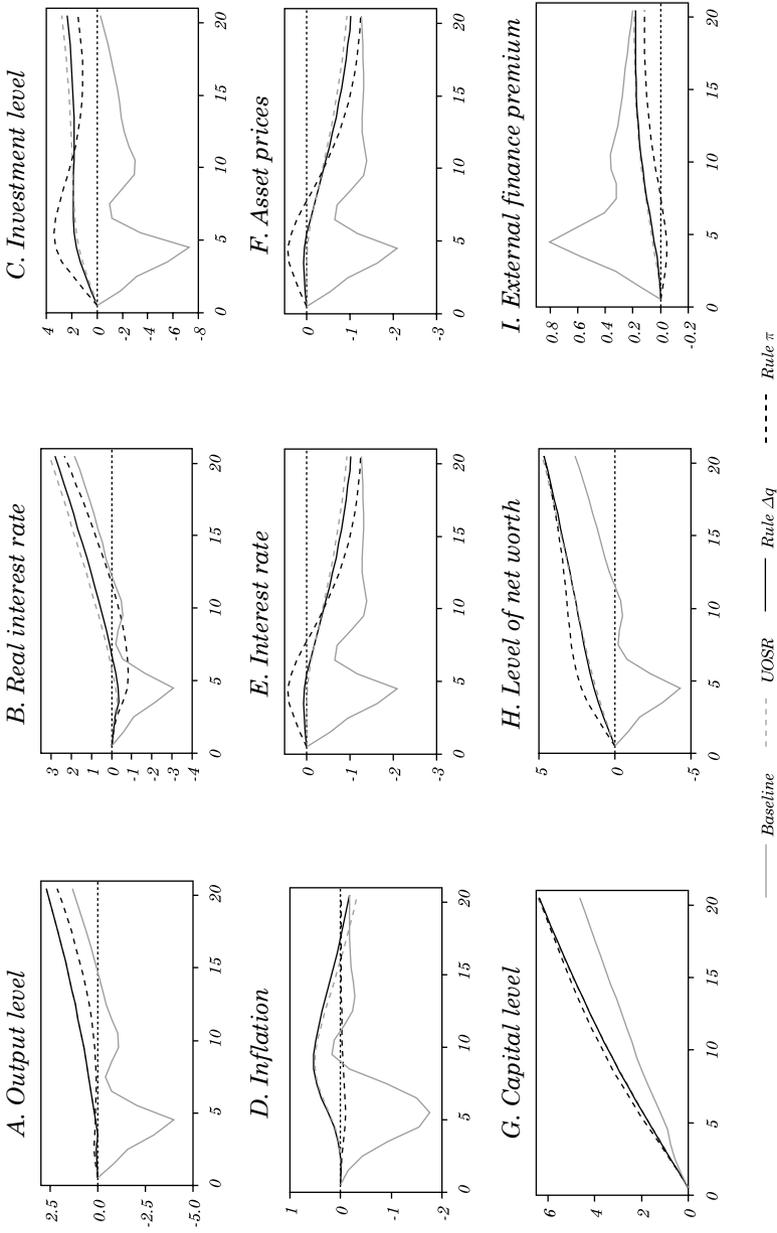
5. CONCLUSIONS

Financial frictions have been shown to play an important role in amplifying business cycle fluctuations. In this paper, we show that the financial accelerator mechanism analyzed by Bernanke, Gertler, and Gilchrist (1999) may render even larger business cycle fluctuations and endogenous asset price bubbles in the presence of small departures from the standard rational expectations assumption used in the literature. These large business cycle fluctuations are amplified in a nonlinear way by the size of the shocks and by the degree of financial fragility in the economy, as determined by capital producers' leverage.

Our preliminary results indicate that even in the presence of endogenous bubbles, responding aggressively to inflation reduces output and inflation volatility. If the central bank adjusts its policy instrument in response to asset price fluctuations, it may

13. For this exercise, we impose $\varphi_i = 0.85$ and $\varphi_q = 0$. We also performed alternative exercises allowing for φ_q different from zero, but the basic conclusion did not change.

Figure 9. Alternative Optimal Simple Rules



Source: Authors' calculations.

reduce output volatility and even inflation volatility in the short run. However, such a monetary policy leads to a surge in inflation several periods after the shocks. A policy that aggressively responds to changes in asset prices may marginally reduce output volatility relative to a policy that reacts aggressively to inflation, but at the cost of generating inflationary pressure.

APPENDIX

Steady State

The return to capital can be expressed as follows:¹⁴

$$R_K = (1 + r)(1 + \rho_K),$$

where ρ_K is the external finance premium in steady state.

The output-to-capital ratio is

$$\frac{Y}{K} = \frac{R_K - (1 - \delta)}{\alpha(1 + g)}.$$

The investment-to-capital ratio is

$$\frac{I}{K} = \frac{g + \delta}{1 + g}.$$

The investment-to-output ratio is

$$\frac{I}{Y} = \frac{I}{K} \left(\frac{Y}{K} \right)^{-1}.$$

The consumption-to-output ratio is

$$\frac{C}{Y} = 1 - \frac{I}{Y}.$$

Finally, the rental rate of capital is

$$r_K = R_K - (1 + \delta).$$

14. When there is no financial accelerator, we assume $\rho_K = 0$.

Table A1. Baseline Calibration

<i>Name</i>	<i>Description</i>	<i>Value</i>
r	Steady-state real interest rate	2.5% (annual basis)
g	Steady-state labor productivity growth rate	1.5% (annual basis)
π	Steady-state inflation rate	2.0% (annual basis)
β	Subjective discount factor	0.99 (annual basis)
σ_L	Inverse of the elasticity of the labor supply	1.0
δ	Depreciation rate of capital	10.0% (annual basis)
φ_I	Elasticity of asset prices with respect to I/K	0.25
α	Capital share in the production technology	0.3
ε_L	Elasticity of substitution among labor varieties	21
ϕ_L	Probability of adjusting nominal wages	0.85
χ_L	Weight of past inflation in wages indexation	0.1
ε	Elasticity of substitution among intermediate goods	11
ϕ	Probability of adjusting prices	0.6
χ	Weight of past inflation in price indexation	0.8
φ_i	Smoothing coefficient in the Taylor rule	0.85
φ_π	Inflation coefficient in the Taylor rule	1.75
φ_y	Output coefficient in the Taylor rule	0.25
ρ_K	Steady-state external finance premium	3.0% (annual basis)
$(K - N)/N$	Steady-state leverage ratio	90%
χ_K	Elasticity of external premium to the leverage ratio	0.065
ρ_Z	AR(1) coefficient of the persistent productivity growth	0.8
γ	Constant-gain parameter	0.025

REFERENCES

- Adam, K. 2005. "Learning to Forecast and Cyclical Behavior of Output and Inflation." *Macroeconomic Dynamics* 9(01): 1–27.
- Adam, K., A. Marcet, and J.P. Nicolini. 2008. "Stock Market Volatility and Learning." Discussion paper 6518. London: Centre for Economic Policy Research.
- Adolfson, M., S. Laséen, J. Lindé, and L.E.O. Svensson. 2008a. "Optimal Monetary Policy in an Operational Medium-Sized DSGE Model." Working paper 225. Stockholm: Sveriges Riksbank.
- . 2008b. "Monetary Policy Trade-offs in an Estimated Open-Economy DSGE Model." Working paper 232. Stockholm: Sveriges Riksbank.
- Ahearne, A., J. Ammer, B. Doyle, L. Kole, and R. Martin. 2005. "House Prices and Monetary Policy: A Cross-Country Study." International finance discussion paper 841. Washington: Board of Governors of the Federal Reserve System.
- Bernanke, B.S. and M. Gertler. 1999. "Monetary Policy and Asset Price Volatility." *Economic Review* 84(4): 17–52. Federal Reserve Bank of Kansas City.
- . 2001. "Should Central Banks Respond to Movements in Asset Prices?" *American Economic Review* 91(2): 253–57.
- Bernanke, B.S., M. Gertler, and S. Gilchrist. 1999. "The Financial Accelerator in a Quantitative Business Cycle Framework." In *Handbook of Macroeconomics*, vol. 1C, edited by J.B. Taylor and M. Woodford. Amsterdam: Elsevier Science, North-Holland.
- Bloom, N. 2009. "The Impact of Uncertainty Shocks." *Econometrica* 77(3): 623–85.
- Borio, C. and P. Lowe. 2002. "Asset Prices, Financial and Monetary Stability: Exploring the Nexus." Working paper 114. Basel: Bank for International Settlements.
- Calvo, G.A. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12(3): 383–98.
- Carlstrom, C. and T. Fuerst. 1997. "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis." *American Economic Review* 87(5): 893–910.
- Cecchetti, S., H. Genberg, J. Lipsky, and S. Wadhvani. 2000. "Asset Prices and Central Bank Policy." Geneva Report on the World Economy 2. Geneva: International Center for Monetary and Banking Studies and Centre for Economic Policy Research.

- Christiano, L., C. Ilut, R. Motto, and M. Rostagno. 2008. "Monetary Policy and Stock Market Boom-Bust Cycles." Working paper 955. Frankfurt: European Central Bank.
- Clarida, R., J. Galí, and M. Gertler. 1998. "Monetary Policy Rules in Practice: Some International Evidence." *European Economic Review* 42(6): 1033–67.
- Dupor, B. 2005. "Stabilizing Non-Fundamental Asset Price Movements under Discretion and Limited Information." *Journal of Monetary Economics* 52(4): 727–47.
- Evans, G.W. and S. Honkapohja. 2001. *Learning and Expectations in Macroeconomics*. Princeton University Press.
- . 2009. "Expectations, Learning, and Monetary Policy: An Overview of Recent Research." In *Monetary Policy under Uncertainty and Learning*, edited by K. Schmidt-Hebbel and C.E. Walsh. Santiago: Central Bank of Chile.
- Faia, E. and T. Monacelli. 2007. "Optimal Interest Rate Rules, Asset Prices, and Credit Frictions." *Journal of Economic Dynamics and Control* 31(10): 3228–54.
- Gilchrist, S. 2004. "Financial Markets and Financial Leverage in a Two-Country World-Economy." In *Banking Market Structure and Monetary Policy*, edited by L.A. Ahumada and J.R. Fuentes. Santiago: Central Bank of Chile.
- Gilchrist, S. and J. Leahy. 2002. "Monetary Policy and Asset Prices." *Journal of Monetary Economics* 49(1): 75–97.
- Gilchrist, S. and M. Saito. 2008. "Asset Prices and Monetary Policy: The Role of Learning." In *Asset Prices and Monetary Policy*, edited by J. Campbell. University of Chicago Press and National Bureau of Economic Research.
- Iacoviello, M. 2005. "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle." *American Economic Review* 95(3): 739–64.
- IMF (International Monetary Fund). 2009. *World Economic Outlook*. Washington, October.
- Justiniano, A. and B. Preston. 2009. "Monetary Policy and Uncertainty in an Empirical Small Open Economy Model." *Journal of Applied Econometrics* 25(1): 93–128.
- Kiyotaki, N. and J. Moore. 1997. "Credit Cycles." *Journal of Political Economy* 105(2): 211–48.
- Marcet, A. and J.P. Nicolini. 2003. "Recurrent Hyperinflations and Learning." *American Economic Review* 93(5): 1476–98.
- Marcet, A. and T. Sargent. 1989. "Convergence of Least Squares

- Learning Mechanisms in Self-Referential Linear Stochastic Models.” *Journal of Economic Theory* 48(2): 337–68.
- Milani, F. 2005. “Adaptive Learning and Inflation Persistence.” Working paper 050607. University of California at Irvine, Department of Economics.
- . 2007. “Expectations, Learning, and Macroeconomic Persistence.” *Journal of Monetary Economics* 54(7): 2065–82.
- Orphanides, A. and J.C. Williams. 2005. “Imperfect Knowledge, Inflation Expectations, and Monetary Policy.” In *The Inflation Targeting Debate*, edited by B.S. Bernanke and M. Woodford. University of Chicago Press.
- . 2008. “Learning, Expectations Formation, and the Pitfalls of Optimal Control Monetary Policy.” *Journal of Monetary Economics* 55 (Supplement 1): S80–S96.
- . 2009. “Imperfect Knowledge and the Pitfalls of Optimal Control Monetary Policy.” In *Monetary Policy under Uncertainty and Learning*, edited by K. Schmidt-Hebbel and C.E. Walsh, pp. 115–44. Santiago: Central Bank of Chile.
- Slobodyan, S. and R. Wouters. 2009. “Learning in an Estimated Medium-Scale DSGE Model.” Working paper 396. Prague: CERGE-EI.
- Tetlow, R. and P. von zur Muehlen. 2002. “Monetary Policy, Asset Prices, and Misspecification: The Robust Approach to Bubbles with Model Uncertainty.” *Computing in Economics and Finance* paper 335. Society for Computational Economics.