Targeting Inflation in an Economy with Staggered Price Setting

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After experiencing high and persistent inflation rates in the 1970s and early 1980s, most industrialized economies entered the new century with a sustained record of low, stable inflation rates. Many commentators attribute the new environment to good luck, in the form of no major supply shocks (at least until the recent hike in oil prices). Others invoke the magic powers of the new economy to explain why inflation has remained subdued despite robust economic growth. A growing body of research, however, points to a dramatic change in central banks' attitude toward inflation, which appears to have had a significant impact on the way monetary policy is conducted.[1]

Many authors consider the adoption of monetary policy strategies that aim, more or less explicitly, at targeting the inflation rate to be a critical factor behind the new era of macroeconomic stability that now seems to characterize the industrialized world.[2] The present paper inquires into the nature and workings of an inflation-targeting regime, using as a reference framework an optimizing monetary business cycle model with staggered price setting. Such a framework, which integrates Keynesian ingredients into a real-business-cycle-type dynamic general equilibrium apparatus, has in recent years become the workhorse for the analysis of the connection between money, inflation, and

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1. See, for example, Clarida, Galí, and Gertler (2000) and Taylor (1999a) for formal econometric evidence of the new anti-inflationary stance of the Fed and other central banks in the 1980s and 1990s.
2. See Bernanke and others (1999) for a detailed account of the experience of several countries that adopted an explicit inflation-targeting regime.

Inflation Targeting: Design, Performance, Challenges, edited by Norman Loayza and Raimundo Soto, Santiago, Chile.

the business cycle, and the assessment of the desirability of alternative monetary policies.

Among other aspects, the novelty of its treatment of the sources and nature of inflation dynamics makes the new optimizing sticky price models a particularly interesting laboratory for the study of inflation-targeting policies. As discussed in section 1 below, inflation and its variations over time are, under the new paradigm, an immediate consequence of price revisions by profit-maximizing firms, in an environment in which the latter are subject to some constraints on the frequency with which they can adjust their prices. Those constraints may cause firms' markups to deviate from the optimal level, thus inducing periodic revisions in prices and, hence, inflation. Consequently, any policy that aims at targeting the aggregate price level will necessarily have to seek to stabilize firms' markups around their profit-maximizing level. This rather general result does not depend on details of the economy's structure.

On the other hand, the specific form of the rule that implements an inflation-targeting policy will generally depend on the structure of the model, the settings for the parameters describing the economic environment, and the properties of the underlying sources of fluctuations. Section 2 derives the interest rate rule that fully stabilizes the rate of inflation for an example economy. I refer to that rule as the optimal inflation-targeting rule. The section also discusses the conditions under which the equilibrium generated by that rule will be unique.

The nature and form of the optimal inflation-targeting rule suggest that its actual implementation would most likely face many difficulties. That consideration leads many authors to propose a variety of simple rules that approximate the outcome of the optimal one and that may be much easier to implement in practice. Sections 3 and 4 analyze the properties of one such rule in the context of the baseline model with staggered price setting developed in the previous section. The rule is a simplified version of the so-called Taylor rule, under which the monetary authority adjusts the interest rate in response to deviations of inflation from target. Section 3 examines the properties of that rule under the assumption that the monetary authority has access to accurate real-time inflation data. Section 4 analyzes the implications of the presence of measurement error in the inflation data used as the basis for interest rate decisions.

1. Sources of Inflation Dynamics

This section lays out a simple model of inflation dynamics in the presence of staggered price setting and discusses the role of markup variations as a source of those dynamics.

1.1 Price Dynamics

The very idea of price stickiness implies some form of dependence of current prices on lagged prices. The exact form of that dependence, and its mathematical representation, hinges on the precise way that sticky prices are modeled.

In an economy with staggered price setting, only a fraction of firms reset prices in any given period. Let me follow Calvo (1983) in assuming that each firm resets its price in any given period only with probability $1 - \theta$, independently of other firms and of the time elapsed since its last price adjustment.[3] By the law of large numbers, a measure $1 - \theta$ of producers reset their prices each period, while a fraction θ keep their prices unchanged. Let p_t denote the log of the aggregate price level, and let p_t^* denote the log of the price set by firms adjusting prices in period t.[4] The evolution of the price level over time can thus be approximated by the log-linear difference equation,

$$p_t = \theta p_{t-1} + (1-\theta) p_t^* . \tag{1}$$

It follows that the rate of price inflation $\pi_t = p_t - p_{t-1}$ will be given by

$$\pi_t = (1-\theta) \left(p_t^* - p_{t-1} \right). \tag{2}$$

Its simplicity notwithstanding, equation 2 turns out to be critical for understanding the source of inflation and its dynamics. Positive (negative) inflation will arise in such an environment if and only if firms adjusting their prices in the current period choose prices that are, on average, above (below) the average level of prices that prevailed in the economy in the previous period. Understanding aggregate inflation and its fluctuations thus requires a model of how and why firms may want to adjust their relative price periodically. I turn next to the optimal choice of that relative price.

1.2 Optimal Price Setting and Inflation Dynamics

In the context of the present model, a firm that is able to reset prices in period t will choose its price to maximize expected discounted profits given technology, factor prices, and the constraint on price adjustment (defined by the reset probability $1 - \theta$). Log-linearization of

3. King and Wolman (1996), Yun (1996), and Woodford (1996) provide a detailed derivation of an optimal price setting rule under the Calvo formalism.

4. Notice that they will all set the same price, since they are assumed to face an identical problem.

the optimal price setting condition around a zero inflation steady state yields the approximate log-linear rule

$$p_t^* = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{mc_{t+k}^n\right\}. \tag{3}$$

In other words, prices are set as a markup over a weighted average of current and expected future (log) nominal marginal costs, $\left\{mc_{t+k}^n\right\}$, where μ is the optimal frictionless markup (that is, the markup they would choose were they able to adjust prices period by period), also expressed in logs. For simplicity I assume that all firms have access to an identical constant returns technology, such that they all face the same marginal cost, independently of the quantity produced.[5]

Let $\mu_t = p_t - mc_t^n$ denote the economy's average markup in period t. Equation 3 can be rewritten as follows:

$$p_t^* - p_{t-1} = \sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\pi_{t+k}\} - (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\hat{\mu}_{t+k}\}, \tag{4}$$

where $\hat{\mu}_t = \mu_t - \mu$ denotes the deviation of the average markup from the frictionless markup (the markup gap, for short).

Equation 4 clearly points to the two factors that underlie the decision by firms currently adjusting prices to deviate from the average price level prevailing in the previous period. The first term captures the firm's willingness to keep up with the aggregate price level during the life of the price, at least in expected terms; in other words, it aims at maintaining the expected relative price unchanged. The second term reflects the wish to adjust that expected relative price, to avoid any anticipated gap between the expected and desired markups, were the firm not to adjust its relative price.

Equations 2 and 4 can be combined into a simple, first-order expectational difference equation for inflation:

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \lambda\hat{\mu}_t, \tag{5}$$

5. To get some intuition for the form of that rule, let $\mu_{t,t+k} \equiv p_t^* - mc_{t+k}^n$ denote the markup in period $t + k$ of a firm that last set its price in period t. Equation 3 can be rewritten as

$$\mu = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\mu_{t,t+k}\},$$

which yields a simple interpretation of the pricing rule: firms set prices at a level at which an appropriate weighted average of anticipated future markups matches the optimal frictionless markup, μ.

where $\lambda \equiv \left[(1-\theta)(1-\beta\theta)\right]/\theta$. [6]

1.3 Inflation Targeting and Markup Stabilization

I now use π^* to denote the target level of inflation under an inflation-targeting regime and assume that such a target remains constant over time. Equation 5 can be rewritten as

$$\pi_t - \pi^* = -(1-\beta)\pi^* + \beta\left[E_t\{\pi_{t+1}\} - \pi^*\right] - \lambda\hat{\mu}_t .$$

A necessary and sufficient condition for attaining the inflation target every period (that is, $\pi_t = \pi^*$ for all t) is given by:

$$\mu_t = \mu - \left(\frac{1-\beta}{\lambda}\right)\pi^* \equiv \mu^*,$$

where μ^* denotes the constant markup that is consistent with a constant inflation rate, π^*. Thus in the environment under consideration, positive (negative) levels of inflation are necessarily associated with average markups below (above) the desired markup, μ.

To derive equation 5, I have not made use of any assumptions on the underlying sources of fluctuations, the properties and characteristics of monetary and fiscal policy, or any aspects of the economy's structure other than the form of staggered price setting and the associated optimal price setting decisions. A number of lessons can be drawn from this simple framework, however, which may inform the design of a monetary policy strategy that aims at targeting inflation. Three principles are worth emphasizing. First, current inflation is a function of current and expected future average markups. Stabilizing inflation necessarily requires that markups be stabilized. Second, a zero inflation target can be achieved by maintaining markups constant at their frictionless level. In that case, all firms will be maximizing profits at current prices and, accordingly, no firm will have an incentive to adjust its price. As a result, the aggregate price level will be stabilized. Third, attaining a positive inflation target requires holding the average markup below its frictionless level. Only in that case will firms adjusting prices in any given period choose to set a price above the average price in the previous period (the latter being a condition for positive inflation).

6. See Sbordone (1999), Galí and Gertler (1999), and Galí, Gertler, and López-Salido (2001) for econometric evidence supporting the empirical relevance of that inflation equation.

2. A Baseline Model with Staggered Price Setting

This section lays out a simple macroeconomic framework in which the model of inflation dynamics developed in the previous section is embedded. For simplicity, and to focus on the essential aspects, the baseline model abstracts from capital accumulation and the external sector. Next I briefly describe the main assumptions and derive the key equilibrium conditions.[7]

2.1 Households

The representative consumer is infinitely lived and seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right), \tag{6}$$

subject to a standard sequence of budget constraints and a solvency condition. N_t denotes hours of work, and C_t is a CES aggregator of the quantities of the different goods consumed:

$$C_t = \left[\int_0^1 C_t(i)^{(\varepsilon-1)/\varepsilon} di \right]^{\varepsilon/(\varepsilon-1)} .$$

The aggregate price index is represented by

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} di \right]^{1/(1-\varepsilon)} ,$$

where $P_t(i)$ denotes the price of good $i \in [0,1]$. The solution to the consumer's problem can be summarized by means of three optimality conditions (two static and one intertemporal), which I represent in log-linearized form, ignoring nonessential constants. Henceforth, lower case letters denote the logarithms of the original variables.

First, the optimal allocation of a given amount of expenditures among the different goods generates the set of demand schedules

$$c_t(i) = -\varepsilon [p_t(i) - p_t] + c_t , \tag{7}$$

for all $i \in [0,1]$.

7. See, for example, King and Wolman (1996), Yun (1996), and Woodford (1996, 2000) for a detailed derivation of the equilibrium conditions in a similar model. Galí and Monacelli (2000) extend the model to an open economy, and a version of the model with capital accumulation is discussed in Woodford (2000).

Second, under the assumption of a perfectly competitive labor market, the supply of hours must satisfy the condition

$$w_t - p_t = \sigma c_t + \varphi n_t, \quad (8)$$

where w_t is the (log) nominal wage.

Finally, the intertemporal optimality condition is given by the Euler equation,

$$c_t = -\frac{1}{\sigma}\left[r_t - E_t\{\pi_{t+1}\} - \rho\right] + E_t\{c_{t+1}\}, \quad (9)$$

where r_t is the yield on a nominally riskless one-period bond (the nominal interest rate, for short) and $\rho \equiv -\log\beta$ is the discount rate.

2.2 Firms

I assume a continuum of firms, each producing a differentiated good with a technology

$$y_t(i) = a_t + n_t(i),$$

where a_t follows an exogenous, unspecified stochastic process.

Total demand for each good is given in levels by

$$Y_t(i) = C_t(i) + G_t(i),$$

where G_t denotes government purchases. For simplicity, I assume that the government consumes a fraction τ_t of the output of each good. Government expenditures are financed through lump sum taxes. Letting $g_t = -\log(1-\tau_t)$, the demand for good i in log-linear form can be rewritten as follows:[8]

$$y_t(i) = c_t(i) + g_t.$$

Given the previous demand schedule, a monopolistically competitive firm that faced no constraints on the frequency of price adjustment would choose a constant optimal gross markup (in logs) equal to $\mu \equiv \log[\varepsilon/(\varepsilon-1)]$.

Let aggregate output be denoted by

$$Y_t \equiv \left[\int_0^1 Y_t(i)^{(\varepsilon-1)/\varepsilon}\, di\right]^{\varepsilon/(\varepsilon-1)}$$

8. One can also reinterpret g_t as a shock to preferences or, more broadly, as any other exogenous component of aggregate demand.

The clearing of all goods markets implies

$$y_t = c_t + g_t , \tag{10}$$

where $y_t \equiv \log Y_t$. Combining the previous market clearing condition with Euler equation 9 yields the equilibrium condition

$$y_t = -\frac{1}{\sigma}\left[r_t - E_t\{\pi_{t+1}\} - \rho\right] + E_t\{y_{t+1}\} - E_t\{\Delta g_{t+1}\} . \tag{11}$$

In addition, and letting $n_t = \log \int_0^1 N_t(i)\,di$, one can derive the following mapping between labor input and output aggregates:[9]

$$n_t = y_t - a_t . \tag{12}$$

The assumption of a constant returns technology implies that all firms face a common nominal marginal cost, given by $w_t - a_t$. The economy's average markup will thus be given by

$$\mu_t = a_t - (w_t - p_t) . \tag{13}$$

Combining equations 8, 12, 10, and 13 generates an expression for the equilibrium average markup in terms of aggregate output, productivity, and government purchases:

$$\mu_t = -(\sigma + \varphi) y_t + (1 + \varphi) a_t + \sigma g_t . \tag{14}$$

In an equilibrium with fully flexible prices, the average markup remains constant at a level μ. The equilibrium level of output is then given by

$$\bar{y}_t = \gamma + \psi_a a_t + \psi_g g_t , \tag{15}$$

where $\psi_a = (1+\varphi)/(\sigma+\varphi)$, $\psi_g = \sigma/(\sigma+\varphi)$, and $\gamma = -\mu/(\sigma+\varphi)$. Henceforth, I refer to the above equilibrium value as the natural level of output or, for short, potential output.

If firms do not adjust prices optimally each period, average markups will no longer be constant. Furthermore, firms' inability to adjust prices optimally every period generally implies the existence of a wedge between output and its natural level. I denote that wedge, or output gap, by $x_t \equiv y_t - \bar{y}_t$. The relationship between the markup gap and the output

9. For nondegenerate distributions of prices across firms, the previous equation holds up only to a first-order approximation. See Yun (1996) and King and Wolman (1996) for a detailed discussion.

gap can be derived from equation 14 and the exogeneity of a_t and g_t:

$$\hat{\mu}_t = (\sigma + \varphi)\ x_t\,. \tag{16}$$

Combining equations 5 and 16 yields the familiar New Keynesian Phillips Curve:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t\,, \tag{17}$$

where $\kappa \equiv \lambda\,(\sigma + \varphi)$.

Finally, equilibrium condition 11 can be rewritten in terms of the output gap:

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma}\left[r_t - E_t\{\pi_{t+1}\} - \rho\right] + \psi_a E_t\{\Delta a_{t+1}\} - (1 - \psi_g) E_t\{\Delta g_{t+1}\},$$

or, equivalently,

$$x_t = -\frac{1}{\sigma}\left[r_t - E_t\{\pi_{t+1}\} - \overline{rr}_t\right] + E_t\{x_{t+1}\}\,, \tag{18}$$

where

$$\overline{rr}_t = \rho + \sigma\psi_a E_t\{\Delta a_{t+1}\} - \sigma(1 - \psi_g) E_t\{\Delta g_{t+1}\}\,. \tag{19}$$

This last equation represents the natural interest rate, that is, the expected real rate of return on a one-period bond that would prevail in an equilibrium under flexible prices.

3. The Optimal Inflation-targeting rule

In the baseline model developed above, a monetary authority seeking to stabilize inflation around a constant target, π^*, can fully attain its goal without facing any costs in terms of output gap instability. Achieving that goal requires that the economy's average markup be kept constant, at a level given by

$$\mu_t = \mu - \left(\frac{1-\beta}{\lambda}\right)\pi^* \equiv \mu^*,$$

for all t. That condition, in turn, corresponds to a constant output gap, as shown by equation 16. Formally, inflation targeting implies that

$$x^* = \left(\frac{1-\beta}{\kappa}\right)\pi^*,$$

for all t. Supporting a permanent non-zero inflation rate thus requires a proportional, permanent deviation of output from its natural level. In other words, the economy is characterized by a non-vertical long-run Phillips curve, at least for levels of inflation that are sufficiently close to zero for the linear approximation to be satisfactory.[10]

The next step is to derive the equilibrium path for the nominal rate that is consistent with a constant inflation π^*. Given equation 18, the implied nominal rate, denoted by r_t^* is given by

$$\begin{aligned} r_t^* &= \overline{rr}_t + \pi^* \\ &= \rho + \pi^* + \sigma\psi_a E_t\{\Delta a_{t+1}\} - \sigma(1-\psi_g) E_t\{\Delta g_{t+1}\}, \end{aligned} \tag{20}$$

where $\psi_a = (1+\varphi)/(\sigma+\varphi)$ and $\psi_g = \sigma/(\sigma+\varphi)$, as above.

The behavior of the equilibrium interest rate can be easily grasped by considering the response of consumption to both technology and fiscal shocks in the flexible price case. An anticipated fiscal expansion, corresponding to $E_t\{\Delta g_{t+1}\} > 0$, is associated, in the flexible price equilibrium, with an expected decline in consumption, which can only be supported with a lower real rate. On the other hand, the anticipation of higher productivity growth (that is, $E_t\{\Delta a_{t+1}\} > 0$) will bring about the expectation that consumption will gradually adjust to its new, higher plateau; supporting that response pattern requires a higher interest rate.

3.1 Implementation: Ruling Out Indeterminacy

Equation 20 cannot be interpreted as a monetary policy rule that the central bank could follow mechanically or that would guarantee the attainment of the optimal allocation. This can be demonstrated by plugging equation 20 into equation 18. The equilibrium dynamics for inflation and the output gap, expressed as deviations from their target levels, can then be represented by means of the stochastic difference equation,

$$\begin{bmatrix} \hat{x}_t \\ \hat{\pi}_t \end{bmatrix} = \mathbf{A_0} \begin{bmatrix} E_t\{\hat{x}_{t+1}\} \\ E_t\{\hat{\pi}_{t+1}\} \end{bmatrix}, \tag{21}$$

where $\hat{x}_t = x_t - x^*$, $\hat{\pi}_t = \pi_t - \pi^*$, and

10. See King and Wolman (1996) for a careful analysis of the steady-state relationship between markups, inflation, and the output gap.

$$\mathbf{A_0} \equiv \frac{1}{\sigma} \begin{bmatrix} \sigma & 1 \\ \kappa\sigma & \kappa + \beta\sigma \end{bmatrix}.$$

Notice that $\hat{x}_t = \hat{\pi}_t = 0$, for all t, always constitutes a solution to equation 21. In other words, an allocation consistent with a rate of inflation that remains constant at its target level is always an equilibrium. However, a necessary and sufficient condition for the uniqueness of such a solution in a system with no predetermined variables like equation 21 is that the two eigenvalues of $\mathbf{A_0}$ lie inside the unit circle.[11] It is easy to check that such a condition is not satisfied in our case. More precisely, while both eigenvalues of $\mathbf{A_0}$ can be shown to be real and positive, only the smallest one lies in the (0,1) interval.[12] A continuum of solutions in the neighborhood of (0,0) thus satisfy the equilibrium conditions (local indeterminacy). Furthermore, one cannot rule out the possibility of equilibria displaying fluctuations driven by self-fulfilling revisions in expectations (stationary sunspot fluctuations).

The previous indeterminacy problem can be avoided—and the uniqueness of the equilibrium with constant inflation restored—by having the central bank follow a rule that would make the interest rate respond to inflation or the output gap (or both) should those variables deviate from their target values. Suppose that the central bank commits itself to following the rule:

$$r_t = \pi^* + \overline{rr}_t + \phi_\pi \hat{\pi}_t + \phi_x \hat{x}_t . \tag{22}$$

In that case, the equilibrium is described by a stochastic difference equation like equation 21, replacing $\mathbf{A_0}$ with

$$\mathbf{A_T} = \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \kappa\sigma & \kappa + \beta(\sigma + \phi_x) \end{bmatrix},$$

where $\Omega = 1/(\sigma + \phi_x + \kappa\phi_\pi)$. If ϕ_π and ϕ_x are restricted to nonnegative values, then a necessary and sufficient condition for $\mathbf{A_T}$ is to have both eigenvalues inside the unit circle, thus implying the uniqueness of the (0,0) solution to equation 21. This is given by[13]

$$\kappa(\phi_\pi - 1) + (1 - \beta)\ \phi_x > 0. \tag{23}$$

11. See, for example, Blanchard and Kahn (1980).
12. Bullard and Mitra (2000) provide a formal proof.
13. Again, see Bullard and Mitra (2000) for a formal proof.

Henceforth, I refer to a rule of the form specified in equation 22, which satisfies the determinacy condition 23, as the optimal inflation-targeting rule.

Once uniqueness is restored, the term $\phi_\pi \hat{\pi}_t + \phi_x \hat{x}_t$ appended to the optimal rule vanishes, implying that $r_t = \pi^* + \overline{rr}_t$, for all t. Stabilization of the output gap and inflation thus requires a credible threat by the central bank to vary the interest rate sufficiently in response to any deviations of inflation or the output gap from target, yet the very existence of that threat makes its effective application unnecessary.

3.2 Implementation: Practical Difficulties

The above analysis suggests that the implementation of a successful inflation-targeting policy is not an easy task. Some of the difficulties can be illustrated in the context of the model.[14] To begin with, the specific form of the optimal inflation-targeting rule will not generally be robust to changes in some of the model characteristics; its correct application thus hinges on knowledge of both the true model and the values taken by all its parameters. Second, the implementation of the rule requires the use of unbiased forecasts of the future path of all underlying exogenous disturbances. It is in that sense that the optimal inflation-targeting rule is forward looking.

Neither condition is likely to be met in practice by any central bank, given the well-known problems associated with measuring variables like total factor productivity, not to mention the practical impossibility of detecting exogenous shifts in some parameters that may be unobservable by nature (for example, parameters describing preferences).[15]The practical difficulties of implementing rules of the sort considered here have led many authors to propose a variety of simple rules—namely, rules that make the policy instrument depend on observable variables and that do not require knowledge of any primitive parameters—and to evaluate their desirability in the context of alternative models.[16]

14. See Blinder (1998) for a general discussion of the practical complications facing central bankers in the design and implementation of monetary policy.

15. Given the likely difficulties in measuring the output gap, the previous argument would also seem to apply to the central bank's need to respond to that variable to avoid the indeterminacy problem. It is clear from equation 23, however, that the equilibrium is unique even if $\phi_x = 0$, so long as $\phi_\pi > 1$.

16. A large number of recent papers seek to analyze the properties and desirability of many such rules. See, for example, the contributions by several authors contained in the Taylor (1999b) volume.

The next section introduces and analyzes the properties of an alternative rule whose practical implementation is likely to be less demanding than the optimal inflation-targeting rule. The rule considered is a simplified version of the well-known Taylor rule. It is shown that the rule can approximate arbitrarily well the desired outcome of inflation stability, without generating any side effects.

4. A Simple, Inflation-Based Interest Rate Rule

This section analyzes the properties of an interest rate rule of the form

$$r_t = r + \phi_\pi \left(\pi_t - \pi^* \right), \tag{24}$$

where $r = \overline{rr} + \pi$ and $\overline{rr} = E\left\{ \overline{rr}_t \right\}$ are, respectively, the steady-state nominal rate and the unconditional mean of the natural interest rate. I also assume that $\phi_\pi > 1$, that is, that the nominal rate responds to current inflation on a more than one-to-one basis. That condition guarantees the local uniqueness of a rational expectations equilibrium. The previous rule is just a simplified version of the rule put forward by John Taylor (1993) as a good characterization of recent U.S. monetary policy. The simplification consists in omitting an output-related term that is present in the original Taylor rule. The justification for that omission is twofold. First, in a model like the one considered here, an inflation-based rule is known to perform better than a more conventional Taylor rule that would have the central bank respond to detrended output as well.[17] Furthermore, it does not perform significantly worse than a rule in which the central bank responds to the true output gap x_t (and which could hardly qualify as a simple rule, given the inherent difficulties in measuring the latter variable). Second, an inflation-based rule would seem to be more tightly connected with an inflation-targeting strategy. In what follows, I refer to the rule above as an inflation-based rule or, for short, a π-rule.

Relative to the optimal inflation-targeting rule derived in the previous section, a rule like that in equation 24 clearly offers practical advantages in terms of transparency, knowledge requirements, implementa-

17. See Rotemberg and Woodford (1999); Levin, Wieland, and Williams (1999); Galí (2000).

tion using real-time data, and so forth. This greater simplicity stems from the fact that under rule 24, the monetary authority does not need to know the contemporaneous value of the natural interest rate, $\overline{rr}_t$, when setting its instrument every period. It should also be clear that a π-rule will not generally succeed in stabilizing inflation fully. As discussed above, to keep inflation constant at its target level, the nominal interest rate would have to change one-for-one with the natural rate, $\overline{rr}_t$. The π-rule will not generate any change in the nominal rate, however, unless inflation deviates from target. Only in the particular case in which the natural interest rate is itself constant will the π-rule in equation 24 succeed in fully stabilizing inflation. In the baseline model developed above, that will be the case if and only if all changes in productivity or government purchases are fully unpredictable.

The previous result, as well as other interesting insights, can be shown more formally by deriving the stochastic difference equation satisfied by the economy's equilibrium when the monetary authority follows the rule given by equation 24. I assume, for simplicity, that the natural interest rate evolves over time according to an exogenous stationary AR(1) process:

$$\overline{rr}_t = \left(1-\rho_r\right)\overline{rr} + \rho_r \overline{rr}_{t-1} + \varepsilon_t ,$$

where $\rho_r \in [0,1)$ and ε_t is white noise with zero mean and variance σ_ε^2. The equilibrium dynamics can then be represented by means of the following system:

$$\begin{bmatrix} \hat{x}_t \\ \hat{\pi}_t \end{bmatrix} = \mathbf{A}_\Pi \begin{bmatrix} E_t\{\hat{x}_{t+1}\} \\ E_t\{\hat{\pi}_{t+1}\} \end{bmatrix} + \mathbf{B}_\Pi \left(\overline{rr}_t - \overline{rr}\right),$$

where $\hat{x}_t = x_t - x^*$, $\hat{\pi}_t = \pi_t - \pi^*$,

$$\mathbf{A}_\Pi = \Theta \begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \sigma\kappa & \kappa+\beta\sigma \end{bmatrix}; \mathbf{B}_\Pi = \Theta \begin{bmatrix} 1 \\ \kappa \end{bmatrix},$$

and $\Theta = 1/\left(\sigma + \kappa\phi_\pi\right)$.

The maintained assumption that $\phi_\pi > 1$ guarantees that both eigenvalues of $\mathbf{A}_\Pi$ lie within the unit circle, thus implying a unique rational expectations equilibrium.[18] Furthermore, it is clear that $\hat{x}_t = \hat{\pi}_t = 0$, for all *t*, will be a solution if and only if $\overline{rr}_t = \overline{rr}$, for all *t*, that is, if and only if the natural interest rate is constant.

18. Notice that $\mathbf{A}_\Pi$ corresponds to $\mathbf{A}_T$ when $\phi_x = 0$, implying that the condition for determinacy specified in equation 23 simplifies to $\phi_\pi > 1$.

How large are the size and persistence of the deviations from the inflation target implied by a simple π-rule, when the natural interest rate varies over time? A number of researchers have praised the performance of similar rules along different criteria and in a variety of models.[19] Here I carry out a simple quantitative exercise that focuses on the ability of the π-rule to stabilize inflation around the target. This involves analyzing the above model quantitatively under three alternative values for ρ_r, the parameter that controls the persistence of the natural interest rate: 0 (no persistence), 1/3 (low persistence), and 3/4 (high persistence). For any given choice of ρ_r, the parameter σ_ε^2 is then set at a level such that the annualized natural real rate has a standard deviation of 1 percent. That calibration is admittedly arbitrary, but it provides a useful benchmark against which to assess the nominal rate volatility implied by the π-rule.

For the remaining parameters, I assume a logarithmic utility for consumption, which corresponds to $\sigma = 1$. I also set $\varphi = 1$, which implies a unit wage elasticity of labor supply. The choice for θ is 0.75; this implies an average price duration of one year, a value in line with both econometric estimates of θ and survey evidence. Finally, I set $\beta = 0.995$.[20]

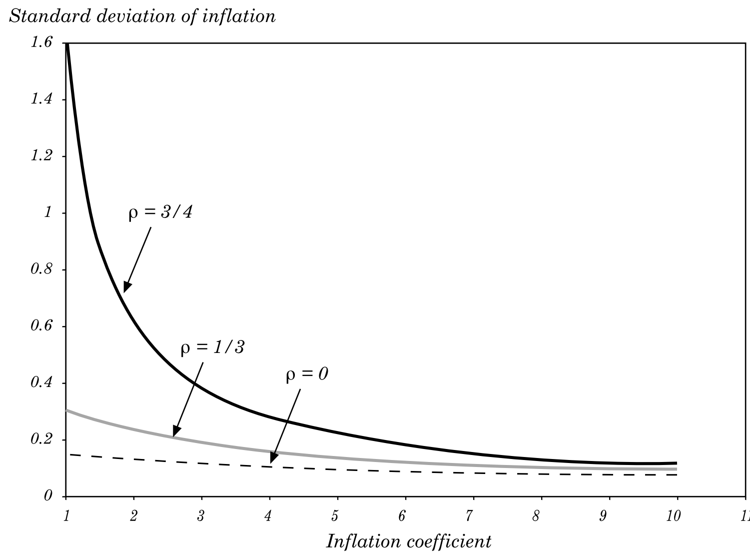
Figures 1 through 3 display a number of statistics that summarize the properties of the equilibrium under a π-rule, for the three configurations of ρ_r and σ_ε^2 considered and a range of values greater than unity for ϕ_π. A number of results are worth stressing. First, higher values for ϕ_π—that is, a more aggressive response to inflation by the monetary authority—have clear stabilizing effects on inflation and the output gap. Thus the standard deviation of both (annualized) inflation and the output gap, shown respectively in figures 1 and 2, decreases monotonically as ϕ_π is raised, independently of ρ_r. The highest value of the inflation coefficient displayed ($\phi_\pi = 10$) causes the residual volatility in either variable to become almost negligible in all cases. In fact, the equilibrium allocation under a π-rule converges to that of the optimal inflation-targeting rule as ϕ_π approaches infinity. That property can be checked analytically by noticing that $\lim_{\phi_\pi \to \infty} \mathbf{B_\Pi} = 0$, while $\lim_{\phi_\pi \to \infty} \mathbf{A_\Pi} = \mathbf{C_\Pi}$ is well defined and bounded, with

$$\mathbf{C_\Pi} = \begin{bmatrix} 0 & -\beta\kappa^{-1} \\ 0 & 0 \end{bmatrix}.$$

19. See, for example, the introduction to the Taylor (1999b) volume for a discussion.

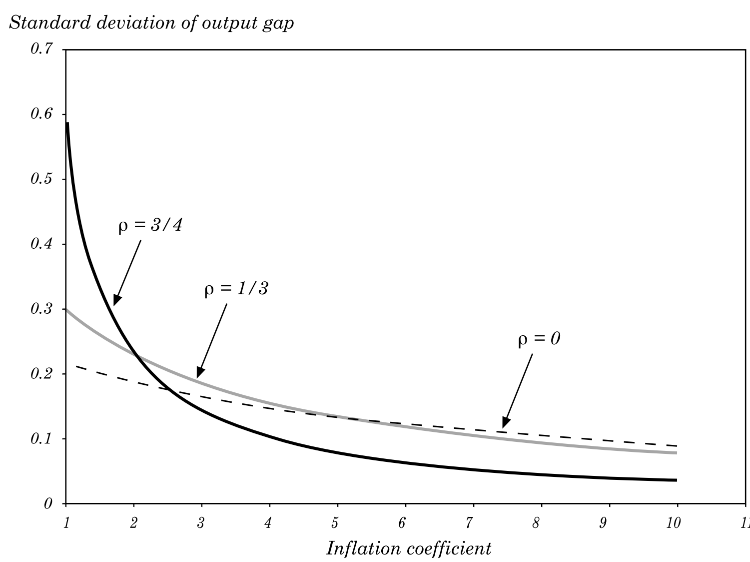
20. Notice that under the assumption of an average annual growth rate of 2 percent—which would correspond to $E\{\Delta a_t\} = 0.005$—the above settings for β and σ are consistent with an average annualized real interest rate, $\overline{rr}$, of 4 percent.

Figure 1. Inflation Volatility under an Inflation-Based Rule



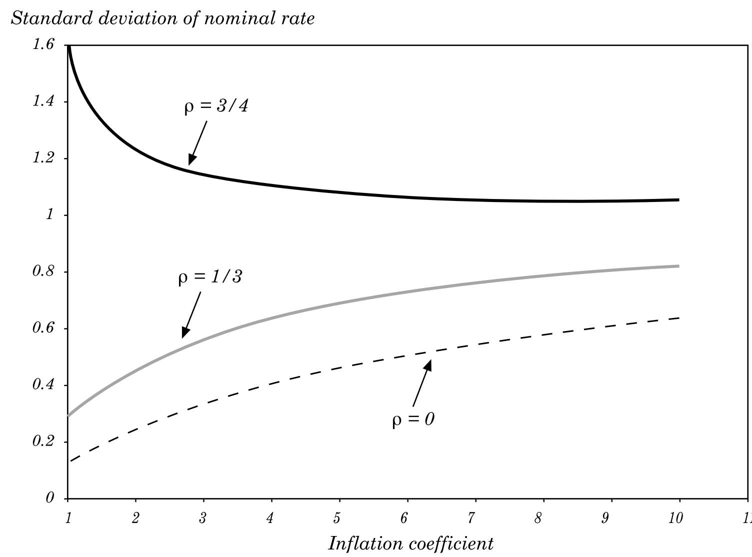
Source: Author's calculations.

Figure 2. Output Gap Volatility under an Inflation-Based Rule



Source: Author's calculations.

Figure 3. Nominal Rate Volatility under an Inflation-Based Rule



Source: Author's calculations.

Second, for any given value of ϕ_π, the volatility of inflation increases with the persistence of the natural interest rate, even though the standard deviation of the latter remains constant, by construction. Thus while the standard deviation of inflation remains very low (less than 15 basis points) and is little sensitive to changes in ϕ_π when $\rho_r = 0$ (no persistence), it behaves quite differently when the natural rate displays more persistence. For $\rho_r = 3/4$, the standard deviation of inflation is as high as 1 percent when $\phi_\pi = 1.5$ (Taylor's original inflation coefficient). Yet, as shown in figure 2, no such monotonic relationship exists between ρ_r and output gap volatility; instead the sign of that relationship seems to depend on the value of the inflation coefficient ϕ_π.

Third, a possible concern with a rule that requires a very strong response of the interest rate to deviations of inflation from target is that it may be a source of high interest rate volatility. This may be particularly worrisome in economies with low inflation targets, since it raises the possibility of hitting the zero bound constraint and perhaps plunging the economy into a liquidity trap. The analysis of the calibrated model helps dispel some of those worries, however. As shown in figure 3, the two scenarios with little or no persistence in the natural rate process ($\rho_r = 0$ and $\rho_r = 1/3$) feature an equilibrium nominal rate that is less volatile than the natural rate itself, for all values of ϕ_π considered.[21] That result no longer holds when fluctuations in the natural real rate are highly persistent ($\rho_r = 3/4$)—the nominal rate is more volatile than its natural counterpart. Interestingly, in the latter case volatility is decreasing in ϕ_π: the stronger the interest rate response to inflation, the lower the volatility of the interest rate. The reason for that seeming paradox is simple. For high values of ρ_r, the gains in inflation stabilization resulting from an increase in ϕ_π more than offset the potential instability resulting from the more aggressive response to a given change in inflation.

How does the nominal rate behave in the limit, as ϕ_π approaches infinity? By construction, its movements over time must match those of the natural rate; in particular its standard deviation must correspond to the latter's (here calibrated at 1 percent). The intuition behind that result is straightforward. Inflation converges to a constant (π^*) as ϕ_π approaches infinity, but as discussed in section 2, the only path for the nominal rate that can support that outcome is given by $r_t = \pi^* + \overline{rr_t}$, for all t. Accordingly, as $\phi_\pi \rightarrow +\infty$ the standard deviation of the nominal rate must converge to that of the natural rate. This common asymptotic behavior is also apparent in figure 3.

Under the maintained assumptions, the analysis points to a very simple policy recommendation for a central bank that seeks to stabilize inflation fluctuations around a target level (and that wishes to follow a simple rule): the outcome associated with the unfeasible optimal inflation-targeting rule can be approximated arbitrarily well by having the central bank commit to a sufficiently strong interest rate response to any deviation of inflation from target. In particular, following a π-rule with a relatively large inflation coefficient would succeed in stabilizing inflation as much as desired.

In practice, however, things are likely to be somewhat more complicated, and even a simple rule with the desirable properties considered here may not perform as well as indicated. The next section analyzes the complications that arise from the presence of error or noise in the real-time inflation measures on which the monetary authority would have to base its decisions if it were to follow a simple π-rule.

21. Recall that the standard deviation of the natural interest rate has been normalized to unity.

5. The Performance of a Simple Inflation-Based Rule in the Presence of Noisy Inflation Data

What would prevent a central bank from responding very strongly to inflation when, as in the model above, that rule leads to an outcome arbitrarily close to that generated by the optimal (but hard to implement) inflation-targeting rule? A frequent argument against the choice of such a policy strategy is that it carries the risk of potentially huge interest rate volatility should inflation deviate from its target. Yet the preceding discussion shows that interest rate volatility remains bounded largely as a result of the lower volatility of inflation resulting from the more aggressive policy. In the limit, as $\phi_\pi \to +\infty$ the standard deviation of the nominal rate must converge to that of the natural rate (1 percent in the calibration above). It is possible, however, that measured inflation could experience some residual variation in spite of the aggressive anti-inflation stance, perhaps due to the presence of measurement error. What, then, would be the consequences of an aggressive inflation-based rule on the volatility of true underlying inflation? Would it lead to excessive instrument instability?

To address that potential problem, I analyzed the equilibrium of the baseline model under the assumption that the monetary authority follows a noise-ridden Taylor rule of the form

$$r_t = r + \phi_\pi \left(\tilde{\pi}_t - \pi^* \right), \tag{25}$$

where $\tilde{\pi}_t$ denotes measured inflation at the time the interest rate decision is made and is given by $\tilde{\pi}_t = \pi_t + \xi_t$

where ξ_t represents the measurement error. Notice that $\tilde{\pi}_t$ can be interpreted as the monetary authority's best estimate of current inflation when the interest rate is set; π_t is then the actual (or revised) level of inflation. The latter is assumed to be known only with some delay, and not to be used as a policy input.

The equilibrium dynamics can now be represented by means of the system

$$\begin{bmatrix} \hat{x}_t \\ \hat{\pi}_t \end{bmatrix} = \mathbf{A}_\Pi + \begin{bmatrix} E_t\{\hat{x}_{t+1}\} \\ E_t\{\hat{\pi}_{t+1}\} \end{bmatrix}, \tilde{B}_\Pi \begin{bmatrix} \overline{rr}_t - \overline{rr} \\ \xi_t \end{bmatrix},$$

where

$$\tilde{B}_\Pi = \Theta \begin{bmatrix} 1 & -\phi_\pi \\ \kappa & -\kappa\phi_\pi \end{bmatrix}.$$

In this system, deviations of inflation and the output gap from target are the result of both fluctuations in the natural interest rate and inflation noise. A close look at the system reveals that the coefficients associated with inflation noise (that is, the elements of the second column of $\tilde{B}_{\Pi}$) have an absolute value that is increasing in ϕ_π. In and of itself, therefore, the presence of noise would call for a mild interest rate response to measured inflation, in order to minimize the volatility of true inflation and the output gap. That observation has to be balanced, however, with the previous finding of a negative relationship between those volatility measures and the strength of the interest rate response to the changes in inflation induced by fluctuations in the natural rate.

The tradeoff can be easily illustrated based on the assumption that both ξ_t and $\overline{rr}_t$ follow independent white noise processes. In that case the variance of inflation is given by

$$\text{var}(\pi_t) = \left(\frac{\kappa}{\sigma + \kappa\phi_\pi}\right)^2 \left[\text{var}(\overline{rr}_t) + \phi_\pi^2 \,\text{var}(\xi_t)\right]. \tag{26}$$

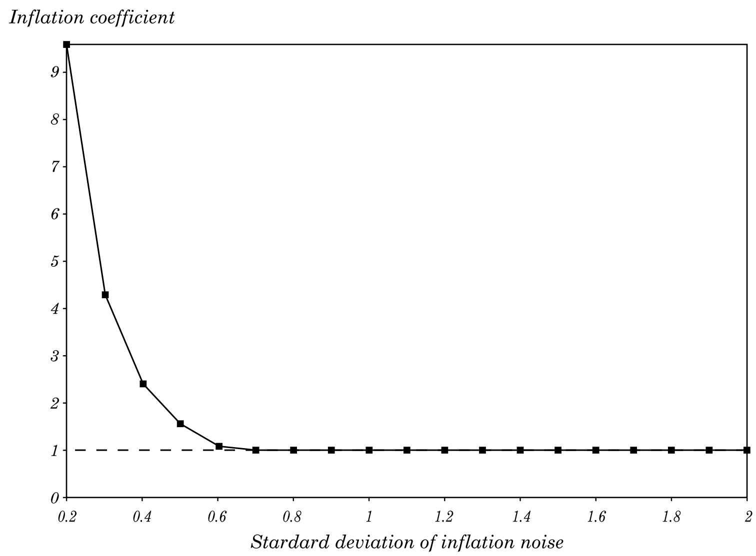
The value of the inflation coefficient, ϕ_π, that minimizes the previous expression subject to the constraint $\phi_\pi \geq 1$ (which guarantees uniqueness) is given by

$$\phi_\pi^* = \max\left\{\frac{\kappa\,\text{var}(\overline{rr}_t)}{\sigma\,\text{var}(\xi_t)}, 1\right\}.$$

Hence, the strength of the interest rate response that minimizes inflation volatility is a function of the volatility of the noise term relative to that of the natural rate. Consistent with the findings of the previous section, the analysis shows that $\lim_{\text{var}(\xi)\to 0} \phi_\pi^* = +\infty$. In other words, as the magnitude of inflation noise vanishes, it is optimal for the central bank to respond more aggressively to deviations of inflation from target. As the importance of the noise term increases, however, the size of the optimal inflation coefficient and thus the strength of the optimal response to deviations of inflation from target decrease monotonically.

The results of several simulations under different values for ρ_r suggest that the previous logic carries over to the case of a persistent natural rate process. Figure 4 illustrates this point by displaying the value of ϕ_π that minimizes the volatility of inflation as a function of the standard deviation of the noise term, under the assumption that $\rho_r = 1/3$. For reasonably low values of noise volatility, the optimal inflation coefficient rapidly attains values above those suggested by Taylor (1993) and others as empirically plausible.

Figure 4. Inflation Stabilization with Noisy Inflation



Source: Author's calculations.

What happens in the limiting case? First, as the inflation coefficient, ϕ_π, approaches infinity,

$$\lim_{\phi_\pi \to \infty} \mathbf{B}_{\Pi} = \begin{bmatrix} 0 & -\kappa^{-1} \\ 0 & -1 \end{bmatrix}.$$

It then follows that, in the limiting case, the output gap and true inflation evolve according to

$$x_t = x^* - \frac{1}{\kappa}\xi_t \text{ and}$$

$$\pi_t = \pi^* - \xi_t ,$$

whereas measured inflation, $\tilde{\pi}_t$, is fully stabilized at a level given by the inflation target, π^*. Furthermore, combining these results with equation 18 shows that the nominal interest rate will evolve in that case according to the process $r_t = \pi^* + \overline{rr}_t + (\sigma/\kappa)\ \xi_t$. Even if the monetary authority changes interest rates very aggressively in response to inflation measures that are partly ridden with error, the volatility of inflation, the output gap, and the nominal rate all remain bounded. Nevertheless, a central bank seeking to minimize the volatility of inflation will find it optimal to choose a finite value for ϕ_π (that is, a more moderate response to inflation deviations from target).

6. Concluding Remarks

The present paper has analyzed the workings of an inflation-targeting regime in the context of an optimizing monetary business cycle model with staggered price setting. Under that paradigm, inflation and its variations over time are a consequence of deviations of markups from their flexible price level. Any policy that aims at stabilizing inflation around a constant target will require that the economy's average markup be stabilized.

While this result is quite general, the specific form of the rule that implements an inflation-targeting policy (the optimal inflation-targeting rule) will generally depend on the structure of the model, the settings for the parameters describing the economic environment, and the properties of the underlying sources of fluctuations. That observation suggests that the actual implementation of the optimal inflation-targeting rule would most likely face many difficulties, and it motivates the search for simple rules and the analysis of their properties. Sections 3 and 4 analyzed the properties of a simplified version of the so-called Taylor rule, which I referred to as the π-rule, under which the monetary authority adjusts the interest rate in response to deviations of inflation from target. Two results are worth emphasizing. First, in the absence of significant measurement error in the inflation data, a simple Taylor rule can approximate the outcome of the optimal inflation-targeting policy arbitrarily well, as long as the interest rate response to movements in inflation is sufficiently strong. Second, if the inflation data is ridden with error, choosing too large a value for the inflation coefficient, ϕ_π, may cause the volatility of inflation to rise. The size of the coefficient that minimizes the volatility of inflation is shown to be finite and inversely related to the volatility of the noise term.

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