Quantitative Easing and Financial Stability

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Since the global financial crisis of 2008–09, many of the leading central banks have dramatically increased the size of their balance sheets and have shifted the composition of the assets that they hold toward larger shares of longer-term securities (as well as toward assets that are riskier in other respects). While many have hailed these policies as contributing significantly to containing the degree of damage to both the countries' financial systems and their real economies resulting from the collapse of confidence in certain types of risky assets, the policies have also been and remain quite controversial. One of the concerns raised by skeptics is that such quantitative easing by central banks may have been supporting countries' banking systems and aggregate demand only by encouraging risk-taking by ultimate borrowers and financial intermediaries in areas that increase the risk of precisely the sort of destructive financial crisis that led to the introduction of these policies in the first place.

The most basic argument for suspecting that such policies create risks to financial stability is simply that, according to proponents of these policies in the central banks (for example, Bernanke, 2012), they represent alternative means of achieving the same kind of relaxation of financial conditions that would, under more ordinary circumstances, be achieved by lowering the central bank's operating target for short-term interest rates—but a means that continues to be available even when short-term nominal interest rates have already reached their effective lower bound and so cannot be lowered to provide further

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stimulus. If one believes that a collateral effect of cuts in short-term interest rates—or perhaps even the main channel through which they affect aggregate demand, as argued by Adrian and Shin (2010)—is an increase in the degree to which intermediaries take more highly leveraged positions in risky assets, thereby increasing the likelihood or severity of a potential financial crisis, then one might suppose that to the extent that quantitative easing policies are effective in relaxing financial conditions in order to stimulate aggregate demand, they should similarly increase risks to financial stability.

One might go further and argue that such policies relax financial conditions by increasing the supply of central bank reserves.[1] An increase in the availability of reserves matters for financial conditions precisely because it relaxes a constraint on the extent to which private financial intermediaries can issue money-like liabilities (which are subject to reserve requirements) as a way of financing their acquisition of more risky and less liquid assets, as in the model of Stein (2012). Under this view of the mechanism by which quantitative easing works, one might suppose that it should be even more inevitably linked to an increase in financial stability risk than expansionary interest rate policy (which, after all, might also increase aggregate demand through channels that do not rely on increased risk-taking by banks).

Finally, some may be particularly suspicious of quantitative easing policies on the grounds that these policies, unlike conventional interest rate policy, relax financial conditions primarily by reducing the risk premiums earned by holding longer-term securities, rather than by lowering the expected path of the risk-free rate.[2] Such a departure from the normal historical pattern of risk premiums as a result of massive central bank purchases may seem a cause for alarm. If the premiums that exist when market pricing is not distorted by the central bank's intervention provide an important signal of the degree of risk that exists in the marketplace, then central bank actions that suppress this signal—not by actually reducing the underlying risks, but by preventing them from being fully reflected in market prices—

1. The term quantitative easing, originally introduced by the Bank of Japan to describe the policy it adopted in 2001 in an attempt to stem the deflationary slump that Japan had suffered in the aftermath of the collapse of an asset bubble in the early 1990s, refers precisely to the intention to increase the monetary base (and hence, it was hoped, the money supply more broadly) by increasing the supply of reserves.

2. Again, see Bernanke (2012) for discussion of this view of how the policies work, though he also discusses the possibility of effects of quantitative easing that result from central bank actions being taken to signal different intentions regarding future interest rate policy.

could distort perceptions of risk in a way that will encourage excessive risk-taking.

The present paper considers the extent to which these are valid grounds for concern about the use of this policy tool by central banks, by analyzing the mechanisms just sketched in the context of an explicit model of the way in which quantitative easing policies influence financial conditions and the way in which monetary policies more generally affect the incentives of financial intermediaries to engage in maturity and liquidity transformation of a kind that increases the risk of financial crisis. It argues, in fact, that the concerns just raised are of little merit. However, it does not reach this conclusion by challenging the view that quantitative easing policies can indeed effectively relax financial conditions (and so achieve effects on aggregate demand that are similar to the effects of conventional interest rate policy); nor does it deny that risks to financial stability are an appropriate concern of monetary policy deliberations or that expansionary interest rate policy tends to increase such risks (among other effects). The model developed here is one in which risk-taking by the financial sector can easily be excessive (in the sense that a restriction on banks' ability to engage in liquidity transformation to the same degree as under laissez-faire would raise welfare); in which, when that is true, a reduction in short-term interest rates through central bank action will worsen the problem by making it even more tempting for banks to finance acquisitions of risky, illiquid assets by issuing short-term safe liabilities; and in which the purchase of longer-term and/or risky assets by the central bank, financed by creating additional reserves (or other short-term safe liabilities, such as reverse repos or central bank bills, which would also be useful in facilitating transactions), will indeed loosen financial conditions, with an effect on aggregate demand that is similar, though not identical to, the effect of a reduction in the central bank's operating target for its policy rate. Nonetheless, the paper shows that quantitative easing policies should not increase risks to financial stability, but rather should tend to reduce them.

The reason for this different conclusion hinges on our conception of the sources of the kind of financial fragility that allowed the recent crisis to occur and the way in which monetary policy can affect the incentives to create a more fragile financial structure. In my view, the fragility that led to the crisis was greatly enhanced by the notable increase in maturity and liquidity transformation in the financial sector in the years immediately prior to the crisis (Brunnermeier, 2009; Adrian and Shin, 2010)—in particular, the significant increase in funding of financial intermediaries by issuance of collateralized short-term debt, such as repos (financing investment banks) or asset-backed commercial paper (issued by structured investment vehicles). Such financing is relatively inexpensive, in the sense that investors will hold the instruments even when they promise a relatively low yield, because of the assurance they provide that the investors will receive payment and can withdraw their funds at any time on short notice if desired. Too much of it is dangerous, however, because it exposes the leveraged institution to funding risk, which may require abrupt deleveraging through a fire sale of relatively illiquid assets. The sudden need to sell relatively illiquid assets to cover a shortfall of funding can substantially depress the price of those assets, requiring even more deleveraging and leading to a margin spiral of the kind described by Shleifer and Vishny (1992, 2011) and Brunnermeier and Pederson (2009).

It is important to ask why such fragile financial structures should arise as an equilibrium phenomenon, in order to understand how monetary policy may increase or decrease the likely degree of fragility. According to the perspective adopted here, investors are attracted to the short-term safe liabilities created by banks or other financial intermediaries because assets with a value that is completely certain are more widely accepted as a means of payment.[3] If an insufficient quantity of such safe assets is supplied by the government (through means that discussed below), investors will pay a money premium for privately issued short-term safe instruments with this feature, as documented by Greenwood, Hanson, and Stein (2010), Krishnamurthy and Vissing-Jorgensen (2012), and Carlson and others (2014). This provides banks with an incentive to obtain a larger fraction of their financing in this way. Moreover, they may choose an excessive amount of this kind of financing, despite the funding risk to which it exposes them, because each individual bank fails to internalize the effects of their collective financing decisions on the degree to which asset prices will be depressed in the event of a fire sale. This gives rise to a pecuniary externality, as a result of which excessive risk is taken in equilibrium (Lorenzoni, 2008; Jeanne and Korinek, 2010; Stein, 2012).

Conventional monetary policy, which cuts short-term nominal interest rates in response to an aggregate demand shortfall, can

3. The role of non-state-contingent payoffs in allowing an asset to be widely acceptable as a means of payment is discussed by Gorton and Pennacchi (1990), Gorton (2010), and Gorton, Lewellen, and Metrick (2012).

arguably exacerbate this problem, as low market yields on short-term safe instruments will further increase the incentive for private issuance of similar liabilities (Adrian and Shin, 2010; Giavazzi and Giovannini, 2012). The question of primary concern in this paper is whether quantitative easing policies, pursued as a means of providing economic stimulus when conventional monetary policy is constrained by the lower bound on short-term nominal interest rates, increase financial stability risks for a similar reason.

In the model proposed here, quantitative easing policies lower the equilibrium real yield on longer-term and risky government liabilities, just as a cut in the central bank's target for the short-term riskless rate will, and this relaxation of financial conditions has a similar expansionary effect on aggregate demand in both cases. Nonetheless, the consequences for financial stability are not the same. In the case of conventional monetary policy, a reduction in the riskless rate also lowers the equilibrium yield on risky assets because if it did not, the increased spread between the two yields would provide an increased incentive for maturity and liquidity transformation on the part of banks, which they pursue until the spread has decreased (because of diminishing returns to further investment in risky assets) to where it is again balanced by the risks associated with overly leveraged investment. (This occurs, in equilibrium, partly through a reduction in the degree to which the spread increases—which means that the expected return on risky assets is reduced—and partly through an increase in the risk of a costly fire-sale liquidation of assets.) In the case of quantitative easing, the equilibrium return on risky assets is reduced, but in this case through a reduction—rather than an increase—in the spread between the two yields. The money premium, which results from a scarcity of safe assets, should be reduced if the central bank asset's purchases increase the supply of safe assets to the public, as argued by Caballero and Farhi (2013) and Carlson and others (2014). Hence, the incentives for the creation of a more fragile financial structure are not increased as much by expansionary monetary policy of this kind.

The idea that quantitative easing policies, when pursued as an additional means of stimulus when the risk-free rate is at the zero lower bound, should increase risks to financial stability because they are analogous to an expansionary policy that relaxes reserve requirements on private issuers of money-like liabilities is also based on a flawed analogy. It is true, in the model of endogenous financial stability risk presented here, that a relaxation of a reserve requirement proportional to banks' issuance of short-term safe liabilities will (under a binding constraint) increase the degree to which excessive liquidity transformation occurs. It is also true that in a conventional textbook account of the way in which monetary policy affects financial conditions, an increase in the supply of reserves by the central bank relaxes the constraint on banks' issuance of additional money-like liabilities ("inside money") implied by the reserve requirement, so that the means through which the central bank implements a reduction in the riskless short-term interest rate is essentially equivalent to a reduction in the reserve requirement. However, this is not a channel through which quantitative easing policies can be effective, when the risk-free rate has already fallen to zero (or more generally, to the level of interest paid on reserves). For in such a case, reserves are necessarily already in sufficiently great supply for banks to be satiated in reserves, so that the opportunity cost of holding them must fall to zero in order for the existing supply to be voluntarily held. Under such circumstances (which is to say, those existing in countries like the United States since the end of 2008), banks' reserve requirements have already ceased to constrain their behavior. Hence, to the extent that quantitative easing policies are of any use at the zero lower bound on short-term interest rates, their effects cannot occur through this traditional channel.

In the model presented here, quantitative easing is effective at the zero lower bound (or more generally, even in the absence of reserve requirements or under circumstances where there is already satiation in reserves); this is because an increase in the supply of safe assets (through issuance of additional short-term safe liabilities by the central bank, used to purchase assets that are not equally money-like) reduces the equilibrium money premium. But whereas a relaxation of a binding reserve requirement would increase banks' issuance of short-term safe liabilities (and hence financial stability risk), a reduction in the money premium should reduce their issuance of such liabilities, so that financial stability risk should, if anything, be reduced.

The idea that a reduction in risk premiums as a result of central bank balance sheet policy should imply a greater danger of excessive risk-taking is similarly mistaken. In the model presented here, quantitative easing achieves its effects (both on the equilibrium required return on risky assets and on aggregate demand) by lowering the equilibrium risk premium—that is, the spread between the required return on risky assets and the riskless rate. But this does not imply the creation of conditions under which it should be more tempting for banks to take on greater risk. To the contrary, the existence of a smaller spread between the expected return on risky assets and the risk-free rate makes it less tempting to finance purchases of risky assets by issuing safe, highly liquid short-term liabilities that need pay only the riskless rate. Hence, again, a correct analysis implies that quantitative easing policies should increase financial stability, rather than threatening it.

The remainder of the paper develops these points in the context of an explicit intertemporal monetary equilibrium model, in which it is possible to clearly trace the general equilibrium determinants of risk premiums, the way in which they are affected by both interest rate policy and the central bank's balance sheet, and the consequences for the endogenous capital structure decisions of banks. Section 1 presents the structure of the model, and section 2 then derives the conditions that must link the various endogenous prices and quantities in an intertemporal equilibrium. Section 3 considers the effects of alternative balance sheet policies on equilibrium variables, focusing on the case of a stationary long-run equilibrium with flexible prices. Section 4 compares the ways in which quantitative easing and adjustments of reserve requirements affect banks' financing decisions. Finally, section 5 compares (somewhat more briefly) the short-run effects of both conventional monetary policy, quantitative easing, and macroprudential policy in the presence of nominal rigidities that allow conventional monetary policy to affect the degree of real economic activity. Section 6 concludes.

1. A Monetary Equilibrium Model with Fire Sales

This section develops a simple model of monetary equilibrium, in which it is possible simultaneously to consider the effects of the central bank's balance sheet on financial conditions (most notably, the equilibrium spread between the expected rate of return on risky assets and the risk-free rate of interest) and the way in which private banks' financing decisions can increase risks to financial stability. An important goal of the analysis is to present a sufficiently explicit model of the objectives and constraints of individual actors to allow welfare analysis of the equilibria associated with alternative policies that is based on the degree of satisfaction of the individual objectives underlying the behavior assumed in the model, as in the modern theory of public finance, rather than judging alternative equilibria on the basis of a more ad hoc criterion.[4]

Risks to financial stability are modeled using a slightly adapted version of the model proposed by Stein (2012). The Stein model is a three-period model in which banks finance their investments in risky assets in the first period; a crisis may occur in the second period, in which banks are unable to roll over their short-term financing and as a result may have to sell illiquid risky assets in a fire sale; and in the third period, the ultimate value of the risky assets is determined. The present model incorporates this model of financial contracting and occasional fire sales of assets into a fairly standard intertemporal general equilibrium model of the demand for money-like assets, namely, the cash-in-advance model of Lucas and Stokey (1987). In this way, the premium earned by money-like assets, which is treated as an exogenous parameter in Stein (2012), can be endogenized, and the effects of central bank policy on this variable can be analyzed, together with the consequences for financial stability.

1.1 Elements of the Model

Like most general equilibrium models of monetary exchange, the Lucas and Stokey (1987) model is an infinite-horizon model, in which the willingness of sellers to accept central bank liabilities as payment for real goods and services in any period depends on the expectation of being able to use those instruments as a means of payment in further transactions in future periods. The state space of the model is kept small (allowing a straightforward characterization of equilibrium, despite random disturbances each period) by assuming a representative household structure; the two sides of each transaction involving payment using cash are assumed to be two members of a household unit with a common objective, which can be thought of as a worker and a shopper. During each period, the worker and shopper from a given household have separate budget constraints (so that cash received by the worker as payment for the sale of produced goods cannot be immediately used by the shopper to purchase goods, in the same market), as is necessary for the cash-in-advance constraint to matter; but at the end of the period, their funds are again pooled in a

4. The proposed framework is further developed in Sergeyev (2016), which considers the interaction between conventional monetary policy and country-specific macroprudential policies in a currency union.

single household budget constraint (so that only the asset positions of households, which are all identical, matter at this point).

I employ a similar device, but increase the number of distinct roles for different household members, in order to introduce additional kinds of financial constraints into the model while retaining the convenience of a representative household. The model assumes that each infinite-lived household is made of four members with different roles during the period: a worker who supplies the inputs used to produce all final goods, and receives the income from the sale of these goods; a shopper who purchases regular goods for consumption by the household and who holds the household's cash balance, for use in such transactions; a banker who buys risky durable goods and issues short-term safe liabilities to finance some of these purchases; and an investor who purchases special final goods and can also bid for the risky durables sold by bankers in the event of a fire sale.[5] As in the Lucas-Stokey model, the different household members have separate budget constraints during the period (which is the significance of referring to them as different people), but pool their budgets at the end of each period in a single household budget constraint.

Four types of final goods are produced each period: durable goods and three types of nondurable goods, called cash goods, credit goods, and special goods. Workers also produce intermediate investment goods that are used as an input in the production of durable goods. Both cash and credit goods are purchased by shoppers; the distinction between the two types of goods is taken from Lucas and Stokey (1987), where the possibility of substitution by consumers between the two types of goods (one subject to the cash-in-advance constraint, the other not) allows the demand for real cash balances to vary with the size of the liquidity premium (opportunity cost of holding cash), for a given level of planned real expenditure. This margin of substitution also results in a distortion in the allocation of resources that depends on the size of the liquidity premium, and I wish to take this distortion into account when considering the welfare effects of changing the size of the central bank's balance sheet.

5. The distinction between bankers, investors, and worker/shopper pairs corresponds to the distinction in the roles of bankers, patient investors, and households in the model of Stein (2012). In the Stein model, these three types of agents are distinct individuals with no sharing of resources among them, rather than members of a single (larger) household; the device of having them pool assets at the end of each period is not needed to simplify the model dynamics, because the model simply ends when the end of the first and only period is reached (in the sense in which the term period is used in this model). In the present model, the representative household device also allows more unambiguous welfare comparisons among equilibria.

The introduction of special goods purchased only by the investor provides an alternative use for the funds available to the investor, so that the amount that investors will spend on risky durables in a fire sale depends on how low the price of the durables falls.[6] The produced durable goods in the model play the role of the risky investment projects in the model of Stein (2012): they require an initial outlay of resources, financed by bankers, in order to allow the production of something that may or may not yield a return later. The device of referring separately to investment goods and to the durable goods produced from them allows investment goods to be treated as perfect substitutes for cash or credit goods on the production side, resulting in a simple specification of workers' disutility of supplying more output, without having to treat durable goods as perfect substitutes for those goods, which would not allow the relative price of durables to rise in a credit boom.

All of the members of a given household are assumed to act so as to maximize a common household objective. Looking forward from the beginning of any period t, the household objective is to maximize

$$\mathrm{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[u(c_{1\tau}, c_{2\tau}) + \tilde{u}(c_{3\tau}) + \gamma \underline{s}_\tau - v(Y_\tau) - w(x_\tau) \right]. \tag{1.1}$$

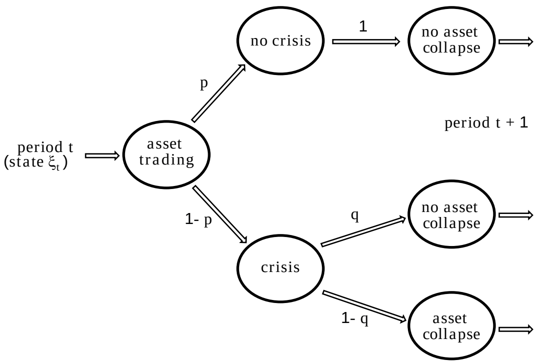
Here c_{1t}, c_{2t} and c_{3t} denote the household's consumption of cash goods, credit goods, and special goods, respectively, in period t; $\underline{s}_t$ denotes the quantity of durables held by the household at the end of period t that have not proven to be worthless, and hence the flow of services in period t from such intact durables; Y_t denotes the household's supply of normal goods (a term used collectively for cash goods, credit goods, and investment goods, which are all perfect substitutes from the standpoint of a producer) in period t; and x_t denotes the household's supply of special goods in period t. The functions $u(\cdot,\cdot)$, $\tilde{u}(\cdot)$, $v(\cdot)$ and $w(\cdot)$ are all increasing functions of each of their arguments; the functions $u(\cdot,\cdot)$and $\tilde{u}(\cdot)$ are strictly concave; and the functions $v(\cdot)$ and $w(\cdot)$ are at least weakly convex. The function $u(\cdot,\cdot)$ also implies that both cash and credit goods are normal goods, in the sense that it will be optimal to increase purchases of both types of goods if a household increases its expenditure on these types of goods on aggregate, while the (effective) relative price of the two types of goods remains the

6. The opportunity of spending on purchases of special goods plays the same role in this model as the possibility of investment in late-arriving projects in the model of Stein (2012).

same.[7] In addition, the discount factor satisfies $0 < \beta < 1$ and $\gamma < 0$. The operator $E_t[\cdot]$ indicates the expectation conditional on information at the beginning of period t.

Each of the infinite sequence of periods $t = 0, 1, 2, ,\ldots$ is subdivided into three subperiods, corresponding to the three periods in the Stein model. The sequence of events and the set of alternative states that may be reached in each period are indicated in figure 1. In subperiod 1, a financial market is open in which bankers issue short-term safe liabilities and acquire risky durables, and households decide on the cash balances to hold for use by the shopper.[8] In subperiod 2, information is revealed about the possibility that the durable goods purchased by the banks will prove to be valueless. With probability p, the no-crisis state is reached, in which it is known with certainty that no collapse in the value of the assets will occur, but with probability $1 - p$, a crisis state is reached, in which it is understood to be possible (though not yet certain) that the assets will prove to be worthless. Finally, in subperiod 3, the value of the risky durables is learned. In both of the no-asset-collapse states, a unit of the durable good produces one unit of services, while in the asset-collapse state (which occurs with probability $1 - q$, conditional on the crisis state being reached), durables provide no service flow.

Figure 1. The Sequential Resolution of Uncertainty within Period *t*



7. The effective relative price is the relative price taking into account the cost to the household of having to hold cash in order to purchase cash goods.

8. This subperiod corresponds both to the first period of the Stein (2012) model, in which risky projects are financed, and to the securities-trading subperiod of the model in section 5 of Lucas and Stokey (1987), in which bonds are priced and hence the liquidity premium on cash is determined.

The various types of goods are produced and sold in subperiod 2. The markets in which the different goods are sold differ in the means of payment that are accepted. It is assumed, as in Lucas and Stokey (1987), that cash goods are sold only for cash that is transferred from the buyer to the seller at that time; the cash balances used for this purpose must have been acquired in subperiod 1 by the household to which that shopper belongs. (The liquidity premium associated with cash is thus determined in the exchange of cash for other financial claims in subperiod 1.) Credit goods are instead sold to shoppers on credit; this means (as in Lucas and Stokey) that accounts are settled between buyers and sellers only at the end of the period, at which point the various household members have again pooled their resources, so that charges by shoppers during the period can be paid out of the income received by workers for goods sold during that same period. The only constraint on the amount of this kind of credit that a household can draw on is assumed to be determined by a no-Ponzi condition (that is, the requirement that a household be able to pay off its debts eventually out of future income, rather than roll it over indefinitely). Investment goods are sold on credit in the same way. Special goods are also assumed to be sold on credit, but in this case, the amount of credit that investors can draw on is limited by the size of the line of credit arranged for them in subperiod 1. In particular, it is assumed that a given credit limit must be negotiated by the household before it learns whether a crisis will occur in subperiod 2 and thus whether investors will have an opportunity to bid on fire sale assets. The existence of the non-state-contingent credit limit for purchases by investors (both their purchases of special goods and their purchases of risky durables liquidated by the bankers in a fire sale) is important in order to capture the idea that only a limited quantity of funds can be mobilized (by potential buyers with the expertise required to evaluate the assets) to bid on the assets sold in a fire sale.[9]

The nature of the cash that can be used to purchase cash goods requires further comment. Unlike Lucas and Stokey, I do not assume that only monetary liabilities of the government constitute cash that is acceptable as a means of payment in this market. Instead, cash is identified with the class of short-term safe instruments (STSIs)

9. In the model of Stein (2012), this limit is ensured by assuming that the patient investors have a budget that is fixed as a parameter of the model. Here this budget is endogenized by allowing it to be chosen optimally by the household in subperiod 1, but it cannot be changed in subperiod 2.

discussed by Carlson and others (2014) in the case of the United States, which includes U.S. Treasury bills (and not simply monetary liabilities of the U.S. Federal Reserve) and certain types of collateralized short-term debt of private financial institutions. The assumption that only these assets can be used to purchase cash goods is intended to stand in for the convenience provided by these special instruments, which accounts for their lower equilibrium yields relative to the short-period holding returns on other assets.[10] The fact that all assets of this type, whether issued by the government (or central bank) or by bankers, are assumed equally to satisfy the constraint is intended to capture the way in which the demand for privately issued STSIs is observed to vary with the supply of publicly issued STSIs, as shown by Carlson and others (2014).

There are, of course, *also* special uses for base money (currency and reserve balances held at the Fed) as a means of payment, of the kind that Lucas and Stokey sought to model. In particular, when the supply of reserves by the Fed is sufficiently restricted, as was chronically the case prior to the financial crisis of 2008, the special convenience of reserve balances in facilitating payments between financial intermediaries results in a spread between the yield on reserves and that on STSIs such as Treasury bills; and the control of this spread by varying the supply of reserves was the focus of monetary policy prior to the crisis. Nonetheless, the spread between the yield on reserves and the Treasury bill rate (or federal funds rate) is not the one of interest here. Under the circumstances in which the Fed has conducted its experiments with quantitative easing, the supply of reserves has been consistently well beyond the level needed to drive the Treasury bill yield down to (or even below) the yield on reserves. Hence, while certain kinds of payments by banks are constrained by their reserve balances, this has not been a binding constraint in the period in which we wish to consider the effects of further changes in the central bank balance sheet. Granting that reserves have special uses that can result in a liquidity premium specific to them (under

10. One interpretation of the cash-in-advance constraint is that it actually represents a constraint on the type of assets that can be held by money-market mutual funds (MMMFs). Such a constraint gives rise to a money premium only to the extent that there are special advantages to investors of holding wealth in MMMFs, such as the ability to move funds quickly from them to make purchases. Rather than explicitly introducing a demand for cash on the part of MMMFs and assuming that households use their MMMF balances to make certain types of purchases, I obtain the same equilibrium money premium more simply by supposing that the STSIs can directly be used as a means of payment in certain transactions.

circumstances no longer relevant at present) does not in any way imply that STSIs cannot *also* have special uses for which other assets will not serve, giving rise to another sort of money premium—one that need not be zero simply because the premium associated with reserve balances has been eliminated.

The acceptability of a financial claim as cash that can be used to purchase cash goods is assumed to depend on its having a value at maturity that is completely certain, rather than being state-contingent. This requires not only that it be a claim to a fixed nominal quantity at a future date, but that it be viewed as completely safe, for one of two possible reasons: either it is a liability of the government (or central bank),[11] or it is collateralized in a way that allows a holder of the claim to be certain of realizing a definite nominal value from it. Bankers can issue liabilities that will be accepted as cash, but these liabilities will have to be backed by specific risky durables as collateral, and holders of the debt has the right to demand payment of the debt at any time, if they cease to remain confident that the collateral will continue to guarantee the fixed value for it.

When bankers purchase risky durables in the first subperiod, they can finance some portion of the purchase price by issuing safe debt (which can be used by the holder during the second subperiod to purchase cash goods), collateralized by the durables that are acquired. If in the second subperiod, the no-crisis state is reached, the durables can continue to serve as collateral for safe debt, as the value of the asset in the third subperiod can in this case be anticipated with certainty. In this case, bankers are able to roll over their short-term collateralized debt and continue to hold the durables. If instead the crisis state is reached, the durables can no longer collateralize safe debt, as there is now a positive probability that the durables will be worthless in the third subperiod. In this case, holders of the safe debt demand repayment in the second subperiod, and the bankers must sell durables in a fire sale, in the amount required to pay off the short-term debt. It is the right to force this liquidation that makes the debt issued by bankers in the first subperiod safe.

To be more specific, suppose that the sale of goods (and in particular, cash goods) occurs at the beginning of the second subperiod: after it

11. A claim on a government need not be completely safe. If, however, a government borrows in its own fiat currency, and if it is committed to ensure that its nominal liabilities are paid with certainty (by monetizing them if necessary), then it is possible for it to issue debt that is correctly viewed as completely safe (in nominal terms).

has been revealed whether the crisis state will occur, but before the decision whether to demand immediate repayment of the short-term debt is made. Thus, at the time that shoppers seek to purchase cash goods, they may hold liabilities issued by bankers that grant the holder the right to demand repayment at any time; it is the fact that the short-term debt has this feature that allows it to be accepted as cash in the market for cash goods. After the market for cash goods has taken place, the holders of the bankers' short-term debt (who may now include the sellers of cash goods) decide whether to demand immediate repayment of the debt. At this point, these holders (whether shoppers or workers) only care about the contribution that the asset will make to the household's pooled end-of-period budget. In the crisis state, they will choose to demand repayment, since this ensures them the face value of the debt, whereas if they do not demand repayment, they will receive the face value of the debt with probability $q < 1$, but will receive nothing if the asset-collapse state occurs. If they demand repayment, they receive a claim on the investors who purchase the collateral in the fire sale; such a claim is assumed to guarantee payment in the end-of-period settlement, if within the bound of the line of credit arranged for the investor in the first subperiod.

The other source of assets that count as cash is the government. Some very short-term government liabilities (Treasury bills) count as cash. In addition, the central bank can issue liabilities that also count as cash. If the central bank increases its supply of SFSIs by purchasing Treasury bills (which are themselves SFSIs), the overall supply of cash will be unchanged. (This again demonstrates that the concept of cash used here differs importantly from that of Lucas and Stokey.) But if the central bank purchases noncash assets (either longer-term Treasury bonds, which are less able to facilitate transactions than are shorter-term bills, or assets subject to other kinds of risk) and finances these purchases by creating new short-term safe liabilities, it can increase the net supply of SFSIs. We are interested in the effects of this latter kind of policy.

1.2 Budget Constraints and Definition of Equilibrium

Each household begins period t with I_{t-1} units of the investment good (purchased in the previous period) and financial wealth A_t, which may represent claims on either the government or other households, and is measured in terms of the quantity of cash that would have the same market value in subperiod 1 trading (even though the assets aggregated in A_t need not all count as cash). In the first subperiod, the investment good is used to produce $F(I_{t-1})$ units of the durable good, which can sold on a competitive market at price Q_t per unit.[12] The banker in each household purchases a quantity s_t of these durables, financed partly from funds provided by the household for this purpose and partly by issuing short-term collateralized debt in quantity D_t. Here D_t is the face value of the debt, the nominal quantity to which the holder is entitled (with certainty) in the settlement of accounts at the end of period t. The price Q_t of the risky asset is quoted in the same (nominal, end-of-period) units; thus, the quantity of funds that the household must provide to the banker is equal to $Q_t s_t - D_t$ in those units.

The household's other uses of its beginning-of-period financial wealth are to acquire cash, in quantity M_t, for use by the shopper and to acquire (longer-term) bonds B_t, which are government liabilities that do not count as cash. The quantity M_t represents the end-of-period nominal value of these safe assets; thus, if interest is earned on cash (as the model allows), M_t represents the value of the household's cash balances inclusive of the interest earned on them, rather than the nominal value at the time that they are acquired.[13] The quantity of bonds B_t is measured in terms of the number of units of cash that have the same market value in subperiod 1 trading (as with the measurement of A_t). Hence, the household's choices of s_t, D_t, M_t, and B_t in the first subperiod are subject to an interim budget constraint,

$$\left(Q_t s_t - D_t\right) + M_t + B_t \leq A_t + Q_t F(I_{t-1}). \tag{1.2}$$

The financing decisions of bankers are also subject to a constraint that safe debt cannot be issued in a quantity beyond that for which

12. We may alternatively suppose that the investment goods are purchased by construction firms that produce the durables and sell them to bankers, and that households simply begin the period owning shares in these construction firms. The explicit introduction of such firms would not change the equilibrium conditions presented below.

13. If cash is equivalent to Treasury bills, M_t represents their face value at maturity, rather than the discounted value at which they are purchased.

they can provide sufficient collateral, given their holdings of the durable s_t.[14] This requires that

$$D_t \le \Gamma_t s_t, \tag{1.3}$$

where Γ_t is the market price of the durable good in the fire sale, should one occur in period t. (Here Γ_t is quoted in terms of the units of nominal value to be delivered by investors in the end-of-period settlement of accounts. Note that while it is not yet known in subperiod 1 whether a crisis will occur, the price Γ_t that will be realized in the fire sale *if* one occurs is perfectly forecastable.) Constraint (1.3) indicates the amount of collateral required to ensure that whichever state is reached in subperiod 2, the value of the collateralized debt will equal D_t, since sale of the collateral in a fire sale will yield at least that amount.

Regardless of the state reached in subperiod 2, the shopper's cash goods purchases must satisfy the cash-in-advance constraint:

$$P_t c_{1t} \le M_t, \tag{1.4}$$

where P_t is the price of normal goods in period t (which may depend on the state reached in subperiod 2), quoted in units of the nominal value to be delivered in the end-of-period settlement. It is this constraint that provides a reason for the household to choose to hold cash balances M_t. The common price for all normal goods follows from the fact that these goods are perfect substitutes from the point of view of their producers (workers) and that all payments that guarantee the same nominal value in the end-of-period settlement are of equal value to the sellers, once the problem of verifying the soundness of payments made in the cash goods market has been solved.[15]

There is no similar constraint on shopper's purchases of credit goods or investment goods, as these are sold on credit. The investor's

14. We might suppose that bankers can also issue debt that is not collateralized—or not collateralized to this extent. But such liabilities would not be treated as cash by the households that acquire them, so that allowing such debt to be issued by a banker would have no consequences any different from allowing the household itself to issue such debt in the first subperiod, in order to finance a larger equity contribution to its banker. Furthermore, allowing households to trade additional kinds of noncash financial liabilities would make no difference for the equilibrium conditions derived here; it would simply allow us to price the additional types of financial claims. The ability of bankers to issue collateralized short-term debt that counts as cash instead matters; this is not a type of claim that a household can issue other than by having its banker issue it (because it must be collateralized by risky durable goods), and issuing such claims has special value because they can relax the cash-in-advance constraint.

15. Cash goods and credit goods sell for the same price in any given period for the same reason in the model of Lucas and Stokey (1987).

purchases c_{3t} of special goods, and purchases s_t^{*d} of durables in the fire sale[16] must, however, satisfy a state-contingent budget constraint:

$$\tilde{P}_t c_{3t} + \eta_t \Gamma_t s_t^{*d} \leq F_t, \tag{1.5}$$

where $\tilde{P}_t$ is the price of special goods (which are quoted in the same units as P_t and which similarly may depend on the state reached in subperiod 2); η_t is an indicator variable for the occurrence of a crisis in period t;[17] and F_t is the line of credit arranged for the investor in subperiod 1, quoted in units of the nominal quantity that the investor can promise to deliver in the end-of-period settlement, and with a value that must be independent of the state that is realized in subperiod 2.[18]

If the crisis state is reached in subperiod 2, the banker offers s_t^{*s} units of the durable goods for sale in the fire sale, the quantity of which must satisfy the bounds,

$$D_t \leq \Gamma_t s_t^{*s} \leq \Gamma_t s_t. \tag{1.6}$$

The first inequality indicates that the banker must liquidate sufficient assets to allow repayment of the short-term debt (given that in this state, the holders will necessarily demand immediate repayment); the second inequality follows from the fact that bankers cannot offer to sell more shares of the durable good than they owns. The range of possible quantity offers defined in equation (1.6) is nonempty only because equation (1.3) has been satisfied; thus, a plan that satisfies equation (1.6) necessarily satisfies equation (1.3), making the earlier constraint technically redundant.

Given these decisions, the durables owned by the household in subperiod 3 will equal

$$\underline{s}_t = s_t + \eta_t \left(s_t^{*d} - s_t^{*s} \right) \tag{1.7}$$

16. We use the notation s_t^* for the quantity of durables liquidated in the fire sale, if one occurs in period t. An additional superscript d is used for the quantity demanded on this market, and a superscript s for the quantity supplied. Note that s_t^{*d} and s_t^{*s} are two independent choice variables for an individual household, and they need not be chosen to be equal, even though in equilibrium they must be equal (given common choices by all households) in order for the market to clear.

17. That is, η_t=1 if a crisis occurs, while η_t=0 if the no-crisis state is reached.

18. Like constraint (1.4), constraint (1.5) is actually two constraints, one for each possible state that may be reached in subperiod 2.

if the durables prove to be valuable, while $\underline{s}_t = 0$ regardless of the household decisions in the asset-collapse state. The household's pooled financial wealth at the end of the period (in nominal units) will be given by

$$W_t = M_t + \left(\frac{R_t^b}{R_t^m}\right) B_t + P_t Y_t - P_t \left(c_{1t} + c_{2t} + I_t\right) + \tilde{P}_t x_t + \eta_t \Gamma_t s_t^{*s} - D_t - F_t + T_t. \tag{1.8}$$

This consists of the household's cash balances at the end of subperiod 1, plus the end-of-period value of the bonds that it holds at the end of subperiod 1, plus additional funds obtained from the sale of both normal goods and special goods in subperiod 2, plus funds raised in the fire sale of assets in the event of a crisis, minus the household's expenditure on normal goods of the various types in subperiod 2, and minus the amounts that it must repay at the end of the period (if not sooner) to pay off the collateralized debt issued by the banker and to pay for the line of credit arranged for the investor, plus the nominal value T_t of net transfers from the government. Because the household must pay F_t regardless of the extent to which the line of credit is used, the investor's expenditure does not need to be subtracted, as it is paid for when F_t is paid.[19] Additionally, bonds that cost the same amount as one unit of cash in subperiod 1 are worth as much as R_t^b / R_t^m units of cash at the end of the period, where R_t^m is the gross nominal yield on cash (assumed to be known when the cash is acquired in subperiod 1, since these assets are riskless in nominal terms), and R_t^b is the gross nominal holding return on bonds (which may depend on the state reached by the end of the period).

Each household is subject to a borrowing limit,

$$W_t \geq \underline{W}_t, \tag{1.9}$$

19. The assumption that F_t must be paid whether or not the full line of credit is used is important because it prevents the household from simply asking for a large line of credit, as much as would be desired in the crisis state, and then not using all of it in the noncrisis state. If that were possible at no cost, the non-state-contingency of the credit available to the investor would have no bite. The assumption that the line of credit must be paid for whether used or not makes this costly and results in the household's wishing ex post in the crisis state that it had provided more funds to the investor—although it also wishes ex post in the noncrisis state that it had provided less credit to the investor. This device implies that the credit available to the investor will be optimal on average, though not optimal in each state because it cannot be state-contingent.

expressed as a lower bound on its net worth after the end-of-period settlement of accounts. I do not further specify the precise value of the borrowing limit, but it can be set tight enough to ensure that any end-of-period net indebtedness can eventually be repaid, while at the same time being loose enough so that the constraint (1.9) never binds in any period. Finally, the household carries into period $t + 1$ the investment goods I_t purchased in subperiod 2 of period t, as well as financial wealth in the amount of

$$A_{t+1} = R^m_{t+1} W_t, \tag{1.10}$$

where the multiplicative factor R^m_{t+1} converts the value of the household's financial wealth at the beginning of period $t + 1$ into an equivalent quantity of cash (measured in terms of the face value of the STSIs rather than their cost in subperiod 1 trading).

A feasible plan for a household is then a specification of the quantities M_t, B_t, s_t, D_t, F_t, s_t^{*s}, for each period t, as a function of the history ξ_t of shocks up until then, and a specification of the quantities c_{1t}, c_{2t}, c_{3t}, I_t, Y_t, and x_t, for each period t, as a function of both ξ_t and η_t (that is, whether a crisis occurs in period t), that satisfies the constraints (1.2)–(1.3) for each possible history ξ_t and the constraints (1.4)–(1.10) for each possible history (ξ_t, η_t), given initial financial wealth A_0 and pre-existing investment goods I_{-1} and also given the state-contingent evolution of the prices, net transfers from the government to households, as well as the borrowing limit. An optimal plan is a feasible plan that maximizes equation (1.1).

Equilibrium requires that all markets for goods and assets clear. Thus, it requires that in the first subperiod of period t,

$$M_t = \tilde{M}_t + D_t, \tag{1.11}$$

$$B_t = B_t^s, \tag{1.12}$$

and

$$s_t = F(I_{t-1}), \tag{1.13}$$

where $\tilde{M}_t$ is the public supply of cash (short-term safe liabilities of the government or of the central bank) and B_t^s is the supply of longer-term government bonds (not held by the central bank). For simplicity, durables are assumed to fully depreciate after supplying a service flow (in the event that there is no asset collapse) in the period in which they are produced and acquired by bankers; thus, the supply of durables to be acquired by bankers in period t is given simply by the new production $F(I_{t-1})$ and is independent of the quantity $\underline{s}_{t-1}$ of valuable durables in the previous period.

Equilibrium also requires that in the second subperiod, if a crisis occurs,

$$s_t^{*d} = s_t^{*s}, \tag{1.14}$$

and that in either the crisis or in the noncrisis state,

$$c_{1t} + c_{2t} + I_t = Y_t, \tag{1.15}$$

and

$$c_{3t} = x_t. \tag{1.16}$$

A flexible-price equilibrium can then be defined as a specification of prices Q_t and Γ_t and cash yield R_t^m for each history ξ_t, and prices P_t and $\tilde{P}_t$ and bond yields R_t^b for each history (ξ_t, η_t) together with a plan (as described above) for the representative household, such that (i) the plan is optimal for the household, given those prices, and (ii) the market-clearing conditions (1.11)–(1.14) are satisfied for each history ξ_t and conditions (1.15)–(1.16) are satisfied for each history (ξ_t, η_t).

1.3 Fiscal Policy and Central Bank Policy

The equilibrium conditions above involve several variables that depend on government policy: the supplies of outside financial assets $\tilde{M}_t$ and B_t^s the net transfers T_t, and the yields R_t^m and R_t^b on the outside financial assets. Fiscal policy determines the evolution of end-of-period claims on the government,

$$L_t \equiv \tilde{M}_t + \left(\frac{R_t^b}{R_t^m}\right) B_t^s + T_t \tag{1.17}$$

by varying state-contingent net transfers to households appropriately. The Treasury also has a debt management decision: at the beginning of each period t, it must decide how much of existing claims on the government will be financed through STSIs (that is, issuance of Treasury bills), as opposed to longer-term debt that cannot be used to satisfy the cash-in-advance constraint. Let $\tilde{M}_t^g$ be Treasury bill issuance by the Treasury in the first subperiod of period t; it follows that the total supply of longer-term debt by the Treasury will equal[20]

$$B_t^g = R_t^m L_{t-1} - \tilde{M}_t^g. \tag{1.18}$$

Of these longer-term securities issued by the Treasury, a quantity B_t^{cb} will be held as assets of the central bank, backing central bank liabilities $\tilde{M}_t^{cb}$ of equal value. I assume that all of these central bank liabilities are STSIs that count as cash. The supply of outside assets to the private sector is then given by

$$\tilde{M}_t \equiv \tilde{M}_t^g + \tilde{M}_t^{cb} \tag{1.19}$$

and

$$B_t^s \equiv B_t^g - B_t^{cb}. \tag{1.20}$$

In equilibrium, the net wealth W_t of the representative household at the end of period t must equal net claims L_t on the government.[21] It then follows from equations (1.10) and (1.18) that the beginning-of-period assets A_t of the representative household must equal

$$A_t = \tilde{M}_t^g + B_t^g.$$

Alternatively, since $\tilde{M}_t^{cb} = B_t^{cb}$

$$A_t = \tilde{M}_t + B_t^s, \tag{1.21}$$

in terms of the supplies of outside assets to the private sector.

At the end of period t, the central bank's assets are worth $(R_t^b / R_t^m)\, B_t^{cb}$, while its liabilities are worth $\tilde{M}_t^{cb} = B_t^{cb}$. In general, these quantities will not be equal; I assume, however, that net balance sheet earnings must be rebated to the Treasury at the end of the period, in a transfer of magnitude

20. Note that liabilities with a market value the same as $\tilde{M}_t^g + B_t^g$ units of cash in subperiod 1 will have a market price of $(\tilde{M}_t^g + B_t^g)/ R_t^m$.

21. A comparison of the definition of W_t in equation (1.8) with the definition of L_t in equation (1.17) shows that the market-clearing conditions imply that $W_t = L_t$.

$$T_t^{cb} = \left(\frac{R_t^b}{R_t^m}\right) B_t^{cb} - \tilde{M}_t^{cb}.$$

A transfer from the central bank to the Treasury allows the Treasury to make a larger transfer to the private sector while achieving the same target for end-of-period claims on the government. However, this does not change formula (1.17) for the size of net transfer that is made to the private sector, because that equation was already written in terms of a consolidated budget constraint for the Treasury and central bank. If instead we write

$$T_t^{g} = L_t - \tilde{M}_t^{g} - \left(\frac{R_t^b}{R_t^m}\right) B_t^{g}$$

for the net transfer from the Treasury required to achieve the target L_t neglecting any transfers from the central bank, then

$$T_t = T_t^{g} + T_t^{cb}.$$

Finally, in addition to choosing the size of its balance sheet, the central bank can choose the nominal interest rate R_t^m paid on its liabilities. In the model, where central bank liabilities (reserves, reverse repos, or central bank bills) are treated as perfect substitutes for all other forms of cash (Treasury bills or STSIs issued by private banks), this policy decision directly determines the equilibrium yield on those other forms of cash, as well.[22] There are thus two independent dimensions of central bank policy each period, each of which can be chosen independently of fiscal policy (that is, of the evolution of both total claims on the government L_t and the supply of short-term safe government liabilities), except to the extent that perhaps B_t^{cb} must be

22. In a more complex model in which reserve balances at the central bank play a special role that other STSIs cannot fulfill and are in sufficiently scarce supply, there will be a spread between the interest rate paid on reserves and the equilibrium yield on other STSIs, although the central bank will still have relatively direct control over the equilibrium yield on STSIs, by varying either the interest rate paid on reserves or the degree of scarcity of reserves. Even before the increased size of central bank balance sheets resulting from the financial crisis, many central banks implemented their interest rate targets largely by varying the interest rate paid on reserve balances, as discussed in Woodford (2003, chap. 1).

no greater than B_t^g.[23] These can alternatively be described as either implementation of the central bank's target for the interest rate paid on cash or variation in the size of its balance sheet holding fixed its target for that interest rate.

There is a further potential dimension of central bank policy, which is choice of the composition of its balance sheet. Above I assumed that the central bank holds only longer-term Treasury securities, but it might also hold Treasury bills on its balance sheet (as indeed the U.S. Federal Reserve does). In this model, however, it is easy to see that central bank acquisition of Treasury bills (financed by issuing central bank liabilities that are perfect substitutes for Treasury bills and pay the same rate of interest) will have no effect on any other aspect of equilibrium. To simplify the algebra, this possibility is not introduced in the notation above.

2. Determinants of Intertemporal Equilibrium

This section characterizes equilibrium in the model just described, with particular attention to the determinants of the supply of and demand for safe assets and the supply of and demand for risky durables, both when originally produced and in the event of a fire sale.

2.1 Conditions for Optimal Behavior

To begin, there are some necessary conditions for optimality of the representative household's behavior. An optimal plan for the household (as defined in the previous section) is one that maximizes a Lagrangian:

$$\begin{aligned}
&\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\Big\{u(c_{1t},c_{2t})+\tilde{u}(c_{3t})+\gamma\Big[\big(1-\eta_t\big)s_t+\eta_t q\left(s_t+s_t^{*d}-s_t^{*s}\right)\Big]-v(Y_t)-w(x_t) \qquad (2.1)\\
&-\phi_{1t}\Big[M_t+B_t+Q_t\left(s_t-F(I_{t-1})\right)-A_t-D_t\Big]\;-\eta_t\phi_{2t}\left(D_t-\Gamma_t s_t^{*s}\right)\\
&-\eta_t\phi_{3t}\left(\Gamma_t s_t^{*s}-\Gamma_t s_t\right)-\phi_{4t}\left(P_t c_{1t}-M_t\right)-\phi_{5t}\left(\eta_t\Gamma_t s_t^{*d}+\tilde{P}_t c_{3t}-F_t\right)\\
&-\phi_{6t}\left[\left(\frac{A_{t+1}}{R_{t+1}^m}\right)-M_t-\left(\frac{R_t^b}{R_t^m}\right)B_t-P_t\left(Y_t-c_{1t}-c_{2t}-I_t\right)-\tilde{P}_t x_t-\eta_t\Gamma_t s_t^{*s}+D_t+F_t-T_t\right]\Bigg\},
\end{aligned}$$

23. In fact, within the logic of the model, there is no problem with allowing B_t^{cb} to exceed B_t^g; this would simply require negative holdings of government bonds by the private sector (issuance of "synthetic" bonds by the private sector), which can already be accommodated in the constraints specified above.

where I have substituted equation (1.7) for $\underline{s}_t$ in the utility function, and equation (1.8) for W_t in equation (1.10), in order to eliminate two variables and constraints from the maximization problem (and thus allow simplification of the Lagrangian). There is also no term corresponding to the constraint (1.9), as in the equilibria discussed below I assume that the borrowing constraint is set so as not to bind in any period.[24]

Differentiating the Lagrangian with respect to the choice variables M_t, B_t, s_t, D_t, s_t, s_t^{*s}, s_t^{*d}, F_t, c_{1t}, c_{2t}, c_{3t}, I_t, Y_t, x_t, and A_{t+1}, respectively, yields the first-order conditions:

$$\phi_{1t} = \mathrm{E}_t\left[\phi_{4t} + \phi_{6t}\right] \tag{2.2}$$

$$\phi_{1t} = \mathrm{E}_t\left[\left(\frac{R_t^b}{R_t^m}\right)\phi_{6t}\right], \tag{2.3}$$

$$\phi_{1t} = (1-p)\phi_{2t} + \mathrm{E}_t\phi_{6t}\,, \tag{2.4}$$

$$\phi_{1t}Q_t = \gamma\left[p + (1-p)q\right] + (1-p)\phi_{3t}\Gamma_t\,, \tag{2.5}$$

$$\gamma q = (\phi_{2t} - \phi_{3t})\Gamma_t + \phi_{6t}^c\Gamma_t\,, \tag{2.6}$$

$$\gamma q = \phi_{5t}^c\Gamma_t\,, \tag{2.7}$$

$$\mathrm{E}_t\phi_{5t} = \mathrm{E}_t\phi_{6t}\,, \tag{2.8}$$

$$u_1(c_{1t}, c_{2t}) = P_t\left(\phi_{4t} + \phi_{6t}\right), \tag{2.9}$$

$$u_2(c_{1t}, c_{2t}) = P_t\phi_{6t}\,, \tag{2.10}$$

$$\tilde{u}'(c_{3t}) = \tilde{P}_t\phi_{5t} \tag{2.11}$$

24. We assume a borrowing limit that constrains the asymptotic behavior of the household's net wealth position far in the future, so as to preclude running a "Ponzi scheme," but that does not constrain the household's borrowing over any finite number of periods.

$$\beta\phi_{1,t+1}Q_{t+1}F'(I_t) = P_t\phi_{6t}\,, \tag{2.12}$$

$$v'(Y_t) = P_t\phi_{6t}\,, \tag{2.13}$$

$$w'(x_t) = \tilde{P}_t\phi_{6t}\,, \tag{2.14}$$

and

$$\phi_{6t} = \beta R^m_{t+1}\phi_{1,t+1}\,, \tag{2.15}$$

for each .

In these conditions, the first seven choice variables (M_t through F_t) must be chosen only as a function of the history ξ_t (that is, the state at the beginning of period t), while the other seven variables (c_{1t} through A_{t+1}) may depend on η_t (that is, whether a crisis occurs in period t) as well as ξ_t. This means that while there is only one condition corresponding to each of the equations (2.2)–(2.8) for each history ξ_t, each of the equations (2.9)–(2.15) actually corresponds to two conditions for each history ξ_t, one for each of the two possible states that may be reached in subperiod 2 (crisis or noncrisis). Similarly, the Lagrange multipliers ϕ_{1t}, ϕ_{2t}, and ϕ_{3t} will each have a single value for each history ξ_t, but the values of the multipliers ϕ_{4t}, ϕ_{5t}, and ϕ_{6t} may differ depending on the state reached in subperiod 2. The conditional expectation $E[\cdot]$ that appears in conditions such as (2.2) refers to the expected value (as of the first subperiod of period t) of variables that may take different values depending which state is reached in subperiod 2.

The superscript c appearing on Lagrange multipliers in equations (2.6)–(2.7) indicates the value of the multiplier in the case that the crisis state occurs in subperiod 2. Thus, condition (2.6) indicates the way in which the values of the multipliers ϕ_{2t} and ϕ_{3t} (which relate to constraints that apply only in the event that the crisis state is reached) depend on the value of the multiplier ϕ_{6t} in the event of a crisis in period t; but this value may be different from the value of ϕ_{6t} if no crisis occurs.

In writing the first-order conditions in this form, I have assumed for simplicity that any random disturbances (other than learning whether or not an asset-collapse occurs, after a crisis state is reached in subperiod 2) are realized in subperiod 2 of some period. Under this assumption, there is no difference between the information set in the first subperiod of period $t+1$ (denoted ξ_{t+1}) and the information set in subperiod 2 of period t.[25] I also assume that while the yield R^b_{t+1} on longer-term government debt may depend on the state reached in subperiod 2 of period $t + 1$, the yield R^m_{t+1} on safe short-term liabilities of the central bank does not; hence, this also must be known as of subperiod 2 of period t. Thus, the central bank's decision about the policy rate R^m_{t+1} (which should actually be regarded as the period t interest rate decision[26]) must be announced in subperiod 2 of period t.[27] Conditions (2.12) and (2.15) can then be written without conditional expectations, as the variables with subscripts $t + 1$ in these equations are ones with values that are already perfectly predictable in subperiod 2 of period t.

In addition to the first-order conditions (2.2)–(2.15), the household's decision variables must satisfy the constraints of the household problem, together with a set of complementary slackness conditions. Condition (2.13), together with the assumption that $v'(Y)>0$ for all possible values of Y, implies that $\phi_{6t} > 0$ necessarily; similarly, given nonsatiation in special goods, condition (2.11) implies that $\phi_{5t} > 0$ necessarily. Because it is associated with an inequality constraint—namely, condition (1.4)—the multiplier ϕ_{4t} is necessarily nonnegative; condition (2.2) then implies that $\phi_{1t} > 0$ necessarily. The remaining multipliers, ϕ_{2t}, ϕ_{3t}, and ϕ_{4t}, are associated with inequality constraints

25. There is, of course, the difference that by the beginning of period $t + 1$, it will be known whether an asset collapse occurred in period t, while this is not yet known in subperiod 2 of period t (in the case that the crisis state is reached). However, because of the assumption of full depreciation of existing durables at the end of each period, while the occurrence of an asset collapse affects household utility, it has no consequences for the assets carried by the household into the following period, the amounts of which are already predictable in subperiod 2 as long as no other random disturbances (such as an unexpected change in the size of net transfers T_t) are allowed to occur in subperiod 3. Policy in periods $t + 1$ and later is also assumed to be independent of whether an asset collapse has occurred in period t. Therefore, the relevant information set for equilibrium determination in subperiod 1 of period $t + 1$ is independent of whether an asset collapse has occurred.

26. R^m_{t+1} is the nominal yield between the settlement of accounts at the end of period t and the settlement of accounts at the end of period $t + 1$ on wealth that is held in the form of cash. This would often be called the period t riskless rate of interest, as it must be determined before the period for which the safe return is guaranteed. I use the notation R^m_{t+1} rather than R_t^m for consistency with the notation R^m_{t+1} for the one-period holding return on longer-term bonds over the same time period; the latter variable is generally not perfectly predictable in subperiod 2 of period t.

27. The model similarly assumes that the Treasury's decision about the Treasury bill supply $\tilde{M}^g_{t+1}$ and the central bank's decision about the size of its balance sheet $\tilde{M}^{cb}_{t+1}$ are announced in subperiod 2 of period t. The Treasury's decision about the size of net transfers T_t and hence the value of total claims on the government L_t at the end of period t are also announced in subperiod 2 of period t.

and so are necessarily nonnegative, but they may be equal to zero if the constraints in question do not bind (as discussed below). If any of these multipliers has a positive value, the corresponding inequality constraint must hold with equality.

2.2 Characterizing Equilibrium

In an equilibrium, all of the necessary conditions for optimality of the household's plan just listed must hold, and in addition, the market-clearing conditions (1.11)–(1.16) must hold. This section draws some further conclusions about relations that must exist among the various endogenous variables in an equilibrium, in order to show how they are affected by central bank policy.

To simplify the discussion, this paper focuses on the case in which any exogenous factors that change over time (apart from the occurrence of crisis states and asset collapses, as depicted in figure 1) are purely deterministic (that is, simply a function of the date t). That is, the exploration of the effects of a temporary disturbance of any other type considers only the case of a shock that occurs in the initial period $t = 0$, with consequences that are perfectly predictable after that. The focus is further restricted to the effects of alternative monetary and fiscal policies that are similarly deterministic; this means that while the model can be used to consider the effects of responding in different ways to a one-time disturbance (in section 5), it does not encompass the effects of responding to the occurrence of a crisis that results in a fire sale of bank assets (or to an asset collapse). The reason is that the concern here is with the consequences for the risks to financial stability of alternative central bank policies *prior* to the occurrence of a crisis; the interesting (but more complex) question of what can be achieved by suitable use of these instruments to respond to a crisis *after* it occurs is left for a later study.

Under this assumption, neither the occurrence of a crisis nor an asset collapse in any period t affects equilibrium determination in subsequent periods, and we obtain an equilibrium in which the variables listed above as functions of the history ξ_t depend only on the date t, and those listed as functions of the history (ξ_t, η_t) depend only on the date t and the value of η_t. Moreover, because the resolution of uncertainty during the period has no effect on equilibrium in later periods, the Lagrange multiplier ϕ_{6t} indicating the shadow value of additional funds in the end-of-period settlement of accounts will be independent of whether a crisis occurs in period t. Consequently, the price P_t of normal goods, the quantities purchased of normal goods (c_{1t}, c_{2t}, I_t), and the quantity Y_t that is produced will all be independent of whether a crisis occurs. Similarly, the Lagrange multiplier ϕ_{4t} associated with the cash-in-advance constraint will have a value that is independent of whether a crisis occurs.

Thus, an equilibrium can be fully described by sequences $\{A_t, M_t, B_t, D_t, F_t, s_t, s_t^*, c_{1t}, c_{2t}, I_t, Y_t, c_{3t}^c, c_{3t}^n\}$ describing the choices of the representative household;[28] sequences $\{Q_t, \Gamma_t, P_t, \tilde{P}_t^c, \tilde{P}_t^n\}$ of prices and sequences $\{R_t^m, R_t^{bc}, R_t^{bn}\}$ of yields on government securities; and sequences $\{\phi_{1t}, \phi_{2t}, \phi_{3t}, \phi_{4t}, \phi_{5t}^c, \phi_{5t}^n, \phi_{6t}\}$ of Lagrange multipliers. Here the superscripts c and n are used to indicate the values that variables take in a given period conditional on whether the crisis state (superscript c) or the noncrisis state (superscript n) is reached; variables without superscripts take values that depend only on the date. For these sequences to represent an equilibrium, they must satisfy all of the equilibrium conditions stated above for each date and for each of the possible states in subperiod 2. Conditional expectations are no longer needed in equilibrium relations such as equation (2.2) or (2.4), and the c superscript is no longer needed in equation (2.6).

2.3 Prices and Quantities Transacted in a Crisis

We turn now to a more compact description of the conditions that must hold in equilibrium. We begin with a discussion of the relations that determine the equilibrium supply of special goods, the degree to which investors are financially constrained, and the price of durable goods in the event of a fire sale.

Conditions (2.11) and (2.14), together with the requirement that in each state, require that

$$\frac{\tilde{u}'(c_{3t}^s)}{w'(c_{3t}^s)} = \tilde{\phi}_{5t}^s \equiv \frac{\phi_{5t}^s}{\phi_{6t}} \tag{2.16}$$

for each possible state s (equal to either c or n) that may be reached in subperiod 2. Since the left-hand side of condition (2.16) is a

28. Here we have reduced the number of separate variables by using a single symbol s_t^* to refer to both s_t^{*s} and s_t^{*d} as these are necessarily equal in any equilibrium, and similarly eliminated separate reference to x_t since it must always be equal to c_{3t} in any equilibrium.

monotonically decreasing function, this equation can be solved uniquely for the demand for special goods in each state,

$$c_{3t}^{s} = c_3(\tilde{\phi}_{5t}^{s}),$$

where $\tilde{\phi}_{kt} \equiv \phi_{kt} / \phi_{6t}$ for any $k \neq 6$, and $c_3(\cdot)$ is the monotonically decreasing function implicitly defined by equation (2.16).

Here $\tilde{\phi}_{5t}^{s}$ measures the degree of financial constraint of investors in state s of subperiod 2. The value $\tilde{\phi}_{5t}^{s} = 1$ would imply no ex post regret in state s about the size of the credit line arranged for the investor, and a demand for special goods that is the same as if there were no constraint separating the funds of the investor from those of the rest of the household; $\tilde{\phi}_{5t}^{s} > 1$ indicates that ex post, the household would wish it had arranged more credit for the investor, while $\tilde{\phi}_{5t}^{s} < 1$ would imply that it would wish it had arranged less. The socially efficient level of production and consumption of special goods in either state is given by the quantity c_3^* such that

$$\frac{\tilde{u}'(c_3^*)}{w'(c_3^*)} = 1.$$

Hence, special goods are underproduced or overproduced in state s according to whether $\tilde{\phi}_{5t}^{s}$ is greater or smaller than 1.

Equation (2.14) can then be used to obtain the implied state-contingent price of special goods (in units of end-of-period marginal utility),

$$\phi_{6t}\tilde{P}_t^{s} = \tilde{p}(\tilde{\phi}_{5t}^{s}) \equiv w'(c_3(\tilde{\phi}_{5t}^{s})),$$

and the implied state-contingent expenditure on special goods (in the same units),

$$\phi_{6t}\tilde{P}_t^{s}c_{3t}^{s} = e_3(\tilde{\phi}_{5t}^{s}) \equiv \tilde{p}(\tilde{\phi}_{5t}^{s})c_3(\tilde{\phi}_{5t}^{s}).$$

Note that $e_3(\tilde{\phi}_5)$ will be a monotonically decreasing function.
Since $\tilde{\phi}_5 > 0$ in each state, budget constraint (1.5) must hold with equality in each state. The fact that F_t must not be state-contingent then implies that the left-hand side of (1.5) must be the same whether a crisis occurs or not, so that in equilibrium,

$$e_3(\tilde{\phi}^n_{5t}) = e_3(\tilde{\phi}^c_{5t}) + \tilde{\Gamma}_t s^*_t \tag{2.17}$$

each period, where $\tilde{\Gamma}_t \equiv \phi_{6t}\Gamma_t$. Moreover, condition (2.8) implies that

$$\left(1-p\right)\tilde{\phi}^c_{5t} + p\tilde{\phi}^n_{5t} = 1.$$

This equation can be solved for $\tilde{\phi}^n_{5t} = \tilde{\phi}^n_5(\tilde{\phi}^c_{5t})$, a monotonically decreasing function with the property that $\tilde{\phi}^n_5(1) = 1$. Substituting this for $\tilde{\phi}^n_{5t}$ in (2.17) yields an equation

$$\tilde{D}(\tilde{\phi}^c_{5t}) = \tilde{\Gamma}_t s^*_t, \tag{2.18}$$

where

$$\tilde{D}(\tilde{\phi}^c_5) \equiv e_3(\tilde{\phi}^n_5(\tilde{\phi}^c_{5t})) - e_3(\tilde{\phi}^c_{5t})$$

is a monotonically increasing function with the property that $\tilde{D}(1) = 0$. Finally, equation (2.7) implies that

$$\tilde{\phi}^c_{5t}\tilde{\Gamma}_t = \gamma q. \tag{2.19}$$

This together with (2.18) implies that

$$\tilde{\phi}^c_{5t}\tilde{D}(\tilde{\phi}^c_{5t}) = \gamma q s^*_t.$$

Since the left-hand side of this equation is a monotonically increasing function of $\tilde{\phi}^c_{5t}$, it can be uniquely solved for

$$\tilde{\phi}^c_{5t} = \tilde{\phi}^c_5(s^*_t), \tag{2.20}$$

where $\tilde{\phi}^c_5(s^*)$ is a monotonically increasing function with the property that $\tilde{\phi}^c_5(0) = 1$.

This solution for the equilibrium value of the multiplier $\tilde{\phi}^c_5$ then allows us to solve for the implied values of $\tilde{\Gamma}_t$, $\tilde{\phi}^n_{5t}$, c^c_{3t}, c^n_{3t}, $\phi_{6t}\tilde{P}^c_t$, and $\phi_{6t}\tilde{P}^n_t$, each as a function of the quantity s^*_t of durable goods that are sold in the fire sale (if one occurs) in period t. We observe that $\tilde{\phi}^c_5$ and c^n_{3t} will be increasing functions of s^*_t and $\phi_{6t}\tilde{P}^n_t$ will be nondecreasing, while $\tilde{\Gamma}_t$, $\tilde{\phi}^n_{5t}$, and c^c_{3t} will be decreasing functions of s^*_t and $\phi_{6t}\tilde{P}^c_t$ will be nonincreasing.

In the case that $s_t^* = 0$ (no assets are sold in a fire sale), $c_{3t}^c = c_{3t}^n = c_3^*$ (the efficient quantity of special goods are produced in both states), $\tilde{\phi}_5^c = \tilde{\phi}_{5t}^n = 1$ (no regret about the size of the line of credit arranged for the investor, in either state), and $\tilde{\Gamma}_t = yq$ (the market price of durables in the crisis state is equal to their fundamental value). Instead, if $s_t^* > 0$ (that is, if any assets are sold in a fire sale), $c_{3t}^c < c_3^* < c_{3t}^n$, $\tilde{\phi}_{5t}^n < 1 < \tilde{\phi}_5^c$, and $\tilde{\Gamma}_t = yq$. This means that special goods are underproduced in the crisis state and overproduced in the noncrisis state, and that ex post, the household wishes it had supplied more credit for its investor if the crisis state occurs, while it wishes that it had supplied less credit if the crisis state does not occur. It also means that if the crisis state occurs, the price at which durables are sold in the fire sale is less than their fundamental value, conditional on reaching that state. Moreover, the size of these distortions is greater the larger is the aggregate value of s_t^*. The fact that households do not take these equilibrium effects into account when choosing their planned value of s_t^{*s} results in a pecuniary externality.

2.4 Implications of the Demand for Safe Assets

We turn next to a discussion of the consequences of the supply of short-term safe instruments for equilibrium purchases of cash and credit goods. We consider first the implications of optimality conditions (2.9)–(2.10), together with the cash-in-advance constraint (1.4) and the associated complementary slackness condition.

Let us first define the demand functions $c_1^*(\lambda)$, $c_2(\lambda)$ as the solution to the problem of choosing c_1 and c_2 to maximize

$$u(c_1, c_2) - \lambda\left(c_1 + c_2\right)$$

for an arbitrary price $\lambda > 0$. Under the assumption that cash and credit goods are both normal goods, both $c_1^*(\lambda)$ and $c_2^*(\lambda)$ must be monotonically decreasing functions.[29] We can then consider the constrained problem

$$\max_{c_1, c_2} u(c_1, c_2) - \lambda\left(c_1 + c_2\right) \quad \text{s.t. } c_1 \leq m, \tag{2.21}$$

29. The paths followed by the two variables as λ is reduced correspond to the "income-expansion path" as a result of increasing the budget available to spend on these two goods, for a fixed relative price (equal prices of the two goods).

where $m > 0$ represents real cash balances available to the household. The solution $c_1(\lambda; m), c_2(\lambda; m)$ to problem (2.21) can be characterized as follows: if $m \leq c_1^*(\lambda)$, then $c_1(\lambda; m) = m$ and $c_2(\lambda; m)$ is implicitly defined by the equation

$$u_2(m, c_2) = \lambda. \tag{2.22}$$

If instead $m \leq c_1^*(\lambda)$ then $c_1(\lambda; m) = c_1^*(\lambda)$ and $c_2(\lambda; m) = c_2^*(\lambda)$.

The Kuhn-Tucker conditions for this latter, constrained problem are easily seen to correspond precisely to conditions (2.9)–(2.10) and constraint (1.4) together with the complementary slackness condition, where the price of normal goods in units of end-of-period marginal utility is given by $\lambda_t \equiv \phi_{6t} P_t$, and available real cash balances are given by $m_t \equiv M_t/P_t$. It follows that the model implies that c_{1t}, c_{2t} must satisfy

$$c_{jt} = c_j(\lambda_t; M_t/P_t)$$

for $j = 1{,}2$ where the functions $c_j(\lambda; m)$ are defined in the previous paragraph.

Associated with this solution will be a value for the normalized Lagrange multiplier $\tilde{\phi}_{4t}$, given by

$$\tilde{\phi}_{4t} = \tilde{\phi}_4(\lambda_t; M_t/P_t),$$

where we define

$$\tilde{\phi}_4(\lambda; m) \equiv \frac{u_1(c_1(\lambda; m), c_2(\lambda; m))}{u_2(c_1(\lambda; m), c_2(\lambda; m))} - 1.$$

The Kuhn-Tucker conditions for the problem (2.21) imply that $\tilde{\phi}_{4t}$ $(\lambda; m)$ for all $m \geq c_1^*(\lambda)$, while $\tilde{\phi}_{4t}$ $(\lambda; m) > 0$ for all $m < c_1^*(\lambda)$. Furthermore, in the latter case (where the cash-in-advance constraint binds), the assumption that both cash goods and credit goods are normal goods implies that $\tilde{\phi}_{4t}(\lambda; m)$ is a decreasing function of λ for fixed m,[30] and a decreasing function of m for fixed λ.[31]

A comparison of equations (2.2) and (2.4) (and recalling that the conditional expectations have been eliminated from both of these conditions) implies that under any optimal plan, it must be the case that $\tilde{\phi}_{4t} = (1-p)\tilde{\phi}_{2t}$. Hence, in any equilibrium where the cash-in-advance constraint binds in some period, so that $\tilde{\phi}_{4t} > 0$, it must also be the case that $\tilde{\phi}_{2t} > 0$, so that the first inequality in equation (1.6) is also a binding constraint, and $D_t = \Gamma_t s_t^*$ (as much collateralized debt is issued by bankers as can be repaid in the event of a crisis, given the quantity of durables that bankers plan to sell in a fire sale). More generally, we can conclude that the normalized Lagrange multiplier $\tilde{\phi}_{2t}$ will be given by

$$\tilde{\phi}_{2t} = \tilde{\phi}_2(\lambda_t; M_t/P_t),$$

where we define

$$\tilde{\phi}_2(\lambda; m) \equiv \frac{\tilde{\phi}_4(\lambda; m)}{(1-p)}.$$

Condition (2.2) implies that the normalized multiplier $\tilde{\phi}_{1t}$ will similarly be given by a function

$$\tilde{\phi}_{1t} = \tilde{\phi}_1(\lambda_t; M_t/P_t), \tag{2.23}$$

where we define

$$\tilde{\phi}_1(\lambda; m) \equiv 1 + \tilde{\phi}_4(\lambda; m).$$

30. Concavity of the utility function implies that increasing c_2 while c_1 remains fixed at m implies a decrease in the marginal utility of credit goods consumption, so that increasing λ with fixed m must correspond to a reduction in the quantity of c_2 that is purchased. In order for the demand m for cash goods to remain the same despite a budget contraction that requires fewer credit goods to be purchased, the relative price of cash goods must decrease (under the assumption of normal goods). This means that u_1/u_2 must decrease, and hence that $\tilde{\phi}_4$ must decrease.

31. In the $\lambda - m$ plane, the level curves of the function $\tilde{\phi}_4$ correspond to income-expansion paths, as the budget for cash and credit goods changes with the relative price of the two types of goods fixed. If the two goods are both normal goods, m must increase along such a path as λ decreases, as discussed above; hence, the level curves must have a negative slope at all points. It then follows that the sign of this partial derivative follows from the sign of the one discussed in the previous footnote.

It follows that $\tilde{\phi}_{1t} > 1$ if and only if the cash-in-advance constraint binds, while it is equal to 1 otherwise. Additionally, both $\tilde{\phi}_{1t}(\lambda; m)$ and $\tilde{\phi}_{2t}(\lambda; m)$ will be decreasing in both arguments, in the region where the cash-in-advance constraint binds.

A comparison of conditions (2.6) and (2.7) similarly implies that under any optimal plan, it must be the case that

$$\tilde{\phi}^{c}_{5t} - 1 = \tilde{\phi}_{2t} - \tilde{\phi}_{3t}. \tag{2.24}$$

This allows solving for the implied value of the normalized multiplier $\tilde{\phi}_{3t}$ as

$$\tilde{\phi}_{3t} = \tilde{\phi}_3(\lambda_t; s^*_t, M_t/P_t),$$

where we define

$$\tilde{\phi}_3(\lambda_t; s^*_t, M_t/P_t) \equiv \tilde{\phi}_2(\lambda_t; M_t/P_t) + 1 - \tilde{\phi}^{c}_5(s^*_t). \tag{2.25}$$

The supply of real cash balances M_t/P_t and the quantity of assets s^*_t sold in the event of a fire sale must be endogenously determined in such a way as to guarantee that in equilibrium, the value of this function is always nonnegative. (The existence of such a solution is shown below.)

Finally, condition (2.5) can be used to determine the equilibrium price of risky durables in the subperiod 1 market. If $\tilde{Q}_t \equiv \phi_{6t} Q_t$ denotes this price in marginal-utility units, then we obtain a solution of the form

$$\tilde{Q}_t = \tilde{Q}(\lambda_t; s^*_t, M_t/P_t),$$

where we define

$$\tilde{Q}(\lambda_t; s^*_t, M_t/P_t) \equiv \frac{\tilde{Q}^* + (1-p)\tilde{\phi}_3(\lambda_t; s^*_t, M_t/P_t)\tilde{\Gamma}(s^*_t)}{\tilde{\phi}_1(\lambda_t; M_t/P_t)}. \tag{2.26}$$

Here, the notation

$$\tilde{Q}^* \equiv \gamma\left[p + (1-p)q\right]$$

is used for the expected marginal utility of the anticipated service flow from a durable purchased in subperiod 1, and

$$\tilde{\Gamma}(s^*) \equiv \frac{\gamma q}{\tilde{\phi}_5^c(s^*)}$$

for the solution for $\tilde{\Gamma}_t$ derived in the previous section.

The fundamental value of a durable purchased in subperiod 1, if the anticipated future service flow were to be valued using the same pricing kernel that is used to price bonds in condition (2.3),[32] would equal[33]

$$\tilde{Q}_t^{fund} \equiv \frac{\tilde{Q}^*}{\tilde{\phi}_{1t}}. \tag{2.27}$$

Thus, equation (2.26) implies that durables will be priced at their fundamental value in subperiod 1 if and only if the second inequality in equation (1.5) is not a binding constraint; that is, the quantity of durables held by bankers (and thus the availability of collateral) does not constrain bankers to issue less collateralized debt than they would otherwise wish. When the constraint binds, so that $\tilde{\phi}_{3t} > 0$ durables are overvalued in subperiod 1. The above discussion of the equilibrium value of $\tilde{\phi}_{3t}$ implies that in order for this to happen, the cash-in-advance constraint must bind (so that $\tilde{\phi}_{2t} > 0$), while the supply of durables (and hence the equilibrium value of s_t^*) must not be too large, so that $\tilde{\phi}_5^c(s_t^*)$ is not too much greater than 1.

2.5 Determinants of the Supply of Safe Assets

We turn now to the endogenous determination of the cash supply M_t, as a result of the financing decisions of bankers. Since $\tilde{\phi}_5^c(s_t^*) > 1$ if $s_t^* > 0$, the left-hand side—and hence also the right-hand side—of equation (2.24) must be positive if any assets will be sold by bankers in the event of a fire sale. But the right-hand side of equation (2.24) can be positive only if $\tilde{\phi}_{2t}$ is positive, which occurs only if the cash-in-

32. That is a general pricing relation for noncash assets, since I make no particular assumption about the nature of the state-contingent return on bonds, only that this asset cannot be used as a means of payment in the cash goods market.

33. Equation (2.3) states that an asset that yields Y_t at the end of period in marginal-utility units should have a price in subperiod 1 of $P_t^Y = \mathrm{E}_t[Y_t] / \phi_{1t}$. For the case of longer-term bonds, $Y_t = \phi_{6t} R_t^b$ and the price in the subperiod 1 market is $P_t^Y = R_t^m$.

advance constraint binds. This, in turn, would require that $D_t = \Gamma_t s_t^*$ as argued in the previous paragraph, and hence that, using equation (11),

$$M_t = \tilde{M}_t + \Gamma_t s_t^*. \tag{2.28}$$

On the other hand, if $s_t^* = 0$, constraint (1.5) requires that $D_t = 0$ as well, so that equation (2.28) must hold in this case, as well. We may thus conclude that in any equilibrium, the total supply of cash will be given by equation (2.28).

It remains to determine the equilibrium value of s_t^*. In marginal-utility units, equation (2.28) can be written

$$\hat{M}_t \equiv \phi_{6t} M_t = \lambda_t \tilde{m}_t + \tilde{\Gamma}_t s_t^*, \tag{2.29}$$

using the notation $\tilde{m}_t \equiv \tilde{M}_t / P_t$ for the real supply of safe assets by the government. Then in any equilibrium where

$$\tilde{m}_t + \frac{\tilde{\Gamma}_t s_t^*}{\lambda_t} > c_1^*(\lambda_t),$$

the cash-in-advance constraint will not bind. However, since this implies that $\tilde{\phi}_{2t} = 0$, equation (2.24) implies that $\tilde{\phi}_{5t}^c$ cannot be greater than 1, which requires that $s_t^* = 0$.

Hence, such an equilibrium occurs if and only if

$$\tilde{m}_t > \tilde{m}^*(\lambda_t) \equiv c_1^*(\lambda_t), \tag{2.30}$$

and involves $\hat{M}_t = \lambda_t \tilde{m}_t$. In this case, equation (2.25) implies that $\tilde{\phi}_{3t} = 0$ so that $\tilde{Q}_t$ is equal to the fundamental value (2.27). In addition, because $s_t^* = 0$, it must be the case that $\tilde{\Gamma}_t = \tilde{\Gamma}(0) = 1$, so that durables are also priced at their fundamental value in subperiod 2, even if the crisis state is reached.

Consider now the possibility of an equilibrium in which the supply of real cash balances is no greater than $c_1^*(\lambda_t)$ (the level required for satiation in cash), but the supply of durables s_t is large enough so that bankers are unconstrained in the amount of collateralized debt that they can issue (so that $\tilde{\phi}_{3t} = 0$). Because of equation (2.24), this requires a value of s_t^* such that

$$\tilde{\phi}_5^c(s_t^*) - 1 = \tilde{\phi}_2(\lambda_t; \tilde{m}_t + \tilde{\Gamma}(s_t^*) s_t^* / \lambda_t). \tag{2.31}$$

It follows from the discussion above that the left-hand side of this equation is an increasing function of s_t^*, while the right-hand side is a nonincreasing function of s_t^* (decreasing until the point at which the cash-in-advance constraint ceases to bind, and constant thereafter).[34] Moreover, the right-hand side is at least as large as the left-hand side if $s_t^* = 0$ given the assumption now that $\tilde{m}_t \leq c_1^*(\lambda_t)$. Hence, there is a unique value of $0 \leq s_t^* < s_t$ that satisfies condition (2.31) if and only if the left-hand side is greater than the right-hand side when $s_t^* = s_t$, which is to say, if and only if

$$\tilde{\phi}_5^c(s_t) - 1 > \tilde{\phi}_2(\lambda_t; \tilde{m}_t + \tilde{\Gamma}(s_t)s_t / \lambda_t). \tag{2.32}$$

Thus, such an equilibrium exists in period t if and only if the outside supply of safe assets $\tilde{m}_t$ fails to satisfy condition (2.30) while the supply of durables s_t does satisfy condition (2.32); in such a case, s_t^* is implicitly defined by condition (2.31), and the total supply of cash is given by condition (2.29). In this case, again $\tilde{\phi}_{3t} = 0$ and hence $\tilde{Q}_t = \tilde{Q}_t^{fund}$. Moreover, if $\tilde{m}_t < c_1^*(\lambda_t)$, the solution must involve $s_t^* > 0$ and hence $\tilde{\Gamma}_t < 1$, so that durables are underpriced in the fire sale in the event of a crisis.

If, instead, $\tilde{m}_t$ does not satisfy condition (2.30) and the supply of durables s_t fails to satisfy condition (2.32), then there can only be an equilibrium in which $s_t^* = s_t$. In this case, the supply of safe assets is given by

$$\hat{M}_t = \lambda_t \tilde{m}_t + \tilde{\Gamma}(s_t)s_t. \tag{2.33}$$

The value of $\tilde{\phi}_{3t}$ is given by equation (2.25), which will be positive in the case of any value of s_t such that the inequality in equation (2.32) is reversed. In any such case, it must be the case that $\tilde{Q}_t > \tilde{Q}_t^{fund}$, so that durables are overvalued in subperiod 1. In addition, the fact that $s_t^* > 0$ implies that $\tilde{\Gamma}_t < 1$ so that durables are *underpriced* in the event of a fire sale, even though they are *overpriced* in subperiod 1. In this case, an asset boom can be followed by a crash.

It is thus possible to completely characterize the equilibrium pricing of risky durables in any period t (both in subperiod 1 and in the event of a crisis) as a function of three quantities: the real supply $\tilde{m}_t$ of safe assets by the government (determined by fiscal policy and central

34. Recall that $\tilde{\Gamma}(s^*)s^* = \tilde{D}(\tilde{\phi}_5^c(s^*))$ is a monotonically increasing function of s*, and that $\tilde{\phi}_2(\lambda; m)$ is a decreasing function of m as long as the cash-in-advance constraint binds, and independent of the value of m for all higher values.

bank asset purchases), the supply of durables s_t (which follows directly from the quantity I_{t-1} of investment goods produced in the previous period), and the marginal utility λ_t that the representative household assigns to additional real end-of-period wealth. The latter quantity depends on expectations about subsequent periods, as discussed next.

In particular, the subperiod 1 equilibrium price of durables, expressed in marginal-utility units, can be written as a function

$$\tilde{\phi}_{1t}\tilde{Q}_t = \varphi(\lambda_t; s_t, \tilde{m}_t)$$

derived in the manner just explained. It is useful for the discussion below to consider how this function depends on the supply of durables s_t. In the case of an outside cash supply satisfying $\tilde{m}_t > c_1^*(\lambda_t)$, or a supply of durables satisfying equation (2.32), in equilibrium it must be the case that $\tilde{\phi}_{3t} = 0$, so that equation (2.26) implies that $\varphi(\lambda_t; s_t, \tilde{m}_t) = \tilde{Q}^*$. Thus, the value of the function is independent of the value of s_t in either of these cases. If instead there are both an outside cash supply below the satiation level and a supply of durables too small to satisfy equation (2.32), the equilibrium supply of safe assets is given by equation (2.33). The right-hand side of this equation is a monotonically increasing function of s_t, so that $M_t / P_t = \hat{M}_t / \lambda_t$ is also an increasing function of s_t.

It follows from this that the equilibrium value of $\tilde{\phi}_{3t}$ given by equation (2.25) will be a monotonically decreasing function of s_t. It then follows from equation (2.26) that $\tilde{\phi}_{1t}\tilde{Q}_t$ will be a monotonically decreasing function of s_t, and hence that the function $\varphi(\lambda_t; s_t, \tilde{m}_t)$ is decreasing in this argument. Thus, in the case that $\tilde{m}_t < c_1^*(\lambda_t)$, the function $\varphi(\lambda_t; s_t, \tilde{m}_t)$ will be a decreasing function of s_t for all supplies of durables too small to satisfy equation (2.32), and will instead be constant at its minimum value of $\tilde{Q}^*$ for all s_t large enough to satisfy equation (2.32). The function is constant (and equal to $\tilde{Q}^*$) whenever $\tilde{m}_t > c_1^*(\lambda_t)$ regardless of the value of s_t.

It will also be useful for the discussion below of intertemporal equilibrium to note that the relative value of funds available in subperiod 1 as opposed to the end of the period will be given by a function of the form

$$\tilde{\phi}_{1t} = \hat{\phi}_1(\lambda_t; s_t, \tilde{m}_t). \tag{2.34}$$

This function depends only on the value of λ_t in the case that $\tilde{m}_t \geq c_1^*(\lambda_t)$, so that there is satiation in cash. It depends on both λ_t and $\tilde{m}_t$ in the case that $\tilde{m}_t < c_1^*(\lambda_t)$ but s_t is large enough to satisfy equation (2.32), but does not depend on s_t, since in this case bankers' collateral constraint does not bind, and s_t^* is independent of the size of s_t. Finally, in the case that $\tilde{m}_t < c_1^*(\lambda_t)$ and s_t is too small to satisfy equation (2.32), the value of the function depends on all three of its arguments. (In this latter case, M_t/P_t will be an increasing function of s_t for given values of the other two arguments, as just discussed; hence $\tilde{\phi}_{1t}$ will be a decreasing function of s_t for s_t in this range.)

2.6 Intertemporal Equilibrium

We now consider the connections between variables in successive periods required for an intertemporal equilibrium. One such connection is given by condition (2.12) for optimal investment demand. Using the solution for the subperiod 1 equilibrium price of durables just derived, condition (2.12) can be written in the alternative form

$$\lambda_t = \beta\varphi(\lambda_{t+1}; F(I_t), \tilde{m}_{t+1})F'(I_t). \tag{2.35}$$

Here I have also used the fact that the supply of durables in period $t+1$ must equal $s_{t+1} = F(I_t)$.

Since the right-hand side of this expression must be a monotonically decreasing function of I_t,[35] condition (2.35) has a unique solution for the equilibrium value of I_t, which can be written in the form

$$I_t = I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1}). \tag{2.36}$$

Because the right-hand side of equation (2.35) is a decreasing function of I_t the function $I(\lambda; \lambda', \tilde{m})$ implicitly defined by this equation will be a monotonically decreasing function of λ. Thus, we obtain a demand curve for investment that is a decreasing function of λ_t, similar to the demands for cash and credit goods as decreasing functions of λ_t that can be derived in the way explained above. But whereas the demands for cash and credit goods depend on s_t and $\tilde{m}_t$ along with the value of λ_t, investment demand depends on expectations regarding the values of λ_{t+1} and $\tilde{m}_{t+1}$ along with the value of λ_t.

35. This relies on the demonstration above that $\varphi(\lambda; s, \tilde{m})$ is a nonincreasing, positive-valued function of s, in addition to the assumption that the function $F(I)$ is strictly concave.

If the solution for the sum of the demands for cash and credit goods is written as

$$c_{1t}+c_{2t} = y(\lambda_t; s_t, \tilde{m}_t),$$

then the aggregate demand for normal goods can be written as

$$Y_t = y(\lambda_t; s_t, \tilde{m}_t)+I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1}). \tag{2.37}$$

In a flexible-price equilibrium (the kind assumed thus far), this quantity of normal goods will also have to be voluntarily supplied, which requires that condition (2.3) be satisfied. Hence, the equilibrium value of λ_t must satisfy

$$v'\left(y(\lambda_t; F(I_{t-1}), \tilde{m}_t)+I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1})\right) = \lambda_t. \tag{2.38}$$

Since the left-hand side of this equation is a nonincreasing function of λ_t (strictly decreasing if $v'' > 0$), there will be a unique solution for λ_t corresponding to given values of $I_{t-1}, \tilde{m}_t, \tilde{m}_{t+1}$, and λ_{t+1}.

In the initial period of the model, the value of I_{t-1} will be given as an initial condition; but in all subsequent periods, the value will be endogenously determined by equation (2.36). Hence, for all periods after the initial period, we obtain an equilibrium relation of the form

$$v'\left(y(\lambda_t; F(I(\lambda_{t-1}; \lambda_t, \tilde{m}_t)), \tilde{m}_t)+I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1})\right) = \lambda_t. \tag{2.39}$$

Given an initial stock of investment goods I_{-1} in period $t = 0$ and a path for $\{\tilde{m}_t\}$ for all $t \geq 0$ (determined by fiscal policy and the central bank's balance sheet policy), an intertemporal equilibrium is then a sequence of anticipated values $\{\lambda_t\}$ for all $t \geq 0$ that satisfy equation (2.38) when $t = 0$ and the second-order nonlinear difference equation (2.39) for all $t \geq 1$.

Given a solution for the path $\{\lambda_t\}$, the associated path for the production of investment goods is given by (2.36) for all $t \geq 0$. This in turn implies a supply of durables s_t for each period $t \geq 0$ using equation (1.13). One then has sequences of values $\{\lambda_t, s_t, \tilde{m}_t\}$ for each of the periods $t \geq 0$. The implied values for the variables s_t^*, M_t / P_t, and so on, as well as for the various normalized Lagrange multipliers, can then be determined for each of these periods using the results derived in the previous sections.

This yields a solution for the allocation of resources, all relative prices and all real asset prices, that involves no reference to any nominal variables, as long as the central bank's balance sheet policy is specified in real terms (since the real supply of outside safe assets is used in the above calculations). In fact, the only element of policy that matters for the determination of real variables in the flexible-price version of the model is the path of $\{\tilde{m}_t\}$. The path of government debt as a whole does not matter for the determination of any variables in the model: Ricardian equivalence obtains (given the assumption of a representative household and lump-sum taxes and transfers), except for the qualification that changes in the government supply of safe assets are not neutral in this model, owing to the cash-in-advance constraint.[36]

Conventional monetary policy (the central bank's control of the interest rate on cash balances R_t^m) is also irrelevant to the determination of real variables, though it can be used to control the general level of prices (the path $\{P_t\}$, and along with it the prices of other goods and assets in monetary units). Condition (2.15) requires that in equilibrium,

$$R_{t+1}^m = \left(1 + r_{t+1}^m\right)\frac{P_{t+1}}{P_t}, \tag{2.40}$$

where

$$1 + r_{t+1}^m \equiv \frac{\lambda_t}{\beta\lambda_{t+1}\hat{\phi}_1(\lambda_{t+1}; s_{t+1}, \tilde{m}_{t+1})}$$

is the equilibrium real return on cash between the end of period t and the end of period $t+1$. Note that the path of the variable $\{r_{t+1}^m\}$ is determined for all $t \geq 0$ by the path of $\{\tilde{m}_t\}$ in the manner discussed above, as with all other real variables. Equation (2.40) then describes the Fisher relation that must hold between the nominal interest rate on cash and the rate of inflation.

The equilibrium paths of the price level $\{P_t\}$ for $t \geq 0$ and of the nominal interest rate $\{R_{t+1}^m\}$ for $t \geq 0$ are jointly determined by the equilibrium relation (2.40) and the reaction function—which may, for example, be of the form $R_{t+1}^m = \psi(P_t / P_{t-1})$ —that specifies how

36. For the same reason, it does not matter exactly what type of liabilities the government issues other than short-term safe assets; and it similarly does not matter, in this model, what type of noncash assets are held on the balance sheet of the central bank.

the central bank's interest rate target responds to variation in the price level. The discussion of how this occurs follows exactly the lines of the discussion of price-level determination in a flexible-price cashless economy in Woodford (2003, chap. 2). While the present model includes a number of financial frictions and other complications not present in the simple model used in that discussion, what matters is that the variable r_{t+1}^m in equation (2.40) evolves in a way that is completely exogenous with respect to the evolution of the price level and independent of the specification of (conventional) monetary policy.

It will simplify the discussion that follows if conventional monetary policy is specified not by a central bank reaction function, but rather by a target path for the price level $\{P_t\}$ for all $t \geq 0$. Since this target path can be achieved by a suitable rule for setting the interest rate R_{t+1}^m—assuming that equation (2.40) does not imply a negative nominal rate at any time,[37] given the target path of prices— the path of the price level is assumed to conform to the target path chosen by the central bank, and equation (2.40) is used to determine the implied equilibrium evolution of the nominal interest rate on cash.

Finally, condition (2.3) requires that the equilibrium expected return on bonds satisfy

$$\mathrm{E}_t[R_t^b] = \hat{\phi}_1(\lambda_t; s_t, \tilde{m}_t)$$

in all periods $t \geq 0$. Given a specification of the character of this alternative form of government debt to determine the relative value of bonds in states *c* and *n*, this relation then completely determines the state-contingent returns on bonds. The solution for equilibrium bond yields is not necessary to solve for any of the other variables discussed earlier; hence, it is not necessary to discuss further the character of bonds or their equilibrium prices.

37. The model as described above would not preclude a negative nominal interest rate in equilibrium, that is, a value $R_{t+1}^m < 1$. It is more realistic, however, to add an assumption that households can demand currency from the central bank at any time in exchange for interest-earning cash, which would for institutional reasons earn a zero nominal interest rate, and that such currency would be acceptable as payment for cash goods. The possibility of holding currency would then preclude equilibria with $R_{t+1}^m < 1$ in any period.

3. The Size of the Central bank Balance Sheet and Stationary Equilibrium

The paper compares the effects of the two dimensions of central bank policy: variation in its target for the interest rate R_t^m paid on cash; and variation in the size of its balance sheet, holding fixed its target for that interest rate. We first compare alternative possible long-run stationary equilibria, in which the inflation rate, the various interest rates, and relative prices are all constant over time, and the real size of the central bank balance sheet and the real supply of Treasury bills by the Treasury are constant over time as well. It can be shown that there exists a two-dimensional family of such stationary equilibria. Moreover, fixing the real supply of Treasury bills, it is still possible to move in both directions within this two-dimensional family of stationary equilibria by varying the two independent dimensions of central bank policy. Thus, even a simple consideration of stationary equilibria allows us to observe the separate effects of the two dimensions of policy.

3.1 Alternative Stationary Equilibria

In a stationary equilibrium, the government pursues a constant inflation target

$$\frac{P_t}{P_{t-1}} = \Pi > 0$$

for all $t \geq 0$, starting from some given initial price level P_{-1}, and chooses to supply a constant quantity of real outside cash balances $\tilde{m}_t = \tilde{m}$ in all periods $t \geq 0$ as well.[38] We further assume that there are no transitory disturbances to preferences, technological possibilities, or financial constraints (so that the equations derived above apply in all periods, with no modifications), and that the economy starts from an initial stock of investment goods I_{-1} that takes the particular value I with the property that starting with this level of investment goods results in an equilibrium in which $I_t = I$ for all $t \geq 0$ as well. In such a case (and for choices of the targets Π and $\tilde{m}$ within suitable ranges),

38. Note that given our assumption of a constantly growing target path for the price level and our assumption that this target is precisely achieved each period, there is no difference between specifying the target path for the supply of outside cash balances as a constant real level or as a nominal target with a constant growth rate equal to the target inflation rate.

there exists an intertemporal equilibrium with the special property that the variables c_{1t}, c_{2t}, c^c_{3t}, c^n_{3t}, s_t, s^*_t, λ_t, $\tilde{Q}_t$, $\tilde{\Lambda}_t$, $\tilde{p}^c_t$, $\tilde{p}^n_t$, $\hat{M}_t$, R^m_t, and the various normalized Lagrange multipliers all have the same constant values for all $t \geq 0$, which are simply denoted c_1, c_2 and so on.

From equation (2.39) it is evident that such a stationary equilibrium must correspond to a constant value λ for the marginal-utility value of end-of-period real income that satisfies

$$v'\left(y(\lambda; F(I(\lambda;\lambda,\tilde{m})),\tilde{m}) + I(\lambda;\lambda,\tilde{m})\right) = \lambda. \tag{3.1}$$

This gives us a single equation to solve for the stationary equilibrium value of λ corresponding to a given stationary target $\tilde{m}$. Given the solution for λ from this equation, the implied stationary value of I is then given by $I = I(\lambda;\, \lambda, \tilde{m})$ which is the value of I_{-1} that must be assumed for the existence of such an equilibrium. Such an equilibrium will obviously involve a constant supply of durables, equal to $s = \mathrm{F}(I)$. These constant values for λ_t, s_t and $\tilde{m}_t$ in all periods can then be used to solve for constant values of all of the other variables listed above, using the methods explained in the previous section.

The constant value of the nominal interest rate on cash will be given by $R^m = (1+r^m(\tilde{m}))\Pi$, where

$$1 + r^m \equiv \frac{1}{\beta\tilde{\phi}_1(\tilde{m})}$$

and $\tilde{\phi}_1(\tilde{m})$ is the stationary value of $\tilde{\phi}_{1t}$, which depends on the value chosen for $\tilde{m}$ as discussed above, but is independent of the choice of Π. Thus, for any choice of $\tilde{m}$, it is possible to choose any value of Π such that

$$\Pi \geq \beta\tilde{\phi}_1(\tilde{m}),$$

so that the required stationary nominal interest rate satisfies $R^m \geq 1$.

There is a stationary equilibrium corresponding to any value $\tilde{m} > 0$, but for all $\tilde{m}$ greater than a critical value m^*, the stationary equilibrium is the same. Here m^* is the level of outside real cash balances required for satiation in cash balances, which is determined as follows. In a stationary equilibrium with satiation in cash balances, it must be the case that $c_1 = c_1^*(\lambda)$ and $c_1 = c_1^*(\lambda)$. In addition, $\tilde{\phi}_1 = 1$ $\tilde{Q} = \tilde{Q}^*$ so that $\varphi(\lambda; s, \tilde{m}) = \tilde{Q}^*$ regardless of the values of λ and s. It follows that the stationary level of investment goods production I must equal $I^*(\lambda)$ the quantity implicitly defined by the equation

$$F'(I) = \frac{\lambda}{\beta \tilde{Q}^*}.$$

From this it follows that the stationary value of λ must satisfy

$$v'\left(c_1^*(\lambda) + c_2^*(\lambda) + I^*(\lambda)\right) = \lambda. \tag{3.2}$$

Since $c_1^*(\lambda)$, $c_2^*(\lambda)$ and $I^*(\lambda)$ are all monotonically decreasing functions, it follows that the left-hand side of equation (3.2) is a nonincreasing function of λ, and the equation must have a unique solution for λ. The associated stationary level of cash balances can be any level greater than or equal to $m^* \equiv c_1^*(\lambda)$. Hence, such a stationary equilibrium exists in the case of any value of $\tilde{m}$ that is greater than or equal to m^*.

Finally, in any stationary equilibrium, the equilibrium real return on longer-term bonds (and indeed, any asset that can neither be used as cash nor used as collateral to issue liabilities that can be used as cash) will equal

$$\frac{E[R^b]}{\Pi} = \frac{R^m \tilde{\phi}_1}{\Pi} = \beta^{-1}.$$

This is independent of both $\tilde{m}$ and Π. Thus, a higher value of $R^m/\Pi = 1 + r^m(\tilde{m})$ corresponds to a reduced spread between the returns on longer-term bonds and those on holding cash. The value of $\tilde{\phi}_1$ (or, more precisely, the log of $\tilde{\phi}_1$) measures this spread.

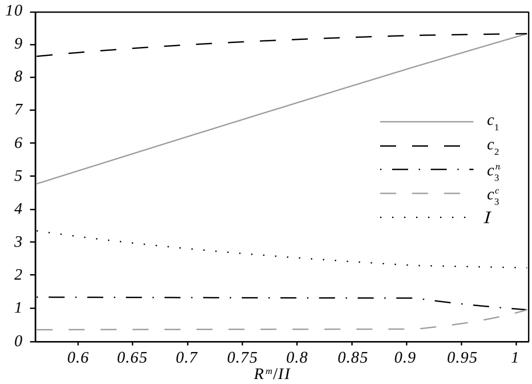
There is thus a two-dimensional family of possible stationary equilibria, which can be indexed by the choice of the two policy variables Π and $\tilde{m}$, which can be independently varied using the two dimensions of central bank policy: conventional monetary policy (interest rate policy) and balance sheet policy (quantitative easing). These two dimensions of monetary policy have quite different effects. In the flexible-price model, interest rate policy has no effect on any real variables, but it can be used (within the limit imposed by the zero lower bound) to control inflation. Balance sheet policy (changing the total supply of outside safe assets by increasing or reducing the quantity of longer-term bonds held by the central bank) can instead affect the steady-state values of all of the real variables in the model, except that further increases in the real supply of outside safe assets beyond the level $\tilde{m} = m^*$ have no further effects.

The possible stationary values of the various real variables that can be achieved by alternative monetary policies can thus be fully characterized by considering the one-parameter family of stationary equilibria corresponding to different values of $\tilde{m}$. These equilibria can be classified into three possible types, according to which of the financial constraints bind. (The three possible cases correspond to the three cases discussed in the treatment of the endogenous determination of the safe asset supply in the previous section.) First, there are equilibria in which the real outside supply of safe assets equals or exceeds the level m^* required for satiation. In these equilibria, the cash-in-advance constraint is slack; bankers finance none of their purchases of durables by issuing collateralized short-term debt (so that the collateral constraint on such issuance is also slack); and as no assets are sold in a fire sale even if the crisis state occurs, there is no ex post regret of the size of investors' credit limit (so that the constraint that this must be fixed in advance also does not bind). Second, there are equilibria in which the real outside supply of safe assets is insufficient. There is some private issuance of safe debt, but the quantity of safe debt issued by bankers is still small enough for the collateral constraint not to bind. Third, there are equilibria in which the incentive for issuance of safe debt by bankers is so strong that their issuance of such liabilities is limited by the availability of suitable collateral. The three cases correspond to different ranges of real outside supply of safe assets: high values of $\tilde{m}$, an intermediate range of values of $\tilde{m}$, and low values of $\tilde{m}$, respectively.

This one-parameter family of stationary equilibria can alternatively be parameterized by the associated value of $R^m/\Pi = 1 + r^m(\tilde{m})$ the stationary gross real rate of return on cash. Values of $\tilde{m}$ increasing from 0 to m^* correspond to values of R^m/Π increasing from some minimum value $1 + r^m(0)$ (which may well be positive, though it will generally correspond to a negative real rate of return) to $1 + r^m(m^*) = \beta^{-1} > 1$ (the point at which the spread between the return on bonds and that on cash is completely eliminated). A numerical example may usefully illustrate how systematic variation in this parameter changes the character of the stationary equilibrium.

Figure 2 shows how the stationary equilibrium values of c_1, c_2, c_3^c, c_3^n, and I vary with alternative stationary values for R^m/Π. The figure thus completely displays the allocation of real resources in each possible equilibrium, and it supplies all of the information needed to evaluate the level of expected utility of the representative household in each case and draw conclusions about the welfare effects of alternative possible long-run policy targets. The values of R^m/Π considered vary from $1 + r^m(0)$ at the left boundary of the figure to $1 + r^m(m^*) = \beta^{-1} > 1$ at the right boundary.

Figure 2. The Allocation of Resources in Alternative Stationary Equilibria Corresponding to Different Constant Values of R^m/Π



In this example, cash and credit goods enter the household's utility function symmetrically, so that in an efficient allocation, equal quantities of the two goods are produced and consumed; thus, a comparison of the magnitudes of c_1 and c_2 indicates the size of the distortion created by the cash-in-advance constraint. There is no distortion ($c_1 = c_2$) at the extreme right of the figure, that is, when $R^m/\Pi = \beta^{-1}$ so that there is no spread between the return on longer-term bonds and cash. Moving left in the figure, as the real return on cash is reduced (meaning that the spread is made progressively larger), the extent to which c_1 is less than c_2 grows progressively greater.

The efficiency of the level of production and consumption of special goods can also be seen directly from the figure. Because both the utility from consuming special goods and the disutility of supplying them are independent of which state occurs in subperiod 2, an efficient allocation requires that c_3^n equal c_3^c and for the parameterization used in this example, the common efficient level of special goods production is equal to 1 (regardless of the level of production and consumption of other goods). Thus, the degree to which c_3^n is greater than c_3^c (and to which the former quantity is greater than 1, while the latter quantity is smaller) indicates the degree to which the production and consumption of special goods is distorted by the fact that investors spend some of their resources on acquiring risky durables in the fire sale that occurs in the crisis state. As one moves from right to left in the figure, bankers' incentive to issue collateralized short-term debt increases, but the consequence is an increasing quantity of durables that must be sold to redeem this debt in the event of a fire sale, increasing the wedge between c_3^n and c_3^c.

The three different possible types of equilibrium correspond to different regions of the horizontal axis in the figure. The possibility of an equilibrium in which the cash-in-advance constraint is slack is represented by the right boundary ($R^m/\Pi = \beta^{-1}$); while this corresponds to an entire range of possible values of $\tilde{m}$ (any $\tilde{m} \geq m^*$), they all correspond to the same real return on cash and the same allocation of resources. The case in which the cash-in-advance constraint binds but bankers' collateral constraint is slack corresponds to values of R^m/Π from around 0.91 to 1.01, while the case in which both constraints bind corresponds to all values of R^m/Π from the left boundary to about 0.91.

In the relatively high-cash-return region, because bankers' collateral constraint does not bind, the quantity of short-term debt issuance by bankers increases relatively rapidly as R^m/Π is decreased, as a consequence of which the wedge between c_3^n and c_3^c increases relatively sharply. However, because durables are still valued at their fundamental value in subperiod 1, the production of durables does not increase greatly. In the lower-cash-return region, further reductions in R^m/Π do not increase debt issuance as rapidly (because now the quantity of debt issued can increase only to the extent that the quantity of durables purchased by bankers also increases enough to provide the required additional collateral), so that the wedge between c_3^n and c_3^c no longer increases so rapidly. Because the ability of durables to allow additional short-term debt issuance increases the price of durables above their fundamental value, the equilibrium production of durables now increases more rapidly with further reductions in R^m/Π.

Figure 3 shows the stationary values of another set of variables, across the same one-parameter family of stationary equilibria: the supply of short-term collateralized debt $\tilde{D}$ (the stationary value of the variable $\tilde{D}_t \equiv \tilde{\phi}_{6t} D_t$), the resulting total supply of cash $\hat{M}$, the upper bound $\tilde{\Gamma}s$ on issuance of short-term debt by bankers given by the expected market value of their assets in the event of a crisis, and for purposes of comparison, the market value $\tilde{Q}s$ of those same assets in subperiod 1.[39] As the equilibrium return on cash falls and

39. Each of these variables is measured in marginal-utility units, as they have a constant value in marginal-utility units in a stationary equilibrium, regardless of the inflation rate. Also, as shown above, the equilibrium relations determining the values of these variables are in many cases simpler when written in terms of the variables expressed in marginal-utility units.

the money premium correspondingly increases (moving from the right boundary of the figure to the left), the issuance of short-term debt by banks increases from an initial value of zero (when the money premium is zero) to progressively higher values. The rate of increase is sharpest in the high-cash-return region, because the upper bound on debt issuance does not bind; after that constraint begins to bind (around $R^m/\Pi = 0.91$), $\tilde{D}$ increases less sharply with further declines in R^m/Π as it can only increase to the extent that $\tilde{\Gamma}s$ also increases. In fact, in the high-cash-return region, $\tilde{\Gamma}s$ decreases as the money premium increases; the reason is that as short-term debt issuance increases, the quantity of assets that must be sold in a fire sale in the event of a crisis increases, depressing the fire-sale value of bankers' assets. Once R^m/Π falls to around 0.91, the constraint comes to bind, both because of the increase in desired debt issuance and the reduction in the value of the collateral available to back such debt. Beyond this point, further increases in the size of the money premium cause $\tilde{\Gamma}s$ to increase, rather than continuing to decrease; this is because the value of relaxing the constraint on short-term debt issuance now contributes to a larger market value of durables in subperiod 1,[40] which induces a larger market supply of durables (as can be seen from the I curve in figure 2), so that $\tilde{\Gamma}s$ increases slightly, even though the fire-sale price $\tilde{\Gamma}s$ continues to fall.

The size of the gap between the solid line indicating the value of $\hat{M}$ and the dashed line indicating the value of $\tilde{D}$ shows how the part of the cash supply that comes from outside safe assets (the value of $\lambda\tilde{m}$, in marginal-utility units) varies across the alternative stationary equilibria. This value decreases monotonically as one proceeds from right to left in the figure, both because $\tilde{D}$ increases and because $\hat{M}$ decreases; the latter effect represents the reduction in the demand for cash balances as the opportunity cost of holding them (that is, the money premium) increases. The fact that the equilibrium relationship between the size of the money premium and the quantity of outside safe assets is monotonic indicates how the choice of a stationary level for the supply of outside safe assets (through the combination of the Treasury's debt-

40. Specifically, the value of $\tilde{\phi}_1\tilde{Q}/\lambda$ increases, which is the ratio of the marginal-utility value of the sale price of a unit of the durable good in subperiod 1, given that payment received in subperiod 1 can be used to acquire cash for use by the shopper, to the marginal-utility value of the sale price of a unit of normal goods in subperiod 2. This relative price determines the incentive to produce additional investment goods, as shown by condition (2.12), and hence the supply of durables. The stationary value of $\tilde{Q}$ does not increase, as can be seen from the $\tilde{Q}s$ curve in this figure.

management policy and the central bank's balance sheet policy) can be used to determine the stationary value of R^m/Π and thus to select which of the stationary equilibria depicted in these figures should occur.

There is a limit to how far R^m/Π can be reduced by shrinking the supply of outside safe assets; at the left edge of the figure, $\tilde{m}$ falls to zero, while R^m/Π is still positive. (This is because this lower bound does not correspond to an opportunity cost high enough to reduce the demand for cash balances to zero; it is only necessary that the demand for cash balances fall to a low enough level that it is no greater than the quantity of safe liabilities that bankers wish to supply, which grows the larger the money premium gets.) However, this lower bound for R^m/Π can easily be well below 1 (as shown in the figure), corresponding to a negative long-run equilibrium short-term real rate. Thus, in the model it is perfectly possible to have an equilibrium short-term real rate that remains negative forever, as a result of a shortage of safe assets; this results in a safety trap in the sense of Caballero and Farhi (2013), in the case that the inflation target Π is too low. An advantage of working with a fully developed monetary equilibrium model, however, is that the existence of a safety trap depends not simply on too low a supply of safe assets (or too great a demand for them), but also on choosing too low an inflation target, just as in the liquidity-trap model of Krugman (1998) and Eggertsson and Woodford (2003).

Figure 4 shows how the degree to which durables are both overvalued in subperiod 1 (and at the time that the decision to divert resources into the production of durables is made) and undervalued in the event of a fire sale varies across the alternative stationary equilibria. The dashed line plots the stationary value of $\tilde{\phi}_1\tilde{Q}/\tilde{Q}^*$, which is to say the ratio of the subperiod 1 market price of durables to their fundamental value.[41] Thus, durables are overvalued in subperiod 1 to the extent that this quantity exceeds 1. As the figure shows, it equals 1 (there is no overvaluation) in the high-cash-return region, given that banks do not wish to acquire additional durables for the sake of being able to issue more collateralized short-term debt. However, for all values of R^m/Π below 0.91, durables are overvalued, and the degree of overvaluation gets progressively higher the larger is the money premium.

41. Alternatively, the quantity plotted is the ratio of Λ^s to its fundamental value $\beta\tilde{Q}^*$, where Λ^s is the marginal-utility valuation assigned to an additional quantity of investment goods sufficient to allow production of an additional unit of durables, so that the demand curve for investment goods can be written as $F'(I)=\lambda/\Lambda^s$.

Figure 3. The Endogenous Supply of Safe Assets in Alternative Stationary Equilibria Corresponding to Different Constant Values of R^m/Π, with Implications for Bank Capital Structure and the Total Supply of Safe Assets

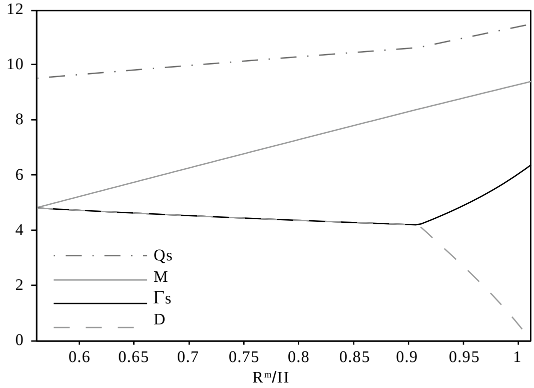
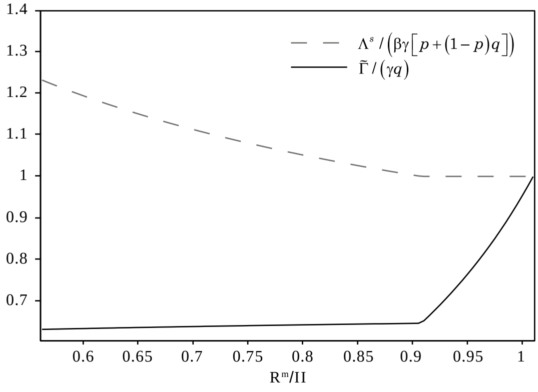


Figure 4. The Initial Overvaluation of Durables and Their Subsequent Undervaluation in the Event of a Crisis,[a] in Alternative Stationary Equilibria Corresponding to Different Constant Values of R^m/Π



a. The dashed line plots the degree of initial overvaluation of durables; the solid line plots the degree of their subsequent undervaluation in the event of a crisis.

The solid line in the same figure plots the stationary value of $\tilde{\Gamma} / \gamma q$, which is the ratio of the fire-sale price of durables to their fundamental value under this contingency (which is smaller than their fundamental value in subperiod 1, since if a crisis occurs the probability that the durables are worthless is higher than previously realized). Thus, durables are undervalued in the fire sale to the extent that this quantity is less than 1. As shown in the figure, durables are undervalued in the fire sale in the case of any $R^m / \Pi = \beta^{-1}$ (corresponding to any $\tilde{m} < m^*$), and the degree of undervaluation increases steadily the larger the money premium. The degree of undervaluation increases especially sharply with increases in the money premium in the high-cash-return region, since in this region s^* (the quantity of assets sold in the fire sale if one occurs) increases relatively sharply with increases in the money premium. Once the constraint that can be no larger than the total quantity s of assets held by bankers becomes binding, s^* increases much less rapidly with further increases in the money premium, and the degree of equilibrium undervaluation correspondingly ceases to increase so rapidly, though it grows somewhat.

Alternatively, the extent to which distortions are created by financial constraints in the alternative stationary equilibria can be measured by looking not at how market valuations differ from fundamental values, but at the extent to which the constraints affect households' decisions, as indicated by the size of the Lagrange multipliers associated with the various constraints. Figure 5 plots the values of the three key (normalized) Lagrange multipliers in the model: $\tilde{\phi}_1$, which indicates a binding cash-in-advance constraint to the extent that it is greater than 1;[42] $\tilde{\phi}_3$, which indicates a binding constraint on the quantity of collateralized short-term debt that bankers can issue to the extent that it is positive; and $\tilde{\phi}_5^c$, which indicates a binding constraint on investors' ability to spend as much in the crisis state as the household would wish ex post, to the extent that it is greater than 1.

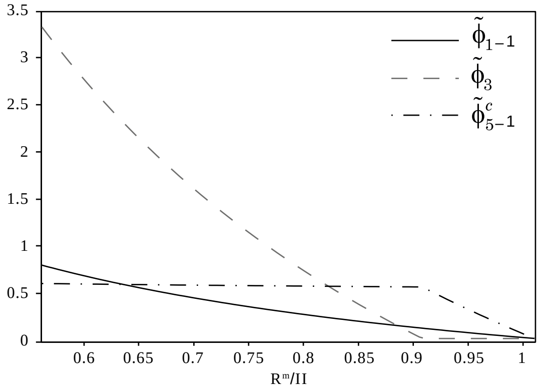
The value of $\tilde{\phi}_1$ is equal to u_1/u_2 the marginal rate of substitution between cash and credit goods, and the more this exceeds 1, the greater the inefficiency of the allocation of expenditure between these two types of goods (which have equal disutility of supply). As the figure shows, the magnitude of this distortion increases steadily as R^m / Π is reduced (which is to say, as the money premium increases), starting from zero distortion when $R^m / \Pi = \beta^{-1}$, so that there is no

42. The quantity $\tilde{\phi}_1 - 1$ plotted in the figure is also the value of $\tilde{\phi}_4$ as well as $1-p$ times the value of $\tilde{\phi}_2$.

money premium. Moreover, the magnitude of the distortion is a convex function of the size of the money premium, so that the rate at which the distortion increases becomes greater for larger values of the money premium.

As explained in section 2.3, the stationary equilibrium values of c_3^c and c_3^n are both monotonic functions of $\tilde{\phi}_5^c$ (the first an increasing function, the latter a decreasing function), with $\tilde{\phi}_5^c = 1$ corresponding to the efficient level of production c_3^* of special goods in both states. Hence, the extent to which $\tilde{\phi}_5^c$ is greater than 1 indicates the degree of inefficiency in the level of production and consumption of special goods (in both states) owing to the possibility of a fire sale of assets by banks. The figure shows that the magnitude of this distortion also increases as $R^m/\,\Pi$ is reduced, starting from zero distortion when $R^m/\,\Pi = \beta^{-1}$. However, the magnitude of this distortion increases sharply with increases in the size of the money premium only in the high-cash-return region; once the availability of collateral becomes a binding constraint on issuance of short-term debt by bankers, the degree of inefficiency in the level of production of special goods increases only gradually with further increases in the size of the money premium.

Figure 5. Lagrange Multipliers Indicating the Degree to Which the Various Financial Constraints Bind, in Alternative Stationary Equilibria Corresponding to Different Constant Values of R^m/Π



Finally, the figure indicates that $\tilde{\phi}_3 > 0$ indicating that the constraint that short-term debt issuance cannot exceed the amount that can be backed by the collateral value of bankers' assets binds, only for values of R^m/Π less than 0.91. Below this point, however, the value of the multiplier rises sharply with further increases in the money premium; this accounts for the increase in the subperiod 1 market price of durables, shown in figure 4, over this same region.

3.2 Consequences of a Larger Central Bank Balance Sheet

We can now consider how a quantitative easing policy that permanently increases the size of the central bank's balance sheet (in real terms, or relative to the size of the economy)—and more specifically, a policy of purchasing longer-term assets and financing these purchases by issuing short-term safe liabilities—affects the economy's long-run equilibrium. To the extent that the effects of the policy are not undercut by an offsetting shift in the maturity composition of the debt issued by the Treasury,[43] such a policy can increase the steady-state level of $\tilde{m}$. If $\tilde{m} < m^*$, so that there is not already satiation of the demand for safe assets even without any creation of safe assets by the private sector, then increasing $\tilde{m}$ will mean moving to a stationary equilibrium with a higher value of R^m/Π, corresponding to a movement further to the right in each of the figures just presented.

This has real effects and, in particular, consequences for financial stability. However, a larger supply of outside safe assets as a result of a policy of quantitative easing should *improve* financial stability. Specifically, whether the economy begins in the low-cash-return or high-cash-return region, a higher value of R^m/Π (and hence a smaller money premium) reduces private issuance of short-term debt $\tilde{D}$. As a consequence, it reduces the quantity s^* of durables that will have to be sold in a fire sale in the event of a crisis and so reduces the severity of the distortions associated with a crisis.[44] Both the degree to which durables are undervalued in the crisis (as shown in figure 4) and the

43. Such a shift in Treasury policy did offset a significant part of the effect of the Fed's asset purchases in recent years, as shown by Greenwood and others (2014).

44. In the simple model presented here, the probability of a crisis is exogenous and so cannot be affected by policy, but policy can affect the severity of a crisis, conditional on the crisis state being reached.

degree of inefficiency in the level of production of special goods (as shown in figure 2) are smaller, the larger the value of R^m/Π.

Thus, from the standpoint of financial stability, a larger central bank balance sheet is clearly to be preferred (at least as far as long-run steady states are concerned). In fact, the other real effects of a quantitative easing policy on the long-run steady state are also beneficial. A higher value of R^m/Π implies that the cash-in-advance constraint binds less tightly (as shown by the value of $\tilde{\phi}_1$ in figure 5), and this results in a more efficient allocation of household expenditure between cash and credit goods (a ratio of c_1/c_2 closer to 1, in figure 2). In the low-cash-return region (where $\tilde{\phi}_3 > 0$), a higher value of R^m/Π also results in less overvaluation of durables in subperiod 1, so that there is less inefficient overproduction of durables (as is also seen in figure 2). Each of these considerations points in the same direction: the equilibrium allocation of resources is more efficient (and the welfare of the representative household is increased) if the real supply of outside safe assets is increased.

The conclusion that expansion of the central bank's balance sheet is associated with a more efficient allocation of resources between cash and credit goods might seem surprising in light of the analysis of Lucas and Stokey (1987), who conclude, in the context of a similar model (but without durable goods production or fire sales), that efficiency in this respect is greater the *lower the rate of growth* of the monetary base—with the highest levels of efficiency (and hence of welfare for the representative household) being achieved only in the case of steady contraction of the size of the central bank's balance sheet. The difference in conclusions results from their assumption that the safe liabilities that count as cash must earn a nominal interest rate of zero (so that $R^m = 1$ is assumed). In that case, steady states with different values of R^m/Π must correspond to different inflation rates Π—whereas here the choice of the inflation target Π is independent of the aspects of policy that determine R^m/Π, within the bound required by the lower bound on nominal interest rates.

Lucas and Stokey conclude, as I do, that relaxation of the cash-in-advance constraint, with a more efficient allocation of expenditure between cash and credit goods, requires a higher value of R^m/Π but in their analysis this requires a lower inflation rate and hence a lower growth rate of the nominal value of outside safe assets $\tilde{M}_t$. In the model presented here, it is also true that in a long-run stationary equilibrium, the growth rate of $\tilde{M}_t$ must equal the inflation rate. However, it is possible for the central bank to control the value of the currency unit other than through its control of the path of $\tilde{M}_t$ (by appropriate variation in R_t^m), so that there is a decision to make about how large $\tilde{M}_t$ should be relative to the level of P_t targeted through interest rate policy, which is separate from the question of the long-run growth rate of the two variables. Thus, it is not correct, more generally, to identify a decision to increase the size of the central bank's balance sheet with a decision to pursue a more *inflationary* policy; in the long run, these are two distinct issues. The short-run consequences of balance sheet expansion are considered in section 5.

4. Quantitative Easing Compared with Macroprudential Policy

Another implication of increasing the supply of central bank reserves through a quantitative easing policy, not discussed in the analysis above, is relaxation of the constraint on private banks' ability to issue money-like liabilities that may result from a requirement that they hold reserves in proportion to their issuance of such liabilities. Such reserve requirements apply (at least in some countries, like the United States) to at least some kinds of short-term safe instruments issued by commercial banks—though not, even in the United States, to the kind of privately issued STSIs that were most responsible for the financial fragility exposed by the recent crisis.[45] Under many traditional textbook accounts of the way that monetary policy affects the economy, the key effect of a central bank open-market operation is precisely to relax this constraint on private bank behavior by increasing the quantity of reserves that are available to satisfy the reserve requirement. This might seem to have important implications for financial stability that would cut in the opposite direction to the analysis above; that is, it might seem that expansion of the central bank's balance sheet should have as an effect, or even as its primary effect, an increase in the extent to which private banks acquire risky assets and finance those assets by issuing money-like liabilities. This is a key theme of the analysis by Stein (2012) and the basis for his proposal that monetary policy decisions be considered from the standpoint of financial-stability regulation.

45. The kinds of liability, such as retail deposits at commercial banks, to which such requirements apply were not subject to highly volatile demand. While these funds could, in principle, be withdrawn on short notice, they were not, probably owing to the existence of deposit insurance; so they were not responsible for any appreciable funding risk.

In the analysis here, I have abstracted from reserve requirements, since even in the United States, these have not been binding constraints on banks' behavior during the Fed's experiments with quantitative easing.[46] The framework can, however, be used to discuss the consequences for financial stability of increasing or decreasing the cost to financial institutions of issuing collateralized short-term debt as a source of financing, even when they hold sufficient assets to provide the collateral for such issuance. This as a separate dimension of policy—*macroprudential* policy—that should be distinguished, conceptually, from both conventional monetary policy (interest rate policy) and central bank balance sheet policy.[47] One might well use instruments of macroprudential policy that affect the ability and/or incentives of banks to issue money-like liabilities that are unrelated to the central bank's balance sheet (and that do not depend on the existence of reserve requirements). Even when the tool that is used is a reserve requirement, one can loosen or tighten this constraint independently of the way one changes the size of the central bank's balance sheet— first, because one can vary the required reserve ratio as well as the supply of reserves; second, because the central bank can vary the supply of STSIs without varying the supply of reserves, if it issues central bank bills or engages in reverse repo transactions,[48] or by varying the quantity of Treasury bills on its own balance sheet.

The effects of varying macroprudential policy are quite different from the effects (considered above) of varying the central bank's supply of outside safe assets, when the latter policy is implemented in a way that has no direct effects on financial institutions' cost of short-term debt issuance. Macroprudential policy can be introduced into the model set out above in the following way. Suppose that a banker who issues short-term debt with face value D_t obtains only $\xi_t D_t$ in additional funds with which to acquire assets in subperiod 1, where $0 \leq \xi_t \leq 1$; the quantity $(1-\xi_t)D_t$ represents a proportional tax on issuance of safe debt, collected by the government. The variable ξ_t (or alternatively the

46. They were not relevant, even earlier, for most of the financing decisions modeled in this paper. As noted earlier, the privately supplied "cash" in this model should be identified primarily with repos or asset-backed commercial paper.

47. Macroprudential policy, modeled in a way similar to that used here, is also compared with conventional monetary policy by Sergeyev (2016), who also discusses Ramsey policy when the two distinct types of policy instruments exist. Sergeyev's discussion of optimal policy does not treat the use of balance sheet policies of the kind that are the central focus here.

48. See Carlson and others (2014) on the usefulness of reverse repo transactions, such as the Fed's proposed ON RRP facility, for this purpose.

tax rate) then represents an instrument of macroprudential policy. the value of ξ_t may be varied from period to period, if the degree to which it is desirable to provide a disincentive to safe debt issuance varies over time; and the choice of the path of $\{\xi_t\}$ is independent of the choice of the path of $\{\tilde{m}_t\}$, the real outside supply of safe assets.

One possible way of implementing such a tax on safe debt issuance is through a reserve requirement. Suppose that a bank that issues safe debt with face value $\hat{D}_t$ is required to hold reserves $H_t \geq k_t \hat{D}_t$ where H_t is the value of the reserves in the end-of-period settlement. Suppose, furthermore, that reserves pay a gross nominal interest rate of $R_t^{cb} \geq R_t^m$ which means that $\theta_t R_t^m / R_t^{cb} \equiv 1$ units of cash must be paid in subperiod 1 to acquire a unit of reserves. Finally, suppose that a bank's reserve balance can be used to pay off its safe debt in subperiod 2, if the holders of the bank's short-term debt are not willing to roll it over, with one unit of reserves serving to retire one unit of short-term debt. Then the bank's collateral constraint again takes the form of equation (1.3), and the assets sold in a fire sale must satisfy constraint (1.6), where now $D_t \equiv \hat{D}_t - H_t$ is short-term debt issuance not covered by the bank's reserve balance. The funds obtained by the bank with which to purchase additional assets in subperiod 1 are only $\hat{D}_t - \theta_t H_t$ owing to the need to acquire reserves with some of the proceeds of the debt issuance. This quantity can alternatively be expressed as $\xi_t D_t$ where

$$\xi_t \equiv \frac{1 - k_t \theta_t}{1 - k_t} \leq 1.$$

If we assume that $k_t \theta_t \leq 1$ so that it is possible for the bank to acquire the required reserves out of the proceeds of its short-term debt issuance,[49] then $\xi_t \geq 0$ as assumed above. Thus, reserve requirements are an example of the kind of macroprudential policy that can be modeled in the way proposed above (in the case that the interest rate paid on reserves is less than the rate paid on cash). In this case, ξ_t can be reduced either by reducing the interest rate R_t^{cb} paid on reserves (relative to the central bank's target for the interest rate paid on cash) or by increasing the required reserve ratio k_t.

The first-order conditions that characterize optimal household behavior are not changed by the introduction of macroprudential policy, except that equation (2.4) now takes the more general form

49. Tighter reserve requirements than this would have no effect, since when $k_t \theta_t = 1$ banks are already completely precluded from raising any funds by issuing short-term debt.

$$\xi_t \phi_{1t} = (1 - p)\phi_{2t} + \mathrm{E}_t \phi_{6t}. \tag{4.1}$$

With this change, the derivation of the conditions for an intertemporal equilibrium proceeds as in section 2. The equilibrium paths of the endogenous variables now depend on the specification of the series $\{P_t, \tilde{m}_t, \xi_t\}$, representing three distinct dimensions of policy: conventional monetary policy; the determination of the outside supply of safe assets by debt management policy and quantitative easing; and macroprudential policy.

This more general version of the model yields a three-parameter family of stationary equilibrium, indexed by stationary values Π, $\tilde{m}$, and ξ. The stationary real allocation of resources depends only the stationary values of $\tilde{m}$ and ξ. The previous section showed how variation in $\tilde{m}$ (or alternatively, in R^m/Π) affects the stationary equilibrium values of real variables and relative prices, for a fixed value of ξ. (That discussion used the assumption that $\xi = 1$, but similar qualitative conclusions would obtain in the case of any fixed value of ξ). Here we consider instead the consequences of varying the stationary value of ξ and in particular, the extent to which the effects of varying the strength of macroprudential policy (perhaps by relaxing or tightening reserve requirements) are equivalent to the effects of variations in the supply of reserves, discussed in the previous section.

Figure 6 shows again the stationary values of the variables plotted in figure 3 (which compares short-term debt issuance by banks with the total supply of cash and with the available collateral to back such issuance), for alternative constant values of $\xi \leq 1$, holding fixed the target that determines the central bank's balance sheet policy (which is here assumed to be a fixed target for the term premium associated with longer-term bonds, or equivalently a fixed value of R^m/Π). In the case shown in the figure, the target for R^m/Π is low enough that, in the absence of any reserve requirement or other regulation of short-term debt issuance by banks (that is, the case $\xi = 1$), the stationary equilibrium is of the low-cash-return type discussed in the previous section; that is, the incentive for short-term debt issuance by banks is great enough for the collateral constraint to bind, resulting in overvaluation and oversupply of durables in subperiod 1. I consider this case for the numerical illustration because it is the case in which there is the most reason to be interested in whether macroprudential policy can reduce the distortions resulting from banks' excessive incentive to issue short-term debt. The corresponding stationary values for the market valuation of durables are shown in figure 7.

Figure 6. Short-Term Debt Issuance by Banks in Alternative Stationary Equilibria Corresponding to Different Constant Values of ξ, for a Fixed Value of R^m/Π

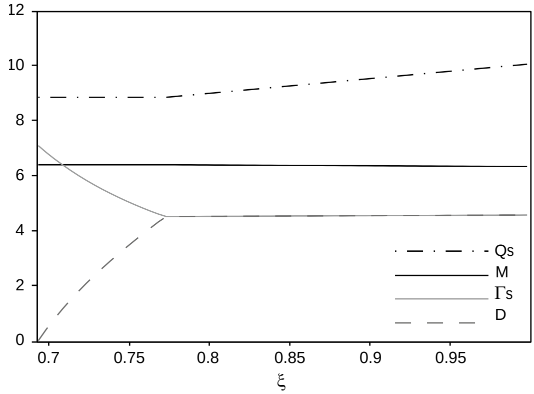
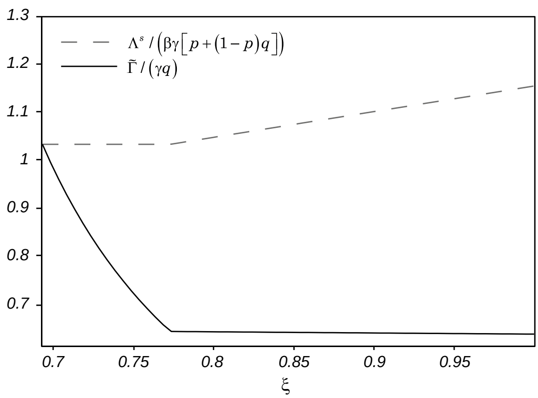


Figure 7. The Initial Overvaluation of Durables and Their Subsequent Undervaluation in the Event of a Crisis,[a] in Alternative Stationary Equilibria Corresponding to Different Constant Values of ξ



a. The dashed line plots the degree of initial overvaluation of durables; the solid line plots the degree of their subsequent undervaluation in the event of a crisis. The figure uses the same fixed value of R^m/Π as in figure 6.

Figure 6 shows that as the tax rate on short-term debt issuance increases (or the effective tax rate, by increasing the required reserve ratio or reducing the rate of interest paid on reserves), which lowers ξ, the stationary value of $\tilde{D}$ falls. For a sufficiently large tax rate (the case of ξ less than 0.77, in the numerical example), the collateral constraint ceases to bind; this implies that durables are no longer overvalued in subperiod 1, as shown in figure 7. In the case of an even larger tax rate (though still less than 100 percent taxation of the proceeds from issuing short-term debt), short-term debt financing of banks is completely driven out ($\tilde{D} = 0$), because the macroprudential tax fully offsets the value of the money premium to issuers of financial claims that can be used as cash. (In the numerical example, this occurs when $\xi = 0.69$, the left boundary of the figures.) When this occurs, bankers no longer have to sell assets in a fire sale, even if the crisis state occurs, and the undervaluation of durables in the crisis state is eliminated, as is also shown in figure 7. Further reductions in ξ below this value are irrelevant, as banks' issuance of short-term debt cannot be further reduced.

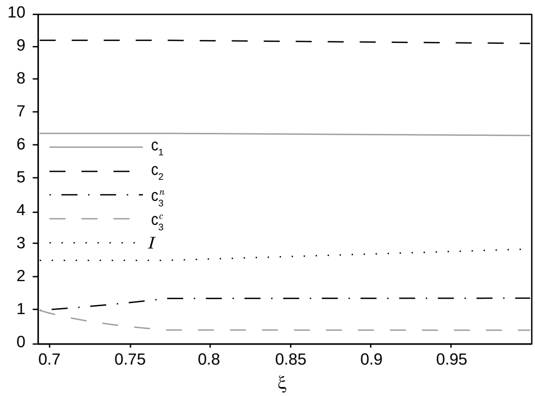
The implications of these alternative equilibria for the allocation of resources are shown in figure 8. Because balance sheet policy is used to fix the value of R^m/Π, the stationary value of $\tilde{\phi}_1$ and hence the stationary value of $\tilde{\phi}_4$ are unaffected by changing ξ. This means that the degree of inefficiency in the allocation of expenditure between cash and credit goods (as measured by the degree to which the marginal rate of substitution u_1/u_2 is greater than 1, the relative cost of producing them) is unaffected, so the equilibrium levels of production of cash and credit goods are little affected. However, as ξ is decreased from 1 (while still greater than 0.77), the degree of inefficient overproduction of investment goods is reduced, owing to the decrease in the degree to which banks are willing to pay to relax their collateral constraints. (Once ξ is less than 0.77, the collateral constraint no longer binds, as shown in figure 6, such that further reductions in ξ produce no further reductions in this distortion.) Moreover, because reductions in ξ reduce short-term debt issuance (as long as ξ remains greater than 0.69) and, with that, the value of s^*, they reduce the degree of inefficiency in the production and consumption of special goods: both c_3^c and c_3^n move closer to the efficient level of 1, which they reach exactly if ξ is reduced to 0.69.

We can now ask to what extent the effects of expanding the supply of central bank liabilities through quantitative easing are equivalent, or even similar, to the effects of relaxing a reserve requirement that limits banks' ability to issue money-like liabilities. In the context of the model, the former sort of policy corresponds to an increase in $\tilde{m}$

(resulting in an increase in R^m/Π if there is not already satiation in cash balances), which can be implemented while keeping ξ fixed; the latter sort of policy corresponds to an increase in ξ (assuming a reserve requirement tight enough to bind), which can be implemented while keeping $\tilde{m}$ fixed or, with an appropriate adjustment of the central bank's balance sheet, while keeping R^m/Π fixed.

A comparison of figures 6–8 with figures 2–4 shows that not only are these two policies not equivalent, their effects are in many respects exactly the opposite. An expansion of the central bank's balance sheet while fixing ξ corresponds to a movement from left to right in figures 2–4: short-term debt issuance by private banks falls, both the overvaluation of durables in subperiod 1 and their undervaluation in the event of a crisis are reduced, the overproduction of durables is reduced, and the level of production of special goods in both the c and n states becomes more nearly efficient. A relaxation of a binding reserve requirement while fixing R^m/Π corresponds instead to a movement from left to right in figures 6–8, which essentially reverses the effects seen in the earlier figures: short-term debt issuance by private banks increases, both the overvaluation of durables in subperiod 1 and their undervaluation in the crisis state increase, the overproduction of durables in increased, and the level of production of special goods is progressively more severely distorted.

Figure 8. The Allocation of Resources under Alternative Stationary Equilibria[a]



a. The figure uses the same set of alternative stationary equilibria shown in figures 6 and 7.

In fact, both an expansion of the outside supply of safe assets and a tightening of reserve requirements (or other forms of macroprudential policy) have similar consequences for financial stability, insofar as both reduce the extent to which banks finance themselves by issuing short-term safe debt. Either of these policies, pursued far enough, will completely eliminate private issuance of money-like claims (the right boundary of figures 2–4 or the left boundary of figures 6–8) and consequently eliminate the distortions resulting from the risk of a fire sale of assets and from the desire of bankers to obtain assets that can be used to collateralize short-term debt issuance. Thus, each of these policies, either of which is welfare-enhancing (when not irrelevant), can serve to some extent as a substitute for the other. However, while a sufficient increase in the outside supply of safe assets would make macroprudential policy unnecessary in the model (since private issuance of money-like claims can be completely eliminated, even if $\xi = 1$), the reverse is not true: even a macroprudential policy of the maximum possible stringency (one that completely prevents private issuance of STSIs) will not eliminate the welfare gains from further expansion of the outside supply of safe assets, since even when $\tilde{D} = 0$ (as in the case with $\xi = 0.69$ in figures 6–8), there will still be inefficient underconsumption of cash goods, owing to the binding cash-in-advance constraint, as long as $\tilde{m} < m^*$.

5. Conventional and Unconventional Monetary Policy in the Presence of Nominal Rigidities

In the analysis thus far, all prices have been assumed to be perfectly flexible and to clear markets each period. In such a model, conventional monetary policy has no real effects, but rather only influences the general level of prices in terms of the monetary unit. It follows that conventional monetary policy has no consequences for financial stability. This establishes a sharp distinction between the effects of conventional monetary policy (interest rate policy) and balance sheet policy, since as shown above, the central bank's balance sheet (specifically, the *real* supply of safe assets by the central bank) does have consequences for financial stability.

Such an analysis is adequate for consideration of the possible long-run stationary equilibria achievable under alternative policies, as in the previous two sections. But it does not suffice for an analysis of the considerations at play when alternative dimensions of monetary policy are used to address short-run macroeconomic stabilization objectives, and this is the context in which central banks' recent experiments with quantitative easing have been conducted. To address the issues raised by recent policies, we need to consider the consequences for financial stability of using quantitative easing as a substitute for an interest rate cut that is prevented by the effective lower bound on short-term nominal interest rates, in a situation where such an interest rate cut would otherwise be desired in order to achieve a higher level of output.

The notion that an interest rate cut would be desired in order to increase real activity only makes sense in the presence of nominal rigidities of some kind. Here I discuss a simple extension of the model presented above, which shows how sticky prices allow conventional monetary policy to have real effects in the short run while only affecting the general price level in the long run. This allows the comparison of the effects of quantitative easing and those of an interest rate cut, with respect to both the effects of these policies on aggregate demand and their consequences for financial stability.

5.1 Equilibrium with a Sticky Price for Normal Goods

Only the price P_t of normal goods must be set in advance, while the prices of special goods, durable goods, and all financial assets are assumed to be perfectly flexible, as above. (Because all three types of normal goods are perfect substitutes from the standpoint of their suppliers, I assume that a single price P_t is posted, at which goods of any of these types can be purchased, and the buyer determines which type of good will be obtained.) For simplicity, I also consider here the case of a single unexpected aggregate shock (apart from the kind of uncertainty represented in figure 1) at some date t, in response to which monetary policy (both interest rate policy and balance sheet policy) may be adjusted; there is no further uncertainty (except for the kind depicted in figure 1) about how the economy will evolve after this shock occurs, and the shock is completely unanticipated prior to its occurrence.

The fact that the shock is completely unexpected means that before it occurs, people expect an equilibrium in which there will never be any random developments except the kind depicted in figure 1. This equilibrium can be assumed to be a stationary equilibrium of the kind described in section 3. In such an equilibrium, the price P_τ of normal goods in any period τ is a deterministic function of time; it does not depend on which state is reached in subperiod 2 of period τ, nor does it depend on the history ξ_τ of states revealed in previous periods. Hence, the same price P_τ is set for normal goods in all periods $\tau \leq t$ as would clear markets in the flexible-price stationary equilibrium analyzed above, even if the price P_τ must be set before subperiod 2 of period τ is reached. For purposes of the present discussion, it is not necessary to define how exactly the predetermined price of normal goods is determined, beyond the assumption that in an environment where the future is perfectly predictable (except for the uncertainty each period depicted in figure 1), the price that is set each period is the one that would clear the market for normal goods.

Let us suppose that period t is one in which no crisis occurs in subperiod 2 (though it is not known up until this time that this would be the case). But let us also suppose that in subperiod 2 of period t, an unexpected shock occurs, as a result of which the utility of cash and credit goods consumption is equal to $\chi u(c_{1t}, c_{2t})$, and the disutility of supplying normal goods is equal to $\chi v(Y_t)$, for some factor $\chi > 0$ that need not equal 1; the other components of the utility function are unaffected by the shock. The factor χ is assumed to take a value different from 1 only in period t (and prior to period t, it is assumed to equal 1 with probability 1 in period t as well). The point of assuming a shock of this particular type is that for a given level of production of investment goods, the efficient level of production and consumption of cash and credit goods would not be changed by the shock χ; however, the real interest rate required to sustain that level of demand will change (will be lower if χ is lower). Hence, the shock χ represents a demand disturbance to which it would be desirable to respond by lowering interest rates, if this is not precluded by the interest rate lower bound.

Both conventional monetary policy and balance sheet policy are allowed to respond to the occurrence of the shock, though their paths are assumed to be perfectly predictable from then on, as with all other exogenous variables. Both R^m_τ and $\tilde{m}_\tau$ are determined in subperiod 2 of period $\tau - 1$. Hence, neither R^m_τ nor $\tilde{m}_t$ can be affected by the value of χ; these variables are both equal to their values in the stationary equilibrium. But R^m_τ and $\tilde{m}_\tau$ can both differ from their stationary equilibrium values in periods $\tau \geq t + 1$.

For simplicity, I consider here only policy responses to the shock of a special sort. We continue to suppose that from period $t + 1$ onward, conventional monetary policy (that is, the choice of $R^m_\tau + 1$ for all $\tau \geq t + 1$) is used to ensure that the path of normal goods prices $\{P_\tau\}$ grows at the constant rate π^* in all periods $\tau \geq t+1$.[50] Moreover, balance sheet policy is used to achieve a real outside supply of cash $m_{\tau+1}$ equal to the stationary equilibrium value $\tilde{m}$ for all $\tau \geq t+1$. The set of alternative monetary policies considered can then be reduced to a two-parameter family, corresponding to different possible choices of R^m_{t+1} and $\tilde{m}_{t+1}$ (both of which must be chosen in subperiod 2 of period t, but which may depend on the value of χ).[51]

Because the price P_t has been fixed in advance, it is assumed to be independent of the value of χ and equal to the price associated with the stationary equilibrium that had previously been expected to continue. Once the shock χ occurs, there is no further uncertainty about how the economy will evolve from then on (except the uncertainty depicted in figure 1). Hence, the price P_τ of normal goods in each period $\tau \geq t+1$ is set so as to clear the market for normal goods in that period. (While P_τ must be set prior to subperiod 2 of period τ, it is not set prior to subperiod 2 of period $\tau-1$.[52]) Thus, in the equilibrium considered in this section, the only period in which the market for normal goods need not clear is period t (the period in which the shock χ occurs); in that period, P_t is set at the level that would clear the market in the event that $\chi = 1$.

More generally, all variables that are determined in subperiod 1 of period t, or earlier, are assumed to be determined as in the equilibrium in which $\chi = 1$ is expected (that is, as in the stationary equilibrium with flexible prices implied by the initial policy). Thus, the values of A_t, M_t, B_t, D_t, F_t, s_t, and Q_t are unaffected by the shock, in addition to P_t and all variables dated $t-1$ or earlier. Instead, the variables c_{1t}, c_{2t}, c_{3t}, I_t, Y_t, x_t, and $\tilde{P}$ as well as all variables dated $t+1$ or later, are determined in a way that takes account of the occurrence of the shock

50. The value of P_{t+1} is set in advance on the basis of expectations about the demand for normal goods in period $t+1$, which will depend on the interest rate R^m_{t+2} because of condition (2.15). Thus, the rule for setting $R^m_{\tau+1}$ in periods $\tau \geq t+1$ can be used to ensure that the market-clearing price for normal goods in all periods $\tau \geq t+1$ is consistent with the inflation target. This desideratum leaves the value of R^m_{t+1} undetermined. The value of P_t reflects expectations about how R^m_{t+1} would be set; but these are expectations about monetary policy in period t that were held prior to the unexpected shock, which may not be confirmed, as a result of the shock.

51. For simplicity, in this section I abstract from the possible use of macroprudential policy, as in sections 1-3; that is, the discussion considers only equilibria in which $\xi_t = 1$ at all times.

52. This means that the length of time for which prices are sticky is limited in the proposed model. A quantitatively realistic model would doubtless need to allow some prices to remain fixed for a longer period, but the simple case considered here suffices to illustrate the qualitative effects of temporary stickiness of prices.

χ.[53] The Lagrange multipliers ϕ_{4t}, ϕ_{5t}, and ϕ_{6t} are jointly determined with this latter set of variables (as well as the Lagrange multipliers for later periods).

The variables that are affected by the shock χ are determined by a system of intertemporal equilibrium conditions of the form stated earlier, with the following exceptions. First, the fact that the suppliers of normal goods must supply whatever quantity of such goods is demanded at the predetermined price P_t means that the first-order condition (2.13) need not be satisfied in period t ex post (that is, after the shock χ occurs). However, the other first-order conditions for optimal household behavior stated above continue to apply, and condition (2.13) also must hold in periods $t + 1$ and later (since normal goods prices in those periods are set in a way that clears the market). Thus, we drop one (but only one) of the conditions that would determine a flexible-price intertemporal equilibrium from subperiod 2 of period t onward, replacing it by the requirement that P_t equal a predetermined value, whether this clears the market for normal goods or not. Second, the partial derivatives $u_i(c_1, c_2)$ in first-order conditions (2.9)–(2.10) are replaced by $\chi u_i(c_1, c_2)$ (for $i = 1,2$) in period t only. All other first-order and market-clearing conditions continue to take the forms stated above.

The demand for cash and credit goods in period t is then given by

$$c_{1t} = c_1(\lambda_t/\chi;\, M_t/P_t)$$

and

$$c_{2t} = c_2(\lambda_t/\chi;\, M_t/P_t),$$

where M_t/P_t is unaffected by the shock. Aggregate demand for normal goods in period t is accordingly

$$Y_t = c_1(\lambda_t/\chi;\, M_t/P_t) + c_2(\lambda_t/\chi;\, M_t/P_t) + I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1}). \tag{5.1}$$

Since $c_1(\lambda;m)$ and $c_2(\lambda;m)$ are both nonincreasing functions of λ and at least c_2 must be decreasing, it follows that aggregate demand is a monotonically increasing function of χ, for given values of λ_t and λ_{t+1}.

53. The variables s_t^* and Γ_t are undefined, as we have assumed that the crisis state does not occur in period t.

Condition (2.40) continues to be a requirement for equilibrium, as a result of which it must be the case that

$$\lambda_t = \beta\lambda_{t+1}\hat{\phi}_1(\lambda_{t+1}; F(I(\lambda_t;\lambda_{t+1},\tilde{m}_{t+1})),\tilde{m}_{t+1})R^m_{t+1}\frac{P_t}{P_{t+1}}. \tag{5.2}$$

This equation indicates how the choice of R^m_{t+1} in period t affects the value of λ_t —and through it aggregate demand Y_t— for given expectations about conditions in period $t+1$. If the price P_t were required to clear the market for normal goods, substitution of equation (5.1) into (2.13) would yield a condition to determine the required value of λ_t in equilibrium; equation (5.2) would then indicate the interest rate R^m_{t+1} required to achieve the price-level target P_t. Under the assumption that P_t is predetermined and need not clear the market, it is possible for R^m_{t+1} to change in response to the shock, resulting in a value of Y_t that need not satisfy the voluntary supply condition (2.13).

In each period from $t+1$ onward, we effectively have flexible prices, so that condition (2.39) is again required for equilibrium. Thus, for any specification of R^m_{t+1} and of the path $\{\tilde{m}_\tau\}$ for all $\tau \geq t+1$, the equilibrium sequence $\{\lambda_t\}$ for $\tau \geq t$ is determined by condition (5.2) and the sequence of conditions of the form (2.39) for each period from $t+1$ onward. Given a solution for the sequence $\{\lambda_\tau\}$, aggregate demand for normal goods is determined by equation (5.1) in period t and by equation (2.37) in each period from $t+1$ onward. The implied equilibrium values of other variables are then determined as described in section 2.

5.2 Real Effects of Conventional and Unconventional Monetary Policy

The effects of quantitative easing can now be compared with those of conventional interest rate policy, as possible responses to a shock χ. If both R^m_{t+1} and the path $\{\tilde{m}_\tau\}$ for $\tau \geq t+1$ remain fixed at the values associated with the stationary equilibrium in which there is no shock, then the values $\lambda_\tau = \bar{\lambda}$ for all $\tau \geq t$ will satisfy condition (2.39) in period t and condition (2.39) for each of the periods $t+1$ and later, where $\bar{\lambda}$ is the constant value of λ_τ in the stationary equilibrium. Aggregate demand for normal goods in period t is then given by equation (5.1). If $\chi = 0$, this implies $Y_t = \bar{Y}$ the constant level of output in the stationary equilibrium. If instead $\chi < 0$, then $Y_t < \bar{Y}$. This reduction in the production of normal goods will be inefficient, since it implies that

$$u_2(c_{1t}, c_{2t}) = \chi\bar{\lambda} < \bar{\lambda} = v'(\bar{Y}) \leq v'(Y_t),$$

so that the marginal utility of additional consumption of normal goods would exceed the marginal disutility of supplying them.

We consider now the extent to which monetary policy can be used to respond to such a shock. In addition to the effects of policy on production and consumption, we are interested in how each of the possible dimensions of central bank policy influence financial conditions. Two measures of financial conditions are especially useful. One is the size of the money premium earned by cash, which can be measured by the extent to which the ratio

$$\frac{E_t[R^b_{t+1}]}{R^m_{t+1}} = \tilde{\phi}_{1,t+1}$$

is greater than one. This is a measure of financial conditions that determines the incentives for short-term debt issuance by banks. Another important measure is the expected one-period real return on longer-term bonds,

$$1 + \bar{r}^b_{t+1} \equiv E_{t+1}\left[\frac{R^b_{t+1}P_t}{P_{t+1}}\right] = \frac{\lambda_t}{\beta\bar{\lambda}}.$$

(This aspect of financial conditions can alternatively be measured by the value of λ_t.) This is the measure of financial conditions that is relevant for determining the aggregate demand for nondurable normal goods, as a result of equation (5.1). Below we analyze the effects of each of the dimensions of policy on both of these measures of financial conditions.

5.2.1 Conventional monetary policy

Conventional monetary policy can be used to mitigate the effects of a χ shock by lowering R_t^m (if this is not prevented by the lower bound on the nominal interest rate). The effects of such policy are most easily seen in the special case that $v(Y)$ is linear, so that $v'(Y) = \bar{\lambda}$ regardless of the value of Y. Then condition (2.39) requires that $\lambda_{t+1} = \bar{\lambda}$ and equation (5.2) reduces to

$$\lambda_t = \beta\bar{\lambda}\hat{\phi}_1(\bar{\lambda}; F(I_t), \tilde{m}_{t+1})R^m_{t+1} / \Pi. \tag{5.3}$$

Here the target gross inflation rate Π has been substituted for P_{t+1}/P_t, on the assumption that interest rate policy in period $t+1$ is used to ensure that the target inflation rate is realized, regardless of other conditions.

For simplicity, I only discuss the case of an equilibrium in which bankers' collateral constraint binds in period $t+1$, so that durables are overvalued in subperiod 1. This is the case in which risks to financial stability are of the greatest concern. It follows from equation (2.29) that

$$M_t / P_t = \tilde{m}_t + \lambda_t^{-1}\tilde{\Gamma}(s_t)s_t$$

This, together with the fact that $\tilde{\phi}_1(\lambda;m)$ is decreasing in both arguments (as shown in section 2), can be used to conclude that $\tilde{\phi}_1(\lambda;s,\tilde{m})$ will be a decreasing function of both s and $\tilde{m}$, for any fixed value of λ.

In addition, equation (2.35) requires that

$$\lambda_t = \beta\varphi(\bar{\lambda}; F(I_t), \tilde{m}_{t+1})F'(I_t). \tag{5.4}$$

This establishes an equilibrium relationship between investment demand I_t and financial conditions as measured by λ_t (although the value of $\tilde{m}_{t+1}$ remains of independent relevance). Equating the right-hand sides of equations (5.3) and (5.4) shows that equilibrium investment I_t must satisfy

$$\hat{\phi}_1(\bar{\lambda}; F(I_t), \tilde{m}_{t+1}) = \frac{\Pi}{\bar{\lambda}R^m_{t+1}}\varphi(\bar{\lambda}; F(I_t), \tilde{m}_{t+1})F'(I_t). \tag{5.5}$$

Again restricting attention to the case where bankers' collateral constraint binds in period $t+1$, the function $\varphi(\lambda;s,\tilde{m})$ can alternatively be expressed in terms of the function $\tilde{\phi}_1$. Note that

$$\begin{aligned}
\tilde{\phi}_{1t}\tilde{Q}_t &= \tilde{Q}^* + \left(1-p\right)\tilde{\phi}_{3t}\tilde{\Gamma}_t \\
&= \tilde{Q}^* + \left(1-p\right)\left[\left(\tilde{\phi}_{2t}+1\right)\tilde{\Gamma}_t - \gamma q\right] \\
&= \gamma p + \left(\tilde{\phi}_{1t} - p\right)\tilde{\Gamma}_t,
\end{aligned}$$

using equations (2.5), (2.6), and (2.4) in succession. It follows that the function φ can be expressed as

$$\varphi(\lambda; s, \tilde{m}) \equiv \hat{\varphi}(\hat{\phi}_1(\lambda; s, \tilde{m}); s), \tag{5.6}$$

where

$$\hat{\varphi}(\tilde{\phi}_1; s) \equiv \gamma p + \left(\tilde{\phi}_1 - p\right)\tilde{\Gamma}(s). \tag{5.7}$$

Condition (5.5) can then be written alternatively in the form

$$\tilde{\phi}_{1,t+1} = \frac{\Pi}{\bar{\lambda} R^m_{t+1}} \hat{\varphi}(\tilde{\phi}_{1,t+1}; F(I_t)) F'(I_t), \tag{5.8}$$

using equation (5.6). This describes a relationship that must exist between investment demand I_t and the money premium $\tilde{\phi}_{1\,t+1}$ in the case of any given specification of conventional monetary policy. This relationship is unaffected by the value of $\tilde{m}_{t+1}$, which is relevant for the discussion of quantitative easing below.

Because $\tilde{\Gamma}(s) \leq \gamma q < \gamma$, equation (5.7) implies that

$$0 < \frac{\partial \hat{\varphi}}{\partial \tilde{\phi}_1} < \frac{\hat{\varphi}}{\tilde{\phi}_1}, \qquad \frac{\partial \hat{\varphi}}{\partial s} < 0.$$

It follows that equation (5.8) implicitly defines a function

$$\tilde{\phi}_{1,t+1} = \bar{\phi}_1(I_t, R^m_{t+1}) \tag{5.9}$$

which is decreasing in both arguments.

This shows, together with equation (2.34), that the equilibrium level of investment I_t must satisfy

$$\hat{\phi}_1(\bar{\lambda}; F(I_t), \tilde{m}_{t+1}) = \bar{\phi}_1(I_t, R^m_{t+1}). \tag{5.10}$$

This can be solved for the effects of monetary policy on investment. Given that $\hat{\phi}_1(\lambda; s, m)$ is a decreasing function of s (when the collateral constraint binds), both sides of equation (5.10) are decreasing functions of I_t.

The comparative statics of I_t in response to a change in either R^m_{t+1} or $\tilde{m}_{t+1}$ then depend on the relative slopes of these two schedules. I assume that in the initial equilibrium, relative to which we wish to consider the effects of a change in monetary policy,

$$\frac{\partial \bar{\phi}_1}{\partial I} < \frac{\partial \hat{\phi}_1}{\partial I} < 0, \tag{5.11}$$

as shown in the upper part of figure 9. In this case, we obtain the conventional signs for the short-run effects of interest rate policy.[54] In particular, because $\bar{\phi}_1(I, R^m)$ is a decreasing function of R^m a reduction of R^m_{t+1} will increase I_t, as shown in the figure.

It also follows from equation (5.6) and the fact that $\hat{\phi}_1(\lambda; s, m)$ is a decreasing function of s that $\varphi(\lambda; s, \tilde{m})$ is also a decreasing function of s. Thus, the right-hand side of equation (5.4) is a decreasing function of I_t, for any fixed value of $\tilde{m}_{t+1}$. Hence, equation (5.4) establishes an inverse relationship between λ_t and I_t that must hold regardless of the value chosen for R^m_{t+1}. This relationship is graphed in the lower part of figure 9. Since a reduction in R^m_{t+1} increases I_t, it must also reduce λ_t, as shown in the figure. This, in turn, will imply an increase in Y_t, because of equation (5.1).[55]

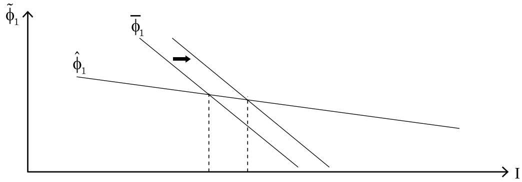
While interest rate policy can be used to stimulate aggregate demand in this way, and so to reduce some of the distortions created by the demand shock χ, it has the side effect of increasing risks to financial stability. Part of the increase in aggregate demand associated with a reduction of λ_t will be an increase in the production of investment goods, as a result of which s_{t+1} will be higher. In the case of an equilibrium in which the collateral constraint binds, this will mean a correspondingly higher value of s^*_{t+1}, as a consequence of which the degree of undervaluation of durables in the event of a crisis and fire sale will be more severe. Thus, in the sticky-price version of the model, it is indeed the case that reducing short-term nominal interest rates increases risk-taking by banks in a way that makes the distortions associated with a crisis more severe, should one occur.

54. If the inequality (5.11) is reversed, the model would imply that a reduction in the interest rate on cash is associated with a decrease, rather than an increase, in aggregate demand. Condition (5.4) establishes an inverse relationship between λ_t and I_t, regardless of whether equation (5.11) holds. Therefore, if a reduction of R^m_{t+1} were associated with a reduction of I_t, this would have to mean an increase in λ_t and hence a decrease in all three terms on the right-hand side of equation (5.1). Such an effect would be contrary to familiar evidence regarding the effects of interest rate policy, and it would also preclude the possibility of a liquidity trap in which the lower bound on nominal interest rates prevents rates from being cut enough to maintain a desired level of real activity.

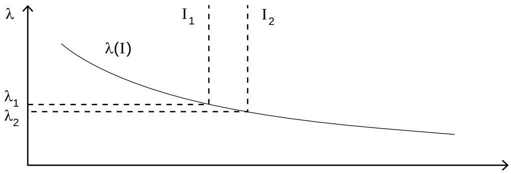
55. The expression $I(\lambda_t; \lambda_{t+1}, \tilde{m}_{t+1})$ in this equation is just the quantity I_t which must increase as shown earlier.

Figure 9. The Effects of a Reduction in the Interest Rate Paid on Safe Assets (R^m_{t+1})

A. Effect on the Money Premium and Equilibrium Investment in Risky Real Assets



B. Effect on Financial Conditions as Measured by λ_t



5.2.2 Effects of unconventional policies

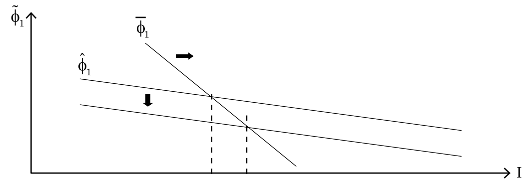
In the event that the lower bound on interest rates prevents R^m_{t+1} from being reduced to the extent that would be necessary to maintain aggregate demand at the desired level, quantitative easing provides an alternative channel through which aggregate demand may be increased. Like conventional interest rate policy, an expansion of the supply of short-term safe assets by the central bank affects aggregate demand by easing financial conditions, as indicated by a reduction in λ_t (which can be thought of as the price of a particular very-long-duration indexed bond).

Consider the effects of an increase in $\tilde{m}_{t+1}$, holding R^m_{t+1} fixed. The effect on equilibrium investment demand can again be determined using equation (5.10). The schedule corresponding to the right-hand side of this equation does not shift as a result of an increase in $\tilde{m}_{t+1}$, but the fact that $\hat{\phi}_1(\lambda; s, m)$ is a decreasing function of $\tilde{m}$ means that the schedule corresponding to the left-hand side of the equation shifts down for each possible value of I_t, as shown now in panel A of figure 10. Then again, assuming that the relative slopes of the two schedules are given by equation (5.11), it is again possible to conclude that I_t must increase while $\tilde{\phi}_{1\,t+1}$ must decrease.

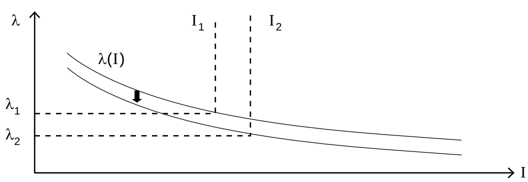
Equation (5.4) can also be used to determine the change in λ_t required by a given size increase in I_t. As argued above, an increase in I_t reduces the right-hand side of this equation, for any given value of $\tilde{m}_{t+1}$. In addition, equation (5.6), together with the result above that $\phi_1(\lambda; s, m)$ is a decreasing function of $\tilde{m}$, implies that the function $\varphi(\lambda; s, \tilde{m})$ is also a decreasing function of $\tilde{m}$. This means that the curve—the graph of equation (5.4)—plotted in panel B of figure 10 shifts down as a result of an increase in $\tilde{m}_{t+1}$. It then follows that λ_t is reduced by an increase in $\tilde{m}_{t+1}$, both because of the decrease in I_t (the shift along the curve) and because of the direct effect of an increase in $\tilde{m}_{t+1}$ (the downward shift of the curve).

Figure 10. The Effects of an Increase in the Central Bank Supply of Safe Liabilities $\tilde{m}_{t+1}$

A. Effect on the Money Premium and Equilibrium Investment in Risky Real Assets



B. Effect on Financial Conditions as Measured by λ_t



It follows that an increase in $\tilde{m}_{t+1}$ must loosen financial conditions, in the sense that λ_t is reduced. This, in turn, means that as in the case of an interest rate cut, Y_t must increase because of equation (5.1). Thus, the effects of quantitative easing are qualitatively similar to those of an interest rate cut: financial conditions are eased, the aggregate demand for normal goods increases (because of an increase in the demand for credit goods and an increase in the demand for investment goods), but at the same time risks to financial stability increase (because of an increase in short-term debt issuance by banks), leading to larger expected distortions in the event that a crisis state occurs in period $t + 1$.

Nonetheless, the two policies do not have quantitatively equivalent effects. A comparison of an interest rate cut (reduction in R^m_{t+1}) and an increase in the net supply of safe assets by the central bank (increase in $\tilde{m}_{t+1}$) that increase the equilibrium demand for investment goods I_t by the same amount shows that the increase in $\tilde{m}_{t+1}$ reduces λ_t by a greater amount.[56] This can be seen from the fact that equation (5.4) must apply in either case. If, by hypothesis, I_t increases by the same amount in both cases, then the only difference in the implied value for λ_t is that $\tilde{m}_{t+1}$ increases in the second case, but remains constant in the first—and this implies a lower value of λ_t in the second case. This means that in the case of quantitative easing, a greater share of the total increase in aggregate demand comes from increased demand for credit goods, as opposed to increased demand for investment goods. Thus, a given degree of aggregate demand stimulus can be achieved with less risk to financial stability if it is brought about through an expansion of the central bank's balance sheet, rather than by cutting the interest rate paid on cash.

We can also consider the effects of aggregate demand stimulus through relaxation of macroprudential constraints (that is, an increase in ξ_{t+1}). Let us generalize the analysis presented in the earlier part of this section to allow for a macroprudential tax (or reserve requirement), so that ξ_t need not equal 1 (as assumed thus far in this section). Conditions (5.1), (5.3), and (5.4) continue to be required for an equilibrium, and the definition of the function $\hat{\phi}_1(\lambda; s, \tilde{m})$ is unchanged; but in equation (5.4), the expression $\varphi(\bar{\lambda}; F(I_t), \tilde{m}_{t+1})$ must be replaced by $\varphi(\bar{\lambda}; F(I_t), \tilde{m}_{t+1}, \xi_{t+1})$, where we define

56. Compare figures 9 and 10. In figure 10, the amount of quantitative easing is chosen so as to achieve the same increase in investment (from I_1 to I_2) as the interest rate cut in figure 9. The reduction in λ_t is instead larger (from λ_1 to $\lambda_3 < \lambda_2$).

$$\varphi(\lambda; s, \tilde{m}, \xi) \equiv \hat{\varphi}(\xi \hat{\phi}_1(\lambda; s, \tilde{m}); s),$$

generalizing (5.6).

Condition (5.8) then takes the more general form

$$\tilde{\phi}_{1,t+1} = \frac{\Pi}{\bar{\lambda} R^m_{t+1}} \hat{\varphi}(\xi_{t+1} \tilde{\phi}_{1,t+1}; F(I_t)) F'(I_t).$$

As above, this implicitly defines a function

$$\tilde{\phi}_{1,t+1} = \bar{\phi}_1(I_t, R^m_{t+1}, \xi_{t+1}), \tag{5.12}$$

where now $\bar{\phi}_1(I, R^m, \xi)$ is decreasing in I and R^m and increasing in ξ. The equilibrium level of investment is again determined by equation (5.10), but now the schedule corresponding to the left-hand side is shifted only by $\tilde{m}_{t+1}$, while the schedule corresponding to the right-hand side is shifted by changes in either R^m_{t+1} or ξ_{t+1}. To a linear approximation (which is to say, in the case of small enough policy changes), an increase in ξ_{t+1} (a relaxation of macroprudential policy, as by reducing the required reserve ratio) has the same effects on I_t and $\tilde{\phi}_{1\,t+1}$ as a certain size of cut in R^m_{t+1}.

Equation (5.12) can also be used to rewrite equation (5.3) in the form

$$\lambda_t = \beta \bar{\lambda} \bar{\phi}_1(I_t, R^m_{t+1}, \xi_{t+1}) R^m_{t+1} / \Pi. \tag{5.13}$$

It follows that since a relaxation of macroprudential policy reduces the value of $\tilde{\phi}_{1\,t+1}$, it must reduce the value of λ_t. Hence, it increases demand for credit goods and so must increase Y_t, like the other two policies just considered. It also follows from equation (5.13), however, that in the case of two policy changes (a cut in R^m_{t+1} or an increase in ξ_{t+1}) that reduce $\tilde{\phi}_{1\,t+1}$ to the same extent and that therefore reduce $\bar{\phi}_1(I_t, R^m_{t+1}, \xi_{t+1})$ to the same extent, the interest rate cut must reduce λ_t by more, thereby stimulating demand for credit goods to a greater extent. Thus, an even greater share of the increase in aggregate demand achieved by relaxing macroprudential policy comes from an increase in investment demand, as opposed to an increase in the demand for credit goods, than in the case of an increase in aggregate demand achieved by cutting the interest rate on cash.

This leads to an ordering of the three types of expansionary policy as follows: for a given degree of increase in aggregate demand, achieving it by increasing $\tilde{m}_{t+1}$ increases I_t the least, achieving it by reducing R^m_{t+1} increases I_t to an intermediate extent, and achieving it by increasing ξ_{t+1} increases I_t (and hence short-term debt issuance by banks and risks to financial stability) the most. One consequence of this is that increasing aggregate demand through monetary policy *need not* involve any increased risks to financial stability at all. For example, one might combine an increase in $\tilde{m}_{t+1}$ with a tightening of macroprudential policy (a reduction of ξ_{t+1}) that exactly offsets the effects of the quantitative easing on desired investment demand, so that there is no net change in I_t. Since the former policy change will reduce λ_t more than the latter policy change increases it, the net effect will be a loosening of financial conditions, with a corresponding increase in the demand for credit goods. Since there is (by hypothesis) no change in investment demand, aggregate demand Y_t will increase; but there will be no associated increase in s_{t+1}, and hence no increase in the severity of the distortions associated with a crisis state in period $t + 1$.

My conclusion is that while quantitative easing *may* increase risks to financial stability in the case that nominal rigidities allow short-run effects of monetary policy on aggregate demand, it *need not* have any such effect. If the increase in the central bank's balance sheet is combined with an increase in the interest rate paid on cash or a tightening of macroprudential policy to a sufficient extent, then it can increase aggregate demand without any adverse consequences for financial stability. It is particularly easy to achieve this outcome by combining the quantitative easing with macroprudential policy, if a suitable macroprudential instrument exists; for in the model, reduction of ξ_{t+1} provides an even greater disincentive to issuance of short-term debt by banks than does raising R^m_{t+1}, for a given degree of reduction in aggregate demand.

These results imply that quantitative easing may be a useful addition to a central bank's monetary policy toolkit, even when interest rate policy is not yet constrained by the effective lower bound on short-term nominal interest rates. In the case of a contractionary shock χ, the effects on aggregate demand can be offset purely through a reduction in R^m_{t+1}, if the lower bound does not prevent the size of rate cut that is needed; but such a response increases risks to financial stability more than necessary. One could alternatively counter the effects of the χ shock by increasing $\tilde{m}_{t+1}$, while leaving R^m_{t+1} unchanged; this would have the advantage of posing less of a threat to financial stability. Even better, one could combine a somewhat larger increase in $\tilde{m}_{t+1}$ with a tightening of macroprudential policy, allowing the effects of the χ shock on aggregate demand to be offset, with even less of an increased risk to financial stability, possibly none at all.

6. Conclusions

We can now assess the validity of the concerns about the consequences of quantitative easing for financial stability sketched in the introduction, in the light of the model just presented. The model is one in which monetary policy does indeed influence risks to financial stability; in particular, policies that loosen financial conditions, either by lowering the central bank's operating target for its policy rate (conventional monetary policy) or by relaxing reserve requirements (or other macroprudential constraints), should each increase the attractiveness of private issuance of money-like liabilities, resulting in increased leverage and as a consequence an increased risk of serious resource misallocation in the event of a funding crisis. This means that there can sometimes be a tension between the monetary policy that would be preferable strictly from the standpoint of aggregate demand management and inflation stabilization, on one hand, and the policy that would minimize risks to financial stability, on the other.

The question is whether it is correct to think of quantitative easing as a policy analogous to these, which poses similar risks to financial stability. The model implies that such an analogy is imperfect. A quantitative easing policy (which increases the public supply of safe assets through the issue of additional safe central bank liabilities, used to purchase assets that do not earn a similar safety premium) similarly increases aggregate demand by lowering the equilibrium rate of return on nonsafe assets. Unlike conventional monetary policy, however, it does this by lowering the equilibrium safety premium (by making safe assets less scarce), rather than by lowering the equilibrium return on safe assets; and this does not have the same consequences for financial stability. Lowering the equilibrium return on risky investments (such as the durable goods modeled here, which one may think of as housing) by lowering the return on safe assets works only insofar as the *increased* spread between the two returns that would result if the return on risky investments did not also fall increases the incentive to finance additional risky investment by issuing safe liabilities, thus increasing the leverage of the banks and the degree to which they engage in liquidity transformation. This results in a reduced equilibrium return on risky investment, but not by enough to fully eliminate the increased spread that induces banks to issue additional safe asset-backed liabilities. This mechanism necessarily increases the risk to financial stability at the same time as it increases aggregate demand. Quantitative easing instead *decreases* the spread between these two returns, at least in the absence of any change in the private supply of safe liabilities. This reduction in the spread reduces the incentive for private issuance of such liabilities. Reduced issuance of safe asset-backed liabilities by banks offsets some of the reduction in the spread, but it does not completely eliminate it, as otherwise banks would not have a reason to reduce their issuance. Hence, in this case, the reduction in the equilibrium return on risky investments is associated with a *reduction* of the incentive for liquidity transformation by banks, rather than an increase.

Similarly, quantitative easing increases the total supply of safe assets and so reduces the safety premium. In contrast to a reduction in reserve requirements (or relaxation of macroprudential policy), it achieves this by increasing the public supply of safe assets (and actually reducing the incentive for private issuance), rather than by increasing the incentive that banks have to finance risky investment by issuing safe asset-backed liabilities. Again, the consequences for the degree of liquidity transformation by the banking sector and the risk to financial stability are entirely different.

Likewise, quantitative easing eases financial conditions by reducing the spread between the required return on risky investments and the return on safe assets. This does not mean, however, that risk premiums are artificially reduced in a way that distorts incentives for prudent behavior, leading to excessive risk-taking. In the model presented here, quantitative easing reduces the safety premium, but it does so because the public supply of safe assets for private investors to hold is increased, not because anyone is misled into underestimating the degree of risk involved in undertaking risky investments. Moreover, the reduced spread reduces the incentive for private issuance of safe liabilities and instead favors financing investment through the issuance of nonsafe liabilities, which is desirable on financial stability grounds. Rather than threatening financial stability by encouraging more risk-taking, it favors stability by encouraging forms of financing that reduce the magnitude of the distortions associated with a funding crisis.

The model was used to compare the effects of three alternative policies that can increase aggregate demand by easing financial conditions: reducing the central bank's operating target for the nominal interest rate on safe assets (that is, conventional monetary policy, on the assumption that the zero lower bound does not yet preclude such easing); relaxing reserve requirements or other macroprudential constraints; and quantitative easing. Among these alternative policies, quantitative easing increases risks to financial stability the least, for any given degree of increase in aggregate demand. Not only does quantitative easing make it possible for a central bank to increase aggregate demand even when conventional monetary policy is constrained by the zero lower bound on nominal interest rates, but, at least in principle, the expansion in aggregate demand can be achieved without the collateral effect of greater risk to financial stability, provided that the increased supply of safe liabilities by the central bank is combined with a sufficient tightening of macroprudential measures. The latter measures alone would reduce aggregate demand, but when combined with quantitative easing, the net effect is an increase in aggregate demand, even when the degree of macroprudential tightening is enough to fully offset any increase in risks to financial stability as a result of the balance sheet policy.

This indicates that a concern for the effects of monetary policy on financial stability need not preclude using quantitative easing to stimulate aggregate demand in circumstances where (as in the United States in the aftermath of the recent crisis) conventional monetary policy is constrained by the zero lower bound. The fact that demand stimulus through quantitative easing poses smaller risks to financial stability than demand stimulus through lowering short-term nominal interest rates suggests that balance sheet policy may be a useful tool of monetary stabilization policy even when a central bank is far from the zero lower bound. In the model presented here, the aggregate demand stimulus achieved by lowering nominal interest rates increases the risk to financial stability more than would a quantitative easing policy that is equally effective in increasing aggregate demand. This implies that even if macroprudential policy is unavailable or ineffective, it should be possible to increase aggregate demand without increasing the risk to financial stability by combining expansionary balance sheet policy with an appropriate *increase* in the policy rate. In such a case, conventional monetary policy would essentially be used for macroprudential purposes (to control the risk to financial stability), while balance sheet policy is used for demand stabilization.

Further study of the effects of quantitative easing policies would therefore seem to be warranted, not simply for the sake of having a more effective policy toolkit for use the next time that conventional policy is again constrained by the zero lower bound, but also, arguably, to improve the conduct of stabilization policy under more normal circumstances as well. The availability of this additional dimension of monetary policy is particularly likely to be of use under circumstances where additional monetary stimulus through interest rate reduction is unattractive owing to concerns about financial stability. Such a situation could easily arise even when interest rates are well above their effective lower bound.

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