A Toolkit for Analyzing Alternative Policies in the Chilean Economy

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As noted by Leeper (1995) "the business pages of leading newspapers give the impression that the effects of alternative monetary policies on the macroeconomy are well understood and predictable." They tend "to write with great certainty that when the monetary authority raises interest rates it slows economic growth, and with it inflation, bidding down stocks and bonds. With equal certainty, press accounts report that the monetary policy responds to economic conditions." Statements like "the recent strength of the economy will prompt the monetary authority to raise interest rates as a preemptive strike against inflation" are not uncommon. With the economy responding to policy and policy responding to the economy, it is hard to tell what causes what. Chile is no exception: similar statements are frequently found in local newspapers.

There is no consensus, however, regarding the interaction of economic conditions and policy. In fact, while several academic papers directly or indirectly try to identify the effects of alternative policies, most of the results found are, to say the least, inconclusive.[1]

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1. Rosende and Herrera (1991), Rojas (1993), Eyzaguirre and Rojas (1996), Morandé and Schmidt-Hebbel (1997), Valdés (1998), Parrado (2001), and some specifications of Cabrera and Lagos (2002) find that output and inflation are affected by innovations in monetary policy. Mendoza and Fernández (1994), Morandé, García, and Johnson (1995), Calvo and Mendoza (1999), and some specifications of Cabrera and Lagos (2002) find complete ineffectiveness of the monetary policy to alter their trajectories.

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The understanding and measurement of the quantitative effects of monetary policy are essential for evaluating the relative merits of alternative policy arrangements, yet few papers address the issue in an integrated and consistent way.[2] This paper does just that by combining sound statistical representations with theoretical models. In contrast, most of the empirical literature focuses on providing statistical descriptions of the data, with no correspondence between the statistical model used to develop the stylized facts that are being explained and a theoretical model that is consistent with them.

The effects that different policy arrangements may have on the economy are usually quantified statistically using vector autoregressions (VARs). While this technique may prove to be valuable for forecasting purposes, it is difficult to obtain a correspondence between the impulse response functions that are derived from it and the economic principles that arise from contesting theories (Hamilton, 1994). As discussed below, the VAR impulse response functions may not have any meaningful interpretation given the identifying restrictions imposed on them (they may come from linear combinations of different shocks), and they cannot provide reliable estimates of the effects of alternative policies. From a theoretical standpoint, in turn, few papers use models derived from first principles to address their empirical implications in an integrated fashion, such that they are subject to the Lucas critique from the get go.

This paper overcomes these shortcomings by integrating statistical models that are able to replicate the intertemporal dynamics of key economic variables with dynamic, stochastic, optimizing models. In the presence of a rival theoretical model, the statistical description of the data provides an objective metric with which to evaluate their merits.

The paper is organized as follows: Section 1 briefly describes the main problems that traditional statistical models have when they are used to quantify the effects of alternative policies. Section 2 presents a statistical model that can be used as a metric with which to compare the empirical implications of alternative theoretical models. Section 3 describes and estimates a simple optimizing model for replicating the stylized facts reported in section 2. Section 4 concludes.

2. Schmidt-Hebbel and Servén (2000) develop a deterministic general equilibrium model in which they impose liquidity constraints and wage rigidities. This model is calibrated and presents exercises regarding the effects of alternative policies.

1. Identified VARs

VAR models have long been used to describe the dynamic interactions of key macroeconomic variables in an economy. Although these models have proved successful as forecasting tools, they rarely can be used to test competing theories and their results cannot be interpreted with sound economic principles. Some economists argue that this occurs because VARs are restricted versions of more structural models, since VARs usually ignore contemporaneous comovements, and they do not explicitly test any stance regarding the economic principles behind the dynamic interactions encountered. Furthermore, VAR models impose arbitrary decompositions on the variance-covariance matrix of the innovations (usually a Cholesky decomposition), making the impulse response functions sensitive to the ordering of the model. Several methodologies have been developed to overcome this shortcoming. However, these functions do not have any direct interpretation in terms of the dynamic consequences of shocks to any of the underlying innovations.[3]

Recently developed models try to overcome the shortcomings of traditional VARs. They are known as structural VARs (SVARs) or identified VARs (IVARs). The main characteristic of these models is that they nest traditional VARs and do not impose orthogonality restrictions among the contemporaneous interactions of the variables in the system. They also provide tools that can be used to conduct inference on the restrictions of competing statistical models and, in principle, provide estimates of the impulse response functions that are supposed to recover the underlying structure of the system.[4] Nevertheless, the robustness of the conclusions drawn from IVAR exercises is questionable, as noted by Cochrane (1998) and more forcefully by Cooley and Dwyer (1998).[5]

3. Pesaran and Shin (1998) develop what they call generalized impulse response functions, which provide impulse response functions that are invariant to the ordering of the unconstrained VAR.

4. Appendix A provides a brief description of the IVAR methodology.

5. The term structural VAR is misleading in the sense that it may give the impression that these statistical objects can be understood as representations of behavioral relations grounded on first principles. This is usually not the case, however, as discussed below. In this paper I prefer to use the term IVAR to make explicit that these models provide tests that can help to decompose impulse response functions in a more formal way, but no structural (behavioral) implications are drawn from them.

1.1 The Usual Practices

As discussed above, several studies attempt to characterize the dynamic consequences of alternative policies in the Chilean economy, but most use traditional VARs and are thus subject to the critiques outlined.[6] With rare exceptions (such as Calvo and Mendoza, 1999, and Valdés, 1998), the studies do not report confidence intervals for the impulse response functions. Furthermore, the studies that do report them rely on asymptotic approximations of the confidence intervals of the model, without performing formal tests for multivariate normality and vector white noise innovations or correcting for biases in the impulse response functions.[7] Confidence intervals based on asymptotic approximations can be deceiving when departures from normality are important, given that normality imposes symmetry on them. Asymmetries may also be present when nonlinear structures are important, and positive and negative shocks may thus imply completely different trajectories. In such cases, confidence intervals for the impulse response functions may still be constructed relying on bootstrapping (Sims and Zha, 1995). However, this practice is, itself, subject to two problems. First, most of the variables included in the unrestricted VAR are usually statistically nonsignificant, but the

6. Valdés (1998) estimated what he termed a semi-structural VAR model, but the identifying assumption he imposes makes it no different from a specific ordering of an unrestricted VAR model, and it is not what I understand as an IVAR. His restrictions correspond to a Cholesky factorization in which the variable used to measure the monetary policy stance comes first, thus making it exactly identified. Obviously, no formal tests against alternative orderings or identifying assumptions can possibly be made in this context. In fact, the impulse response of that model can be directly computed without estimating the parameters with the methodology described in appendix A. Other examples of such a practice can be found in García (2001) and Cabrera and Lagos (2002). Parrado (2001) uses the IVAR methodology, but his results are subject to other problems.

7. When estimating VARs or IVARs, it is often forgotten that they need to be correctly specified prior to conducting impulse response exercises. As a minimum, vector white noise innovations are needed—that is, innovations that are orthogonal not only to their own past, but also to the past of the other innovations of the system. Furthermore, given that the construction of confidence intervals for the impulse response functions generally rely on asymptotic approximations, formal tests for multivariate normality of the residuals should be conducted. In the former case, Ljung and Box tests cannot be applied, as they rely on univariate specifications; Wilks, Portmanteu or LRT tests should be employed (see Lütkepohl, 1991, for details). This fact is independent of the information criteria chosen to select a model, given that it is used only to account for parsimony. In the latter case, Jarque-Bera univariate tests for normality are not appropriate, and multivariate specification should be used (Doornik and Hansen, 1994).

bootstrapped model takes their point estimates as given and thus unnecessarily inflates the confidence intervals. Second, and more importantly, the confidence intervals usually considered are constructed using Efron's suggestion, although these intervals do not have the correct coverage if the distribution under consideration is asymmetric, as is well documented. Given that bootstrapping is used precisely for these purposes, Hall's confidence intervals are better suited for dealing with departures from normality.

Another important consideration that has to be taken into account is the way in which some of the previous studies deal with nonstationarities. As Sims, Stock, and Watson (1990) demonstrate, VAR estimates with some integrated series are super consistent, but they have nonstandard asymptotic distributions. Impulse response functions from this type of series can be constructed from Monte Carlo or bootstrap approximations (methods that are now readily available and can be routinely performed). If the nonstationarity comes from deterministic trends, however, incorrectly differentiating the series may impose nontrivial dynamics on the model. In particular, this would incorporate a unit-root-type of vector moving average (MA) process in the series, such that the use of ordinary least squares (OLS) estimation would not be advisable.[8] Care should thus be given to when and when not to differentiate a variable prior to estimating the VAR.

1.2 Unit Roots and Impulse Response Functions

With the exception of Parrado (2001), all of the studies discussed in the first section choose to differentiate the variable that captures the economy's level of activity (usually the monthly economic activity index, or Imacec). Typical examples of the unbelievable dynamics that result from impulse response functions using first differences on the scale variable can be found in Valdés (1998) and García (2001). Even if one assumes that their models were correctly specified in terms of lag selection and normality and that there were no biases associated with the estimated parameter, they find a significantly negative effect of what they refer to as the monetary policy innovation to the first difference of the scale variable. If the monetary policy innovation has

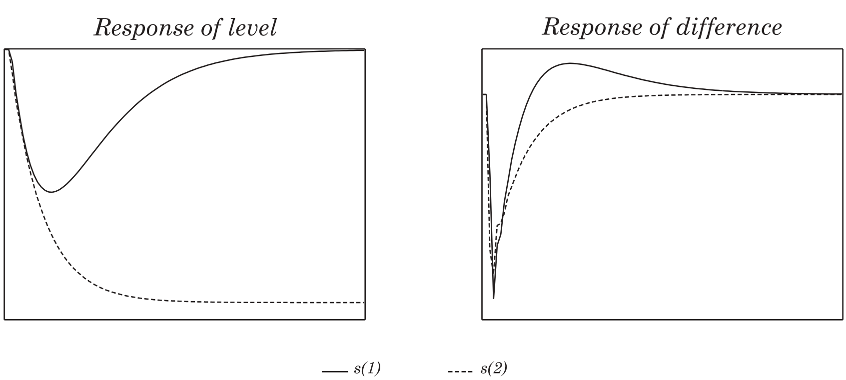
8. In this case, exact (unconditional) maximum likelihood estimation of the parameters should be conducted; this practice is rarely (if ever) followed, given that it is computationally demanding. Chumacero (2001a) describes a computationally efficient way of dealing with this problem.

a negative, though transitory, effect on the growth rate of the scale variable, what are the implications for the level of the series?

Figure 1 shows the implications for both levels and growth rates of a unit shock on the innovations estimated in that case.[9] For the sake of comparison, I consider two types of shocks. The first, termed *s*(1), corresponds to the effect (in both levels and differences) of a transitory shock when the scale variable is modeled in levels, thus making the shocks transitory and causing the level to revert to its deterministic trend. The second, *s*(2), corresponds to the same exercise when the scale variable is modeled in differences. The *s*(2) shocks on monetary policy have increasing and permanent effects on the level of the series even when the shock is not very persistent in terms of growth rates.

Which type of shock is correct? Chumacero and Quiroz (1996) and Chumacero (2000) find no evidence to support the practice of differentiating series such as Imacec.[10] Even in that case, it is important to consider the implications of the shocks for the levels of scale variables once a unit root is introduced. As figure 1 makes clear, such a mighty power of the monetary policy is difficult to rationalize even in the most extreme of Keynesian models.

Figure 1. Implications of Different Impulse Response Functions



9. The parameters are not chosen to match exactly the impulse response of the studies in which significant effects are found, but are arbitrarily chosen to demonstrate the effects for the level of the series. The essence of the results would not change if the actual impulse response functions reported were used.

10. Chumacero (2001b) shows, at both the theoretical and empirical levels, that it is unlikely for a unit root to be present in scale variables such as Imacec.

1.3 Ordering, Causality, and Interpretation

An even more important problem with these results involves their interpretation. As mentioned, Valdés (1998) and Cabrera and Lagos (2002) use a specific ordering in the construction of the impulse response functions of their VARs, in which the monetary policy innovation is not caused by any other innovation. Although it is common practice to order VARs according to Granger causality results of the variables in levels, the decomposition of the variance-covariance matrix has little to do with that ordering. In fact, there is no theoretical basis for justifying a specific ordering of the impulse response functions generated from a Cholesky decomposition based on Granger causality, as they may have no relation with the order of precedence of the levels. More fundamentally, it is not difficult to imagine a theoretical economy in which none of the monetary variables has any effect whatsoever on the real sector but which presents the dynamics that supposedly justify the results of Valdés (1998) or Cabrera and Lagos (2002).

Consider for example, the case of a closed endowment economy with a representative agent that is interested in maximizing the following:

$$\varepsilon_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) ,$$

subject to

$$y_t + \left(1 + r_{t-1}\right) b_{t-1} \geq c_t + b_t ,$$

where y is the level of the endowment, c is the level of consumption, b_t is the demand of a risk-free private bond that pays off a net return of r_t in the following period (this return is known at t), $u(\cdot)$ is strictly increasing and strictly concave, β is the subjective discount factor, and ε_t denotes the expectation operator conditional on the information available at time t.

Under the conditions stated above, the gross return on this asset is given by

$$\left(1 + r_t\right)^{-1} = \beta \varepsilon_t \left[\frac{u'\left(y_{t+1}\right)}{u'\left(y_t\right)} \right], \tag{1}$$

which simply states that the gross return of the asset is a function of the intertemporal marginal rate of substitution (stochastic discount factor).

Consider now a special case of equation (1), in which I impose a constant relative risk aversion (CRRA) utility function with the Arrow-Pratt relative risk aversion coefficient denoted by γ (inverse of the intertemporal elasticity of substitution). Equation (1) can then be expressed as

$$\left(1+r_t\right)^{-1} = \beta\varepsilon_t\left(\frac{y_{t+1}}{y_t}\right)^{-\gamma} . \tag{2}$$

The return on the asset is determined by solving equation (2), which requires explicitly introducing a law of motion for the endowment process. Consider two of such cases. The first assumes that the log of the endowment is difference stationary (DS) and the second that it is trend stationary (TS):

Case 1 (DS): $\Delta \ln y_{t+1} = \alpha + \sum_{i=0}^{k} \delta_i \Delta \ln y_{t-i} + w_{t+1}$,

where, $w_t \sim N\left(0,\sigma_w^2\right)$, and

Case 2 (TS): $\ln y_{t+1} = \eta + \alpha t + \sum_{i=0}^{l} \delta_i \ln y_{t-i} + v_{t+1}$,

where $v_t \sim N\left(0, \sigma_v^2\right)$.

In both equations, *w* and *v* are innovations, and *k* and *l* denote the number of lags necessary to produce them. Under these assumptions, the return on the asset can be computed as follows:

$$r_t \cong \ln\left(1+r_t\right) = \begin{cases} a_\varepsilon + \gamma\sum_{i=0}^{k}\delta_i\Delta\ln y_{t-i} & (\text{DS}) \\ a_v + \gamma\sum_{i=0}^{l}\delta_i\Delta\ln y_{t-i} - \gamma v_t & (\text{TS}) \end{cases}$$

where $a_i = \alpha\gamma - \ln\beta - 0.5\,\gamma^2\,\sigma_i^2$ for i = *w*, *v*.

This example shows that Granger causality and VAR results may be completely misleading when one attempts to identify impulse response functions as effects of alternative policies. I therefore focus on rather simple dynamics that help build the case. Using a first-order autoregressive, or AR(1), process for DS, I compactly characterize the dynamics of the system by

$$\begin{bmatrix} \Delta \ln y_{t+1} \\ \ln(1+r_t) \end{bmatrix} = \begin{bmatrix} \alpha \\ a_w \end{bmatrix} + \begin{bmatrix} \delta & 0 \\ \delta\gamma & 0 \end{bmatrix} \begin{bmatrix} \Delta \ln y_t \\ \ln(1+r_{t-1}) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w_{t+1} \\ w_t \end{bmatrix}. \tag{3}$$

Since r_t is known at period t, VAR estimates and Granger causality tests would typically be made in a system that comprises $\Delta \ln y_t$ and $\ln(1 + r_t)$. How would Granger causality results from a system like this look? Given that r_t is an exact function of the growth rate of the endowment in period t, there should be a strong contemporaneous correlation between variables whose sign will depend exclusively on the value of δ. (In fact, the contemporaneous correlation should be –1 or 1, because the relationship among these variables is deterministic.) VAR models and Granger causality tests typically rely on regressions of lagged values of the variables, so Granger causality tests will display bidirectional Granger causality between the asset return and the growth rate.

If y is TS, $\Delta \ln y$ has a unit root in its MA component. With a pure trend stationary process, the system can be conveniently expressed as:

$$\begin{bmatrix} \Delta \ln y_{t+1} \\ \ln(1+r_t) \end{bmatrix} = \begin{bmatrix} \alpha \\ a_v \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \ln y_t \\ \ln(1+r_{t-1}) \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & -\gamma \end{bmatrix} \begin{bmatrix} v_{t+1} \\ v_t \end{bmatrix}. \tag{4}$$

Since both variables are functions of innovations, there is strong evidence in favor of unidirectional causality from the asset returns to the growth rate! The endowment presents a combination of two independent innovations, such that the contemporaneous correlation between growth and the asset return should be negative (on average), but possibly nonsignificant and rather small.

This exercise shows that not all that glitters is gold. In both cases, I find statistical evidence in favor of Granger causality from the asset return to the growth rate of the endowment, even though there is no real (economic) causation whatsoever in that direction in this simple setup. If anything, the endowment is the causal variable (in the real sense). Thus an econometrician who chooses to interpret Granger causality tests and VAR results mechanically may completely misinterpret the actual structure of the economy.

It would not be difficult to replicate impulse response functions such as those described in figure 1 for economies such as those represented by equations (3) and (4) if the analyst takes the leap of faith that the policy instrument used by the monetary authority is the real interest rate (comparable to r). This cannot be the case, however, as demonstrated below. Even if it were, and the authority's instrument accurately reflected the real return of a risk-free bond, impulse response and Granger causality results cannot be interpreted as useful tools for identifying the effects of alternative policies. Economics—and not pure statistics—must be used to do so.

As the examples make clear, VARs or IVARs cannot be used to identify the effects of alternative policies. However, well-specified time series models can be used to provide a (statistically) objective metric for comparing alternative theoretical models (that are robust to the Lucas critique, at least in principle). I discuss this issue in the next section.

2. The Metric

This section presents the results of a nine-variable IVAR model for the Chilean economy. The model is intended to provide a good statistical description of the variables included, but I am careful not to provide a structural interpretation of it. Special attention is given to testing the proper order of the model and verifying whether the innovations are jointly Gaussian.

The variables taken into consideration correspond to monthly time series from January 1985 to July 2001 of the log of the industrial production index of the United States (y^*); the log of the first difference of the U.S. wholesale price index (WPI) (p^*); the log of real money holdings in the United States (m^*); the log of the federal funds rate (i^*); the log of the real exchange rate (e); the log of Chile's monthly activity index (y); the log of the first difference of the Chilean consumer price index (p); the log of domestic real money holdings (m); and the log of the monetary policy rate set by the Central Bank of Chile and denominated in UF (d).[11] In all cases, a quadratic trend was included to take into account possible smooth changes in trends over time.

11. The *Unidad de Fomento,* or UF, is a unit of account indexed to past and present inflation rates; it is widely used in Chile.

2.1 Parsimony

The first step in estimating the IVAR is to compute the unrestricted VAR. This computation is done following the usual OLS regressions for each variable on the system and choosing the optimal lags. Privilege must be given to a representation that is able to obtain innovations prior to reducing it to a parsimonious representation.

Model selection based on the Akaike information criterion (AIC) tends to choose models that are less parsimonious than the Bayesian (BIC), Hannan-Quinn (HQC), or final prediction error (FPE) criteria. In this case, AIC prefers a model with thirteen lags, while BIC and HQC choose only one lag. Finally, FPE prefers a VAR(2) model. Extensive likelihood ratio tests on the residuals show that even a VAR(1) is able to produce residuals that can be characterized as vector white noise processes but that present important departures from Gaussianity.

Table 1 shows the effect of a phenomenon that is often overlooked in practice. Since all models consider the dynamic interactions of nine variables, increasing the number of lags has nontrivial effects on the parsimony and accuracy of the estimation. In particular, more than 50 percent of the parameters are not statistically significant at standard levels even in the simple unconstrained VAR(1) models. Ignoring this fact may cause any unconstrained version of the model to induce spurious dynamics that are not present in the data. Furthermore, even small-order VAR models—such as a VAR(4)—have a huge saturation ratio (that is, the ratio between the number of parameters estimated in each equation and the sample size). In that particular example, more than 20 percent of the sample is compromised in estimating the parameters of each equation.

Table 1. Implications of the Choice of Different Lags

Order	*Number of parameters*	*Saturation ratio*	*Percent of insignificant variables*
1	108	0.061	0.509
2	189	0.107	0.614
3	270	0.154	0.704
4	351	0.201	0.729
5	432	0.249	0.771
6	513	0.300	0.791
7	594	0.346	0.806
8	675	0.395	0.824
9	756	0.444	0.783
10	837	0.495	0.808
11	918	0.545	0.849
12	999	0.597	0.822

Source: Author's calculations.

Not accounting for parsimony not only affects inference when obtaining bootstrapped confidence intervals, but it may also substantially modify the impulse response functions themselves. Figure 2 shows that this is indeed the case. Even when the traditional Cholesky decomposition is used to compute the impulse response functions, the apparent responses of the variables to the interest rate innovation are enhanced under the unconstrained VAR(2) model, which ignores the fact that more than 61 percent of the variables are redundant.[12]

2.2 Choice of the Impulse response Function

Once the VAR model is estimated, identification tests can be performed to assess the characteristics of the $\mathbf{B}_0$ matrix that best fits the data if IVAR models are to be considered. Recall that different specifications of this matrix will modify the impulse response functions nontrivially, so this point requires special attention. I used likelihood ratio tests, AIC, and BIC in the line of Leeper (1995) and Leeper, Sims, and Zha (1996) while testing these specifications.[13]

The preferred specification for $\mathbf{B}_0$ has a similar structure to the Cholesky decomposition. Contrary to the identifying assumption of Morandé and Schmidt-Hebbel (1997) and Valdés (1998), however, monetary instruments should be ordered precisely in the opposite direction, tending to react contemporaneously to innovations in the price equation and output equation. One important feature of using IVAR models is that inference on the contemporaneous associations of the innovations can be performed once the parameters of $\mathbf{B}_0$ are estimated by maximum likelihood. If this is done, most of the variables considered in $\mathbf{B}_0$ cannot be considered as statistically significant. Parrado (2001) imposes a different structure on his IVAR model, but most of the variables he considers are also insignificant.

Thus, IVAR models also impose arbitrary decompositions on the impulse response functions that result when statistically insignificant parameters are considered. I therefore report the impulse response functions that are obtained using Pesaran and Shin's (1998) methodology. In contrast with the Cholesky decomposition, generalized impulse response functions do not depend on the ordering of the equations; how-

12. As discussed above, the FPE criteria chooses the VAR(2) model, while BIC and HQ prefer the VAR(1) model. Nevertheless, the parsimonious VAR model in the case of the VAR(1) fails to produce vector white noise errors. I therefore conduct all the following exercises using a VAR(2) as the baseline model.

13. See appendix A for details.

ever, like the Cholesky decomposition, generalized impulse responses are exactly identified and tests for reductions cannot be performed.

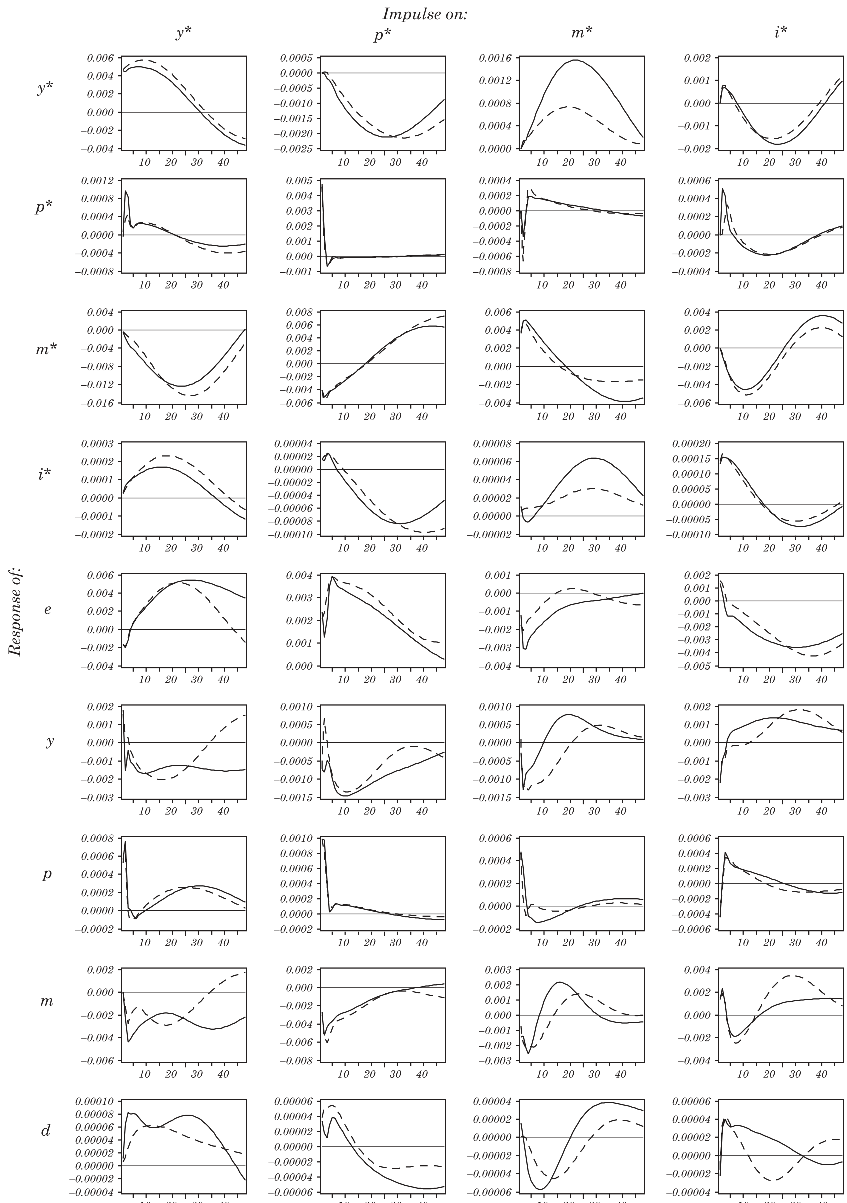
Figure 3 presents the generalized impulse response functions for four years (forty-eight months). Efron's 95 percent confidence intervals are also presented. As discussed above, Efron's bootstrapped confidence intervals may not have the correct coverage in the presence of asymmetries. Since this is precisely one of the sources of departures from normality, Efron's confidence intervals are not advisable.

The last two columns of figure 3 represent the effects of potential candidates for a measure of the monetary policy innovation. If innovations to *m* are considered monetary policy, surprise changes in the stock of money are persistent and predict subsequent movements in both inflation and output. The latter, however, is very short lived and dies out almost instantaneously. On the other hand, inflation increases only after a few periods (it is not statistically significant initially), and the response to *m* innovations in this system show what is sometimes called the liquidity puzzle: interest rates do not decline when *m* jumps upward. The liquidity effect, which hypothesizes that the policy-induced increased liquidity of a monetary expansion should lower interest rates, seems not to be present if innovations to *m* are considered measures of monetary policy stance.

The liquidity effect is not problematic if the innovations in the UF interest rates are considered monetary policy (last column). In this case, the initial shock can be interpreted as a monetary contraction. Here, the liquidity effect is strong until the fifteenth month and then eventually dies out. In terms of output, the *d* shock has a short-lived effect of contracting *y*. The problem with this shock arises when the effect on *p* is analyzed: monetary contractions tend to increase prices steadily. This results is very common in the literature (Leeper, Sims, and Zha, 1996) and has been labeled the price puzzle. Interpreting either column eight or column nine as a monetary contraction therefore requires accepting that monetary contractions produce inflation, which seems as unlikely as the notion that monetary expansions fail to lower interest rates.[14] However, if *d* is considered the monetary policy instrument, the ninth column shows very plausible responses. That is, interest rates increase with positive shock on output and inflation.

14. The price puzzle is also present in the impulse response functions of Valdés (1998) when the inflation target is not considered. Several authors conclude that the price puzzle can be eliminated if terms of trade or the price of oil is included (Parrado, 2001). In the present case, however, including terms of trade did not change my results.

Figure 2. Effects of Not Considering Parsimony in Cholesky Impulse Response Functions[a]



Source: Author's calculations.

a. Continuous line: parsimonious VAR; dashed line: unconstrained VAR.

Figure 2. (continued)

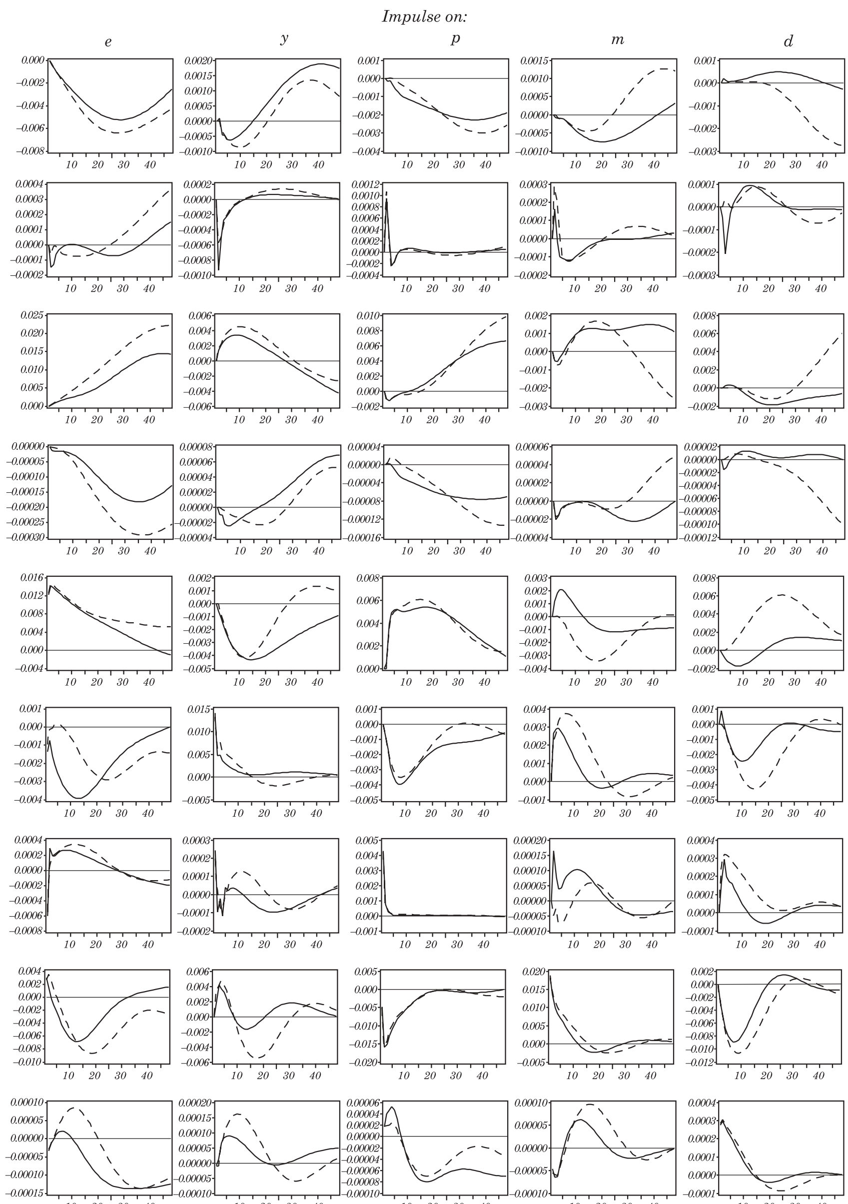
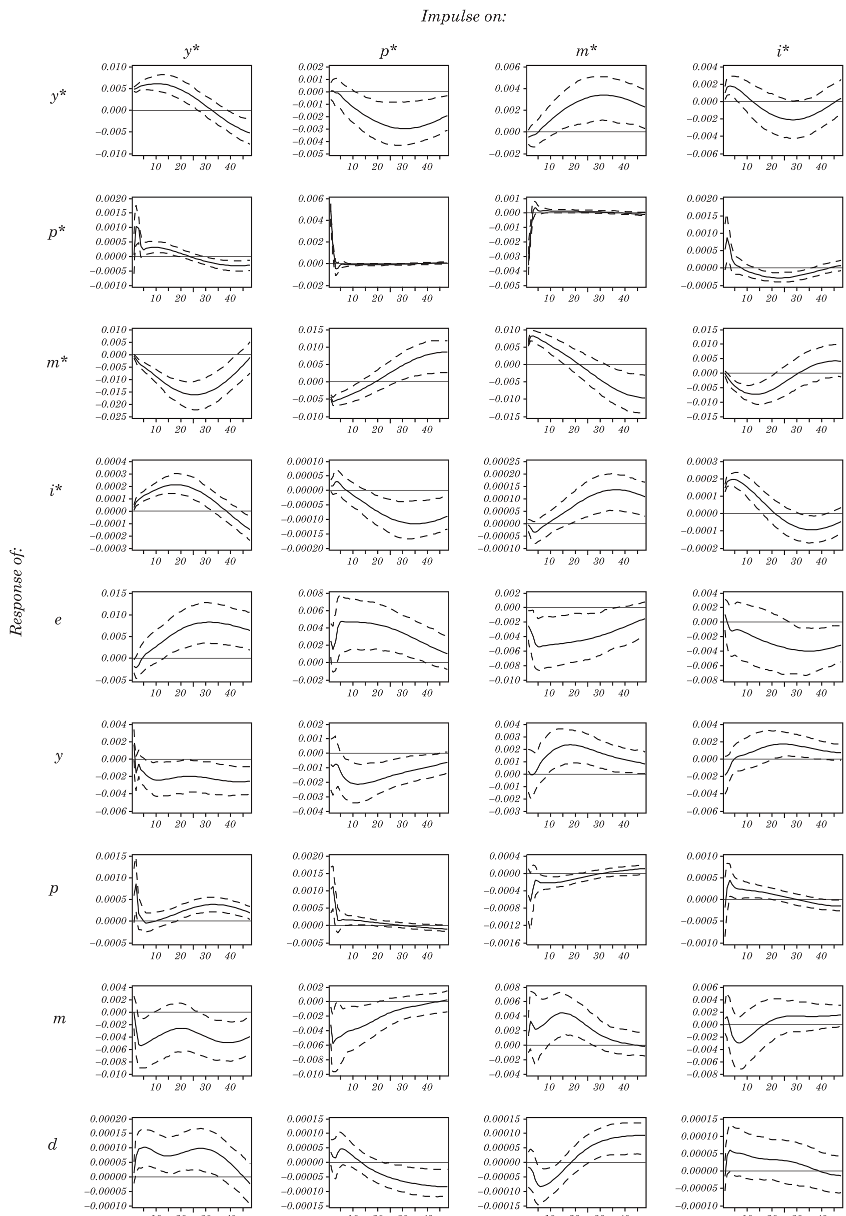


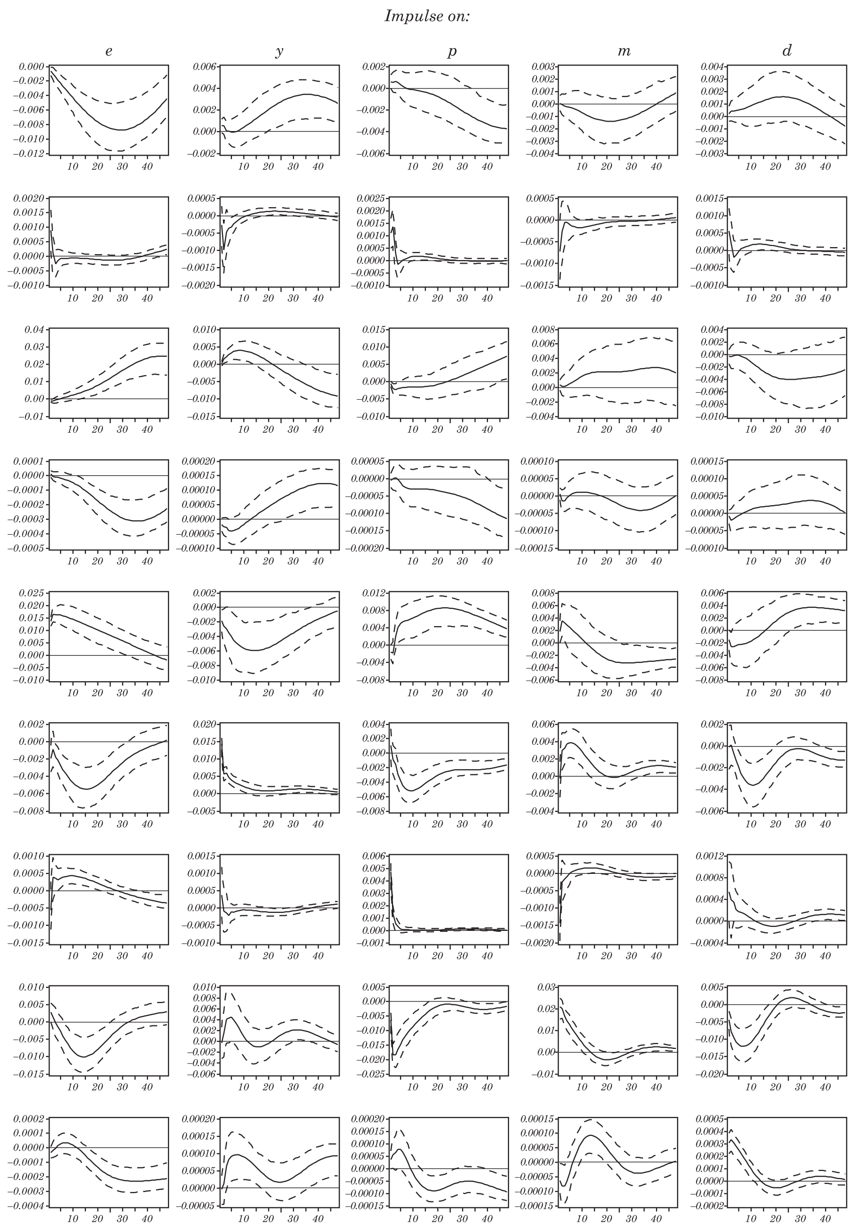
Figure 3. Generalized Impulse Response Functions[a]



Source: Author's calculations.

a. Hall's bootstrapped confidence intervals in parenthesis.

Figure 3. (continued)



The results from both IVAR and generalized impulse response functions show that care should be taken in interpreting these type of innovations as monetary policy. Of course, traditional VAR modeling renders even more implausible the dynamic responses of prices and output (as shown with the example of the s(2) shocks). The following section develops a theoretical model that helps explain why IVAR modeling is not sufficient for characterizing the effects of alternative policies and that this statistical exercise alone is not sufficient.

However, VAR estimates provide excellent statistical characterizations of the dynamic interactions of the variables considered. The difficulty of translating these statistical objects into structural economic models should not constitute a surprise. Thus, this model should only be considered a statistical representation, and no structural interpretation should be attempted. My results are simply a metric with which to obtain estimates of deep parameters of internally consistent dynamic models by using efficient method of moments (EMM).[15] I then compare the responses of the variables in the theoretical model with those generated by the IVARs.

3. The Model

This section develops a simple optimizing model, whose empirical implications will be compared with those of the econometric model described above. To make the model computationally manageable and to gain insights into the characteristics of the data that the model is and is not able to replicate, I introduce stringent assumptions in the stylized economy that is modeled. Needless to say, many of the assumptions come directly from the observed dynamic interactions of key variables in the IVAR estimated above.

I consider a simple economy in which agents try to maximize the expected discounted time-separable utility function:

$$\varepsilon_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{ht}, c_{mt}, \frac{M_t}{P_{ht}} \right), \tag{5}$$

where c_{mt} is the consumption of tradable (importable) goods in period t, c_{ht} is the consumption of nontradable goods in period t, M_t denotes

15. See appendix D.

the nominal money stock that the individual acquires at the beginning of period t and then holds through the end of the period, P_{ht} is the price level of nontradable goods, β is the subjective discount factor, ε_t denotes the conditional expectation on information available on period t, and $u(\cdot)$ is a strictly increasing and strictly concave function in all its arguments.

A few observations are in order. I introduced money in the utility function to make money valuable in general equilibrium. Implicit in this assumption is that money may be valuable both as a store of wealth and a medium of exchange. Feenstra (1986) shows that this specification is equivalent to one derived from the literature of transaction costs. Of course, cash-in-advance constraints are merely special cases of the transaction costs technologies.[16]

Equation (5) is maximized subject to the budget constraint:

$$\begin{aligned} & q_{ht} + \frac{(1+T_{mt})P^*_{mt}E_t q_{mt}}{P_{ht}} + (1+r_{t-1})b_{t-1} + \frac{(1+i_{t-1})B_{t-1}}{P_{ht}} \\ & + \frac{(1+d_{t-1})D_{t-1}U_t}{P_{ht}} + \frac{M_{t-1}}{P_{ht}} \\ & \geq c_{ht} + \frac{(1+T_{mt})P^*_{mt}E_t c_{mt}}{P_{ht}} + b_t + \frac{B_t}{P_{ht}} + \frac{D_t U_t}{P_{ht}} + \frac{M_t}{P_{ht}} + \frac{Z_t}{P_{ht}} , \end{aligned} \tag{6}$$

where E is the nominal exchange rate, T_m is the import tariff of a tradable good (q_m) that can be acquired in a competitive international market with price denoted by P^*_m, and q_h denotes the output of the nontradable good produced in the economy and sold at price P_h. Agents may also acquire indexed private bonds (in terms of nontradable goods, b) with sure return r that are in zero net supply, nominal government bonds (B) with (net) nominal return i, government bonds indexed to the UF (D) with net return d, and money balances that are carried to the next period. Finally, Z denotes lump sum taxes (or transfers) levied by the government. As a first approximation, the outputs of the different sectors of this economy will be characterized as stochastic endowments, thus making resource allocations irrelevant.

The problem of the representative consumer can then be summarized by the value function that satisfies

16. The neutrality found on the previous section may tempt one to work with specifications such as cash-in-advance. However, that type of specification imposes the assumption of constant velocity, which is not supported by the data.

$$\upsilon(s_t)=\max_{\{c_h,c_m,b,B,D,M\}}\left[u\left(c_h,c_m,\frac{M}{P_h}\right)+\varepsilon\beta v(s_{t+1})\right],$$

subject to equation (6) and the perceived laws of motion of the states s.[17]

The governments' budget constraint is given by

$$\frac{T_{mt}P^*_{mt}E_t(c_{mt}-q_{mt})}{P_{ht}}+\frac{B_t}{P_{ht}}+\frac{D_tU_t}{P_{ht}}+\frac{M_t}{P_{ht}}+\frac{Z_t}{P_{ht}}+\frac{B^*_t}{P_{ht}}$$

$$=\alpha_t q_t+\frac{(1-\alpha_t)P^*_{mt}E_t}{P_{ht}}g_t+\frac{(1+i_{t-1})B_{t-1}}{P_{ht}}+\frac{(1+d_{t-1})D_{t-1}U_t}{P_{ht}}$$

$$+\frac{M_{t-1}}{P_{ht}}+\frac{(1+i_{t-1})}{P_{ht}},$$

where g is the level of government expenditure (in terms of nontradables), α is the fraction of government expenditures destined to the consumption of nontradables, and B^* is the supply of government bonds to foreign investors.

Finally, I consider a representative foreign investor who solves a dynamic portfolio allocation problem, maximizing the following expected discounted utility:

$$\varepsilon_0\sum_{t=0}^{\infty}\beta^{*t}\omega\left(c^*_{mt},\frac{M^*_t}{P^*_{mt}}\right)$$

subject to the constraint:

$$q^*_{mt}+\frac{(1+i_{t-1})B^*_{t-1}}{E_tP^*_{mt}}+\frac{(1+i^*_{t-1})b^*_{t-1}}{P^*_{mt}}+\frac{M^*_{t-1}}{P_{ht}}$$
$$\geq c^*_{mt}+\frac{B^*_t}{E_tP^*_{mt}}+\frac{b^*_t}{P^*_{mt}}+\frac{M^*_t}{P^*_{mt}}+\frac{Z^*_t}{P^*_{mt}}, \qquad (7)$$

where q^*_m is a stochastic endowment, c^*_m is the level of consumption of a composite good by the foreign representative agent, B^* is the demand of bonds supplied by the domestic government, and b^* is the demand for international bonds (issued by the foreign authority) that

17. I define $s_t=(q_{ht}, q_{mt}, r_{t-1}, i_{t-1}, d_{t-1}, b_{t-1}, B_{t-1}, D_{t-1}, M_{t-1}, P_{ht}, P^*_{mt}, E_t)$.

yield a return of i^*. The other variables are analogous to those in the domestic economy.

The foreign investor's value function satisfies

$$\upsilon^*\left(s_t^*\right) = \max_{\{c_m^*, b^*, B^*, M^*\}} \left[\omega\left(c_m^*, \frac{M^*}{P_m^*} \right) + \varepsilon\beta^* v^*\left(s_{t+1}^*\right) \right],$$

subject to equation (7) and the perceived laws of motion of the states s^*.[18]

The foreign government satisfies the following constraint:

$$\frac{b_t^*}{P_{mt}^*} + \frac{M_t^*}{P_{mt}^*} + \frac{Z_t^*}{P_{mt}^*} = g_t^* + \frac{\left(1+i_{t-1}^*\right) b_{t-1}^*}{P_{mt}^*} + \frac{M_{t-1}^*}{P_{mt}^*} \ .$$

The market-clearing conditions for the tradable and nontradable markets are as follows:

$$q_{ht} = c_{ht} + \alpha_t g_t \ ,$$

$$CA_t \equiv -\left(B_t^* - B_{t-1}^*\right) = P_{mt}^* E_t \left[q_{mt} - c_{mt} - \left(1-\alpha_t\right) g_t \right] - i_{t-1} B_{t-1}^* \text{ , and}$$

$$CA_t^* \equiv \left(B_t^* - B_{t-1}^*\right) = P_{mt}^* E_t \left(q_{mt}^* - c_{mt}^* - g_t^* \right) + i_{t-1} B_{t-1}^* = -CA_t \ ,$$

where the first equation describes the equilibrium in the domestic nontradable goods market; the second presents the equilibrium in the domestic tradable goods market, which shows that the current account balance must be compensated by the capital account balance; and the third equation shows the equilibrium condition of the foreign economy's goods market (expressed in terms of domestic currency), which displays a condition analogous the second. In general equilibrium, the current account balance of one economy is the negative of the other; thus the supply and demand of the tradable goods equate.

I define a recursive competitive equilibrium for these economies as a set of prices and policy functions such that markets clear. To find the policy functions compatible with the market-clearing conditions, one must solve the problems faced by each of the agents in these economies.

Appendix B demonstrates that the first-order conditions of both optimization problems can be used to obtain the value of the real

18. I define $s^*_t = (q^*_{mt}, i_{t-1}, i^*_{t-1}, b^*_{t-1}, B^*_{t-1}, M^*_{t-1}, P^*_{mt}, E_t)$.

exchange rate, e (defined as a relative price between tradables and nontradables), the equilibrium real interest rate for the risk-free indexed private bond, the nominal interest rate of the risk-free government bond, the demand for domestic real balances, the foreign nominal interest rate, the no-arbitrage condition, and the demand for foreign real money holdings.

3.1 An Example

The following example shows how prices and real variables are determined, thereby offering insight into the characteristics of the economy under consideration. I consider that the consumer has a Cobb-Douglas CRRA utility function of the form,

$$u\left(c_h, c_m, \frac{M}{P_h}\right) = \frac{\left[c_h^{\varphi} c_m^{\delta} \left(M/P_h\right)^{1-\varphi-\delta}\right]^{1-\gamma}}{1-\gamma} \ ,$$

where γ is the Arrow-Pratt CRRA coefficient. In the particular case that $\gamma \rightarrow 1$, the last equation can be conveniently expressed as

$$u\left(c_h, c_m, \frac{M}{p_h}\right) = \varphi \ln c_h + \delta \ln c_m + (1-\varphi-\delta) \ln \frac{M}{P_h} \ ,$$

which is the functional form used for this example.

Suppose further that the domestic endowments follow first-order Markov processes that are independent of foreign and domestic nominal variables. Domestic government expenditures are a constant fraction of the production of nontradables and are partially financed by imposing a fixed import tariff on the tradable good. The monetary authority in the domestic economy sets a UF-indexed interest rate by supplying the amount of bonds that the foreign and domestic markets are willing to take at the referred rate. I introduce more structure to the domestic monetary policy as needed.

Using equations (B.11) through (B.16) from appendix B, the equilibrium conditions in this case are as follows:

$$e_t \equiv \frac{P_{mt}^{*} E_t}{P_{ht}} = \frac{\delta\left(q_{ht} - \alpha_t g_t\right)}{\left(1+T_{mt}\right)\varphi\left[q_{mt} - \left(1-\alpha_t\right) g_t - N\right]_t} \ , \quad (8)$$

$$1 = \beta\left(1 + r_t\right)\varepsilon_t\left(\frac{q_{ht} - \alpha_t g_t}{q_{h,t+1} - \alpha_{t+1} g_{t+1}}\right), \tag{9}$$

$$1 = \beta\left(1 + d_t\right)\frac{U_{t+1}}{U_t}\varepsilon_t\left(\frac{q_{ht} - \alpha_t g_t}{q_{h,t+1} - \alpha_{t+1} g_{t+1}}\frac{P_{ht}}{P_{h,t+1}}\right), \tag{10}$$

$$1 = \beta\left(1 + i_t\right)\varepsilon_t\left(\frac{q_{ht} - \alpha_t g_t}{q_{h,t+1} - \alpha_{t+1} g_{t+1}}\frac{P_{ht}}{P_{h,t+1}}\right), \text{ and} \tag{11}$$

$$\frac{M_t}{P_{ht}} = \frac{1 - \varphi - \delta}{\varphi}\left(q_{ht} - \alpha_t g_t\right)\frac{1 + i_t}{i_t}\ , \tag{12}$$

where N is defined as the net amount of foreign private capital inflows in terms of the tradable good (current account deficit plus payments of interests).

While the results of the previous equations depend on the simple parameterization chosen, their qualitative implications will hold for a wide variety of functional forms for preferences. Equation (8) confirms several beliefs in popular culture. The real exchange rate appreciates with a decline in the productivity of nontradables, an increase in the productivity of tradables, increased net capital inflows, and trade protection. As Arrau, Quiroz, and Chumacero (1992) show, an increase in government expenditure has an ambiguous effect on the real exchange, depending not only on its propensity to consume nontradable goods, but also on the structure of private consumption.

Equation (9) presents the condition that determines the value of the real interest rate in this economy. Contrary to several claims, the monetary authority is not capable of affecting the real interest rate directly. In this economy, the real interest rate displays a positive relation with the growth rate of the nontradables sector in the long run. This means that an economy that is growing at a faster rate than another economy will have higher autarkic real interest rates. If there are limitations to the free flow of capital from one economy to the other, the economy that is growing faster will have a higher interest rate. This rate may have no relation with the real interest rate set by the monetary authority, and thus the claim that the Central Bank sets the real interest rate is fundamentally incorrect.[19] What, then, did the monetary authority set with instruments indexed to the UF?

Equations (10) and (11) have the answer. Both instruments must be arbitraged, given that the terms in the expectation operators coincide. It thus does not matter whether the authority sets a nominal or a UF-indexed rate. Nevertheless, the difference between the law of motion of the UF and the actual price level shows that the real interest rate will have fundamental differences with the UF-indexed rate. The difference between them is that the latter instrument and truly indexed bonds are contingent on the actual realization of inflation, whereas the former is (or at least should be) set considering the expected inflation rate. This difference between the two equations is precisely the same as that which prevents the Fischer equation from holding in the presence of uncertainty. The only case in which it would hold (on average) is when the inflation process is independent of the intertemporal marginal rate of substitution. This condition is not likely to hold because the reaction function of the monetary authority (particularly in Chile) is extremely dependent on its perception of the business cycle and the growth rate the economy. If the monetary authority sets an inflation target, equations (10) and (11) show that it must be consistent with the chosen interest rate. Thus, either of these equations will help one solve for the inflation rate, consistent with perceptions on the evolution of the economy (intertemporal marginal rate of substitution) and the monetary authority's policy rule.

Finally, equation (12) determines the demand for real money holdings. This equation is independent of parameters that may characterize the monetary authority's policy rule. If the policy rule changes, however, badly specified money demand equations may find evidence of instability even when there is none.

Several monetary policy arrangements can be examined even in this simple case. For example, if the authority sets the nominal exchange rate, no arbitrage conditions with the foreign investor will determine the nominal interest rate that is consistent with this policy.

19. Prior to the last quarter of 2001, the Central Bank of Chile set its policy rated with UF-indexed instruments. This fact led many specialists and nonspecialists to claim that the Central Bank sets the real interest rate, although this claim is fundamentally false.

Likewise, equation (12) will endogenously determine the money stock consistent with this policy.

3.2 Results

Section 2 examined how the problems of using VAR and IVAR impulse response functions to identify the effects of monetary policy are ill-conditioned practices. Regardless of the method used, it is difficult to rationalize several of the results that are supposed to capture the effects of monetary policy. Furthermore, as the theoretical model discussed above shows, several dynamic interactions between variables depend on the particular specification for the policy rule of the government.

My estimated model closes with a Taylor rule for the monetary authority:

$$\ln\left(1+i_{t+1}\right)=a_0+a_1\ln\left(\frac{y_t}{y_s}\right)+a_2\varepsilon_t\ln\left(\frac{1+\pi_{t+1}}{1+\pi_s}\right)+a_3\ln\left(1+i_t\right)+w_{t+1},$$

where y_s and π_s denote the steady state values for output and inflation.

The methodological steps are, first, to use the gradients of the VAR(2) discussed on section 2 as the matching conditions for the theoretical model. Second, I estimate the parameters of the theoretical model using EMM and the gradients of the VAR model as the metric. Third, once the estimates of the model are obtained, I undertake a long simulation of the theoretical model, estimate a VAR(2) with it, and derive the generalized impulse response function. Finally, I shock the theoretical model with a transitory innovation to the domestic interest rate and obtain the true impulse response function.[20]

Figure 4 presents the results of comparing the impulse response functions that are obtained with the VAR(2) estimated with artificial data and the impulse response functions that come from the theoretical model. Several features are worth mentioning. First, the impulse response functions estimated with simulated data are broadly consistent with the data; that is, the model also produces a price puzzle, a small contraction of the level of activity, a slight appreciation, and strong liquidity effects. Second, the overidentifying restrictions test does not reject the null hypothesis that the model captures

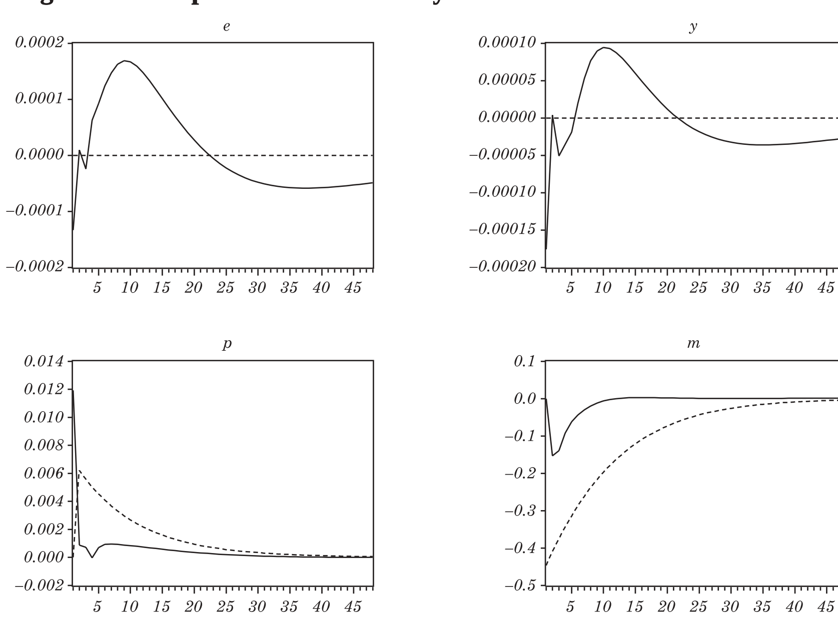
20. Here, true means the impulse response function that is consistent with the theoretical model and not the statistical object.

the dynamic interactions of the variables involved. Third, and most importantly, even though the model replicates the impulse response functions of the data, the true response of the variables with respect to a monetary innovation have little to do with the responses that come from the VAR.

By construction, the model presents a dichotomy between real and nominal variables. Monetary policy thus has no effect on the real exchange rate and output, even when the impulse response functions of the model show nonneutralities. This is so because interest rates carry information about the future evolution of the economy: a higher interest rate signals lower output today relative to the long run, since interest rates Granger cause output in this model (as it did in the example of section 1). It is not surprising, therefore, that the impulse response functions may show spurious responses of output to interest rate innovations.

A case for neutrality (or near neutrality) can be made precisely because of the presence of the price puzzle. Models that display important nonneutralities (with Phillips curves and such) would have a very difficult time trying to explain this puzzle. In the case at hand,

Figure 4. Response to Monetary Innovations[a]



Source: Author's calculations.

a. Continuous line: generalized impulse response; dashed line: true impulse response.

however, the theoretical model predicts that inflation and interest rates should be positively correlated because of the inability of nominal variables to affect real variables. Thus, if the real interest rate remains basically constant, prices must move in the same direction as the nominal interest rate innovation. This follows simply from the interaction of the feedback of the Taylor rule and the dichotomy with real variables.

The model thus shows that only a few dimensions of the impulse response functions are truly consistent with the responses of fundamental models. The idea that VARs can help identify the effects of monetary policy is naïve.

4. Concluding Remarks

The objectives of this paper were twofold: first, to show that the traditional practice of quantifying the effects of monetary policy from the impulse response functions of VARs or IVARs is misleading, since it is impossible to recover structural shocks independently of the structure chosen; and second, to construct a simple metric with which to compare competing theoretical models. This second objective is important because the theoretical and empirical literature in the field of macroeconomics has not reached a consensus on which metric to use to judge a model's success in capturing key features of the data. This paper shows that such a metric can be constructed and that the statistical object that comes from it can help economists understand which features of different theoretical models are important and which are not.

The statistical model is a nine-variable VAR(2) whose impulse response functions cannot be directly considered structural. Furthermore, one has to concede that deflationary policies are inflationary and the money demand depends positively on interest rates.

The theoretical model that is estimated is broadly consistent with the VAR(2) model, but it has striking implications. First, by construction, it displays neutrality of the monetary policy. Second, because of this feature, it is not difficult to replicate impulse response functions that appear to be consistent with nonneutralities. Third, the price puzzle is only a puzzle for a model in which important nonneutralities are a major driving force. Finally, even when the model has built-in nonneutralities, they must not have first-order implications in order for the model to be consistent with the statistical object chosen.

APPENDIX A
Estimation and Inference in an IVAR

This appendix presents a brief summary of the techniques used to estimate IVAR models, their differences with traditional VARs, and the methods developed to test their specification.

Consider a model of the type

$$\mathbf{B}_0\mathbf{y}_t = \mathbf{k} + \mathbf{B}_1\mathbf{y}_{t-1} + \ldots + \mathbf{B}_p\mathbf{y}_{t-p} + \mathbf{u}_t \;, \tag{A.1}$$

where $\mathbf{y}_t$ is an n vector, $\mathbf{k}$ is an n vector of constants, $\mathbf{B}_i$ is an $n \times n$ matrix (i = 0,.., p), and $\mathbf{u}_t$ is an n vector white noise process with (diagonal) variance-covariance matrix $\mathbf{D}$. Premultiplying equation (A.1) by the inverse of $\mathbf{B}_0$, I obtain

$$\mathbf{y}_t = \mathbf{c} + \mathbf{C}_1\mathbf{y}_{t-1} + \ldots + \mathbf{C}_p\mathbf{y}_{t-p} + \mathbf{e}_t \;, \tag{A.2}$$

where, given that $\mathbf{u}$ is a vector white noise process and that $\mathbf{e} = \mathbf{B}_0^{1}\mathbf{u}$, $\mathbf{e}$ is also a vector white noise process with variance-covariance matrix $\boldsymbol{\Omega}$. Equation (A.2) is precisely the representation generally used in VAR models, thus making it interpretable as a reduced form of equation (A.1). The only case in which the VAR model would be equivalent to equation (A.1) is when $\mathbf{B}_0$ is an identity matrix. If some of the off-diagonal elements of this matrix are different from zero, the error terms on $\mathbf{e}$ will be formed by linear combinations of the structural innovation, $\mathbf{u}$. Thus, the impulse response functions estimated with equation (A.2) cannot be interpreted as the dynamic response of the variables in the system to the underlying innovations.

To recover the structural parameters of equation (A.1), I use the parameters estimated from equation (A.2) to obtain an estimate of $\boldsymbol{\Omega}$; with it, I then solve the nonlinear system:

$$\Omega = \mathbf{B}_0^{-1}\mathbf{D}\left(\mathbf{B}_0^{-1}\right)' \;, \tag{A.3}$$

or the log-likelihood function that relates both variance-covariance matrices:[21]

$$\ell\left(\mathbf{B}_0, \mathbf{D}, \hat{\Omega}\right) \propto \frac{T}{2}\ln\left(\mid \mathbf{B}_0 \mid^2 - \ln \mid \mathbf{D} \mid - tr\left[\mathbf{B}_0'\mathbf{D}^{-1}\mathbf{B}_0\hat{\Omega}\right] \right). \tag{A.4}$$

21. See Hamilton (1994) for details.

One advantage of this approach is that the variance-covariance matrix of the parameters that solve equation (A.3) are readily available. Once the structural parameters are recovered, inference based on likelihood ratio (LRT) or Wald tests can be conducted as usual.

One important issue as that of identification. Since $\mathbf{\Omega}$ is symmetric, there are $n(n + 1)/2$ distinct parameters in the variance-covariance matrix of the residuals of equation (A.2). Given that $\mathbf{D}$ is diagonal, there are at most $n(n - 1)/2$ parameters that can be estimated in $\mathbf{B}_0$ if the order condition of identification is to be satisfied. Thus, some restrictions (which one hopes are testable) must be imposed. If z is used to denote the number of parameters estimated in $\mathbf{B}_0$, the number of overidentifying restrictions is $r = [n(n - 1)/2] - z$. In that case, the LRT test for overidentifying restrictions is simply

$$\mathbf{LRT} = 2\left(-\frac{T}{2}\ln|\hat{\Omega}| - \frac{t}{2}n - \ell^{*}\right) \xrightarrow{D} X_r^2 \ , \tag{A.5}$$

where ℓ^{*} is the value of the log-likelihood function that maximizes equation (A.4).

This methodology not only provides a robust way of estimating the effects of orthogonal innovations to the system, but it may also be a useful tool for discriminating among models. Recall that VARs impose a somewhat arbitrary ordering of the variables in the system, which will affect the resulting impulse response functions, while IVARs provide formal tests under which to contrast alternative orderings and contemporaneous relations among variables. This feature may constitute a valuable intermediate stage that provides insights with respect to the theoretical models that can and cannot be used to replicate the dynamic interactions among variables. Nevertheless, as IVARs heavily rely on the identifying assumptions imposed on them, they are useful only as intermediate devices between data and theory.

A healthy practice, whether using VARs or IVARs, is to compute standard errors (and thus confidence intervals) associated with the impulse response functions. Traditional econometric packages rely on the asymptotic distribution of the impulse response functions or on Monte Carlo experiments (based on the maintained hypothesis of Gaussian innovations) to construct them. These methods may, however, provide poor approximations of the confidence intervals in small samples, even with the assumption of normality (owing to the small sample bias of the OLS estimators). Another important problem with this type of confidence intervals is that they are symmetric (owing to the assumption of normality). In finite samples, the innovations may have important departures from normality (typically leptokurtosis) and may not be symmetric (if there is skewness), such that the Monte Carlo approximation may not be advisable. In this case, bootstrap methods may be used to replicate the empirical distribution of the innovations. Sims and Zha (1995) also advise that constructing confidence intervals may help to correct the pervasive nature of the biases implicit in the VAR estimation. This can be done, again, with bootstrapping.[22]

22. Christiano, Eichenbaum, and Evans (1996) provide a detailed description of the algorithm used to construct both the impulse response functions and confidence intervals with bootstrapping. Sims and Zha (1995) describe the algorithm used for bias reduction.

Appendix B
Equilibrium Conditions for the Theoretical Model

This appendix derives the first-order conditions for the optimization problems of the domestic representative consumer and the representative foreign investor. These conditions are later used to describe the characteristics of the equilibrium conditions of the economies presented in the theoretical model.

The first-order conditions with respect to c_{ht}, c_{mt}, b_t, B_t, D_t, and M_t for the representative domestic consumer are the following:

$$u'_{c_{ht}} - \lambda_t = 0 \ , \tag{B.1}$$

$$u'_{c_{ht}} - \lambda_t \left(1 + T_{mt}\right) \frac{P^*_{mt} E_t}{P_{ht}} = 0 \ , \tag{B.2}$$

$$\lambda_t - \beta \varepsilon_t \upsilon'_{b_t} = 0 \ , \tag{B.3}$$

$$\lambda_t - \beta P_{ht} \varepsilon_t \upsilon'_{B_t} = 0 \ , \tag{B.4}$$

$$\lambda_t - \beta \frac{P_{ht}}{U_t} \varepsilon_t \upsilon'_{D_t} = 0 \ , \text{ and} \tag{B.5}$$

$$u'_{\frac{M_t}{P_{ht}}} \frac{1}{P_{ht}} - \frac{\lambda_t}{P_{ht}} + \beta \varepsilon_t \upsilon'_{M_t} = 0 \ , \tag{B.6}$$

where λ is the dynamic multiplier associated with the constraint described in equation (6) of the main text. The corresponding envelope conditions are

$$\upsilon'_{b_{t-1}} = \lambda_t \left(1 + r_{t-1}\right) , \tag{B.7}$$

$$\upsilon'_{B_{t-1}} = \frac{\lambda_t}{P_{ht}} \left(1 + i_{t-1}\right) , \tag{B.8}$$

$$\upsilon'_{D_{t-1}} = \frac{\lambda_t U_t}{P_{ht}} \left(1 + d_{t-1}\right) , \text{ and} \tag{B.9}$$

$$\upsilon'_{M_{t-1}} = \frac{\lambda_t}{P_{ht}} \ . \tag{B.10}$$

Combining equations (B.1) and(B.2) shows that the real exchange rate (e)—defined as the relative price between tradables and nontradables—is given by the ratio of marginal utilities between the consumption of both goods, corrected by the import tariff. That is,

$$e_t \equiv \frac{P^*_{mt} E_t}{P_{ht}} = \frac{u'_{c_{mt}}}{\left(1+T_{mt}\right) u'_{c_{ht}}} \,. \tag{B.11}$$

Combining equations (B.3) and (B.7) establishes the condition that determines the equilibrium real interest rate for the risk-free indexed private bond, while combining equations (B.4) and (B.8) yields the equilibrium condition that determines the nominal interest rate of the risk-free government bond. Finally, combining equations (B.5) and (B.9) gives the equilibrium interest rate for the UF indexed bond. That is,

$$1 = \beta\left(1+r_t\right)\varepsilon_t \frac{u'_{c_{h,t+1}}}{u'_{c_{ht}}} \,, \tag{B.12}$$

$$1 = \beta\left(1+i_t\right)\varepsilon_t \left(\frac{u'_{c_{h,t+1}}}{u'_{c_{ht}}} \frac{P_{ht}}{P_{h,t+1}} \right), \text{ and} \tag{B.13}$$

$$1 = \beta\left(1+d_t\right)\frac{U_{t+1}}{U_t}\varepsilon_t \left(\frac{u'_{c_{h,t+1}}}{u'_{c_{ht}}} \frac{P_{ht}}{P_{h,t+1}} \right). \tag{B.14}$$

In equation (B.14), U_{t+1}/U_t is known at period t, given that

$$\frac{U_{t+1}}{U_t} = \left[\frac{P_t}{P_{t-1}}\right]^a \left[\frac{P_{t-1}}{P_{t-2}}\right]^{1-a} \,, \tag{B.15}$$

which can be approximated as 21/30.[23]

23. The UF has a deterministic law of motion that depends on a weighted average of present and past inflation. With the assumption that a typical month has thirty days, the variation of the UF from the last day of month t to the last day of month $t+1$ is given by equation (B.15), where P is the consumption-based price level derived in appendix C.

Finally, combining equations (B.6) and (B.10), and using equation (B.13), I derive the condition that determines the demand for real balances:

$$\frac{u'_{M_t/P_{ht}}}{u'_{c_{ht}}} = \frac{i_t}{1+i_t} \ . \tag{B.16}$$

These equations, combined with the market-clearing conditions and the policy rules followed by the public sector (as well as the functional form of preferences), determine the temporal trajectory of these variables.

The first-order conditions with respect to c^*_{mt}, B^*_t, b^*_t, and M^*_t for the representative foreign investor are the following:

$$\omega'_{c^*_{mt}} - \lambda_t = 0 \ , \tag{B.17}$$

$$\lambda^*_t - \beta^* P^*_{mt} E_t \varepsilon_t \upsilon^{*\prime}_{B^*_t} = 0 \ , \tag{B.18}$$

$$\lambda^*_t - \beta^* P^*_{mt} \varepsilon_t \upsilon^{*\prime}_{b^*_t} = 0 \text{ , and} \tag{B.19}$$

$$\omega'_{M^*_t/P^*_{mt}} \frac{1}{P^*_{mt}} - \frac{\lambda^*_t}{P^*_{mt}} + \beta^* \varepsilon_t \upsilon^{*\prime}_{M^*_t} = 0 \ . \tag{B.20}$$

where λ^* is the dynamic multiplier associated with equation (7) in the main text. The envelope conditions for this problem are given by

$$\upsilon^{*\prime}_{B^*_{t-1}} = \frac{\lambda^*_t}{P^*_{mt} E_t}\left(1 + i_{t-1}\right) , \tag{B.21}$$

$$\upsilon^{*\prime}_{b^*_{t-1}} = \frac{\lambda^*_t}{P^*_{mt}}\left(1 + i^*_{t-1}\right) \text{, and} \tag{B.22}$$

$$\upsilon^{*\prime}_{M^*_{t-1}} = \frac{\lambda^*_t}{P^*_{mt}} \ . \tag{B.23}$$

As in the previous problem, the equilibrium conditions for the foreign nominal interest rate, the no-arbitrage condition, and the demand for foreign real money balances can be found by combining the envelope and the first-order conditions, which yield

$$1 = \beta^{*}\left(1 + i_t\right)\varepsilon_t\left(\frac{\omega'_{c^*_{m,t+1}}}{\omega'_{c^*_{mt}}}\frac{P^*_{mt}}{P^*_{m,t+1}}\frac{E_t}{E_{t+1}}\right), \tag{B.24}$$

$$1 = \beta\left(1 + i^*_t\right)\varepsilon_t\left(\frac{\omega'_{c^*_{m,t+1}}}{\omega'_{c^*_{mt}}}\frac{P^*_{mt}}{P^*_{m,t+1}}\right), \text{ and} \tag{B.25}$$

$$\frac{\omega'_{M^*_t/P^*}}{\omega'_{c^*_{mt}}} = \frac{i^*_t}{1 + i^*_t}\ . \tag{B.26}$$

The Euler equations of both problems were solved considering binding constraints (because of the assumption that both utility functions are strictly increasing). The values of these variables will be determined in general equilibrium by their interaction with the market-clearing conditions and the laws of motion of the states (including government policies).

With relatively mild conditions, existence and uniqueness for the value functions of both problems can be demonstrated using Blackwell's conditions for contraction and contraction mapping theorem arguments (see Altug and Labadie, 1995; Stokey, Lucas, and Prescott, 1989).

Appendix C
The Aggregate Consumption-based Price Index

This appendix derives the aggregate consumption-based price index for the constant elasticity of substitution (CES) utility function, extending the derivations of Obstfeld and Rogoff (1996) for an economy with a tradable good, a nontradable good, and money.

Let $c = f(c_h, c_m, n)$ be a composite consumption good that is a linear-homogeneous function of c_h, c_m, and n, where $n = M/P_h$. I am interested in finding a price index associated with c that will indicate how much of the good a consumer can obtain from a given level of expenditure Z (denominated in domestic currency).

I define the aggregate consumption-based price index, P, as the minimum expenditure $Z = P_h c_h + (1+T_m) P_m^* E c_m + Wn$, such that $c = f(c_h, c_m, n) = 1$, given P_h, P_m^*, E, T_m and W. In this case W denotes the user cost of holding currency; the closed-form expression is derived below.

Consider the CES consumption index of the following form:

$$c = \left[\varphi^{1/\theta} c_h^{(\theta-1)/\theta} + \delta^{1/\theta} c_m^{(\theta-1)/\theta} + (1-\varphi-\delta)^{1/\theta} n^{(\theta-1)/\theta}\right]^{\theta/(\theta-1)}; \qquad \text{(C.1)}$$
$$\varphi, \delta \in (0,1),\ \theta > 0\ .$$

The highest value of the index, c, for a given value of Z (found by substituting the demands for each good in equation (C.1)) is given by

$$\left\langle \varphi^{1/\theta}\left[\frac{\varphi Z}{P_h D}\right]^{(\theta-1)/\theta} + \delta^{1/\theta}\left\{\frac{\delta Z}{P_h\left[\dfrac{P_h}{P_m^* E(1+T_m)}\right]^{\theta} D}\right\}^{(\theta-1)/\theta} + A \right\rangle^{\theta/(\theta-1)}, \qquad \text{(C.2)}$$

where

$$A = (1-\varphi-\delta)^{1/\theta}\left[\frac{(1-\varphi-\delta)Z}{P_h (P_h/W)^{\theta} D}\right]^{(\theta-1)/\theta},$$

$$D = \varphi + \delta\left(\frac{P_m^* E}{P_h}\right)^{1-\theta} + (1-\varphi-\delta)\left(\frac{W}{P_h}\right)^{1-\theta}.$$

Defining P as the minimum expenditure needed to obtain $c = 1$, I solve for P by imposing $P = Z$ and equating equation (C.2) to 1. After trivial manipulations and using equation (B.11), I find that the price index is given by

$$P_t = P_{ht}\left\{\varphi + \delta\left[e_t\left(1+T_{mt}\right)\right]^{1-\theta} + \left(1-\varphi-\delta\right)W^{1-\theta}\right\}^{1/(1-\theta)} .$$

W results from the ratio of the marginal utility of real money holdings and the marginal utility of consumption of nontradables. It is thus correct to infer that W_t is simply the right-hand side of equation (B.16). The consumption based price index adopts the form

$$P_t = P_{ht}\left\{\varphi + \delta\left[e_t\left(1+T_{mt}\right)\right]^{1-\theta} + \left(1-\varphi-\delta\right)\left(\frac{i_t}{1+i_t}\right)^{1-\theta}\right\}^{1/(1-\theta)} . \tag{C.3}$$

Expression (C.3) can be used to compute the evolution of the general price level, once the other prices are determined.

If equation (C.1) were Cobb-Douglas, (that is, if $\theta = 1$), then equation (C.4) would be

$$P_t = P_{ht}\left\{1 + \left[e_t\left(1+T_{mt}\right)\right]^{\delta} + \left(\frac{i_t}{1+i_t}\right)^{1-\varphi-\theta}\right\} . \tag{C.4}$$

A trivial extension of equation (C.3) for the case of the foreign economy is analyzed by Obstfeld and Rogoff (1996). In that case, the consumption-based price index is defined as

$$P_t^* = P_{mt}^*\left[\varphi^* + \left(1-\varphi^*\right)\left(\frac{i_t^*}{1+i_t^*}\right)^{1-\theta^*}\right]^{1/(1-\theta^*)} , \tag{C.5}$$

where the values of the parameters with superscripts correspond to those of the foreign consumers.

APPENDIX D
The Efficient Method of Moments

This appendix, which is based on Chumacero (1997), presents a brief introduction to the type of efficient method of moments (EMM) estimators that are used in the paper.[24]

Consider a stationary stochastic process $p(y_t \mid x_t, \rho)$ that describes y_t in terms of exogenous variables, x_t, and structural parameters, ρ, which the researcher is interested in estimating. Consider an auxiliary model $f(y_t \mid x_t, \theta)$ that has an analytical expression whereas the $p(.)$ density may not. Gallant and Tauchen (1996) propose using the scores

$$\left. \frac{\partial \ln f\left(y_t \middle| x_t, \theta\right)}{\partial \theta} \right|_{\hat{\theta}_T} ,$$

where

$$\hat{\theta}_T = \arg\max_{\theta \in \Theta} \sum_{t=1}^{T} \ln f\left(y_t \middle| x_t, \theta\right)$$

is the quasi-maximum likelihood estimator of $f(.)$ for a sample of size T, to generate the generalized method of moments (GMM) conditions,

$$m_N(\rho) = \iint (\partial/\partial\theta) \ln f\left(y \middle| x, \hat{\theta}_T\right) p\left(y \middle| x, \rho\right) dy \; p\left(x \middle| \rho\right) dx \, .$$

In cases in which analytical expressions for these integrals are not available, simulation may be required to compute them. In that case I define the moments as

$$m_N(\rho) = \sum_{n=1}^{N} (\partial/\partial\theta) \ln f\left(\tilde{y}_n \middle| \tilde{x}_n, \hat{\theta}_T\right),$$

where N is the sample size of the Monte Carlo integral approximation drawn from one sample of y, x generated for a given value of ρ in the structural model.

24. The interested reader is referred to Gallant and Tauchen (1996) for a complete and formal treatment of this and other setups in which EMM can be applied. Chumacero (1997) presents Monte Carlo evidence showing that this technique is superior to GMM in several dimensions.

The GMM estimator of ρ with an efficient weighting matrix is given by

$$\hat{\rho}_N = \underset{\rho \in \Re}{\arg\min}\ m'_N(\rho)\hat{W}_T^{-1}m_N(\rho)\ , \qquad \text{(D.1)}$$

where, if the auxiliary model constitutes a good statistical description of the data-generating process of y, the outer product of the gradients (OPG) can be used in the weighting matrix; that is,

$$\hat{W} = \frac{1}{T}\sum_{t=1}^{T}\left[(\partial/\partial\theta)\ln f\left(y_t \middle| x_t, \hat{\theta}_T\right)\right]\left[(\partial/\partial\theta)\ln f\left(y_t \middle| x_t, \hat{\theta}_T\right)\right]' . \qquad \text{(D.2)}$$

Gallant and Tauchen (1996) demonstrate the strong convergence and asymptotic normality of the estimator presented in equation (D.1), as well as the asymptotic distribution of the objective function that $\hat{\rho}_N$ minimizes. That is, let k be the dimension of ρ and q the dimension of θ; then

$$T\ J_T = Tm'_N(\hat{\rho})\hat{W}_T^{-1}m_N(\hat{\rho}) \xrightarrow{D} \chi^2_{q-k}\ ,$$

which corresponds to the familiar overidentifying restrictions test described by Hansen (1982). As in GMM, identification requires that $q > k$. Statistical inference may thus be carried out in identical fashion as in GMM. However, depending on the complexity of the auxiliary model, Wald-type tests based on the variance-covariance matrix obtained by differentiating the moments may be difficult to construct.

One major advantage of EMM is that the econometrician does not need to observe directly all the variables in the structural model to compute ρ. This feature is extremely attractive, because in many cases the poor small sample performance of GMM estimators is due to the limited amount of observations that the econometrician has for estimating the structural model.

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