

# OPTIMAL MONETARY POLICY IN A SMALL, OPEN ECONOMY: A GENERAL-EQUILIBRIUM ANALYSIS

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The two central issues in monetary policy are separated by time horizon. The first relates to the short run: what is the appropriate monetary policy across the business cycle? The second relates to the long run: what is the optimal long-run rate of inflation? This paper explores these classic issues from the vantage point of a small, open economy. "Smallness" opens up both possibilities and pitfalls for the policymaker, because several important variables (such as the foreign interest rate, the real exchange rate, and the worldwide real rate of interest) are now exogenous from the perspective of the home country.

The short-run focus of monetary policy is to determine how best to respond to real shocks buffeting the economy. These real shocks include fiscal disturbances, fluctuations in multifactor productivity, and movements in real exchange rates. The central banker must make decisions in real time in the face of severe data problems. In particular, it is typically unlikely that the central bank can respond directly to fundamental shocks; instead it must respond to movements in endogenous variables. For example, the central bank will not immediately observe movements in productivity but instead must respond to movements in the rate of inflation.

These informational restrictions imply that the central bank must use a fairly simple policy rule, namely, a reaction function linking movements in the nominal interest rate to these movements in endogenous variables. One such rule is the celebrated Taylor (1993)

rule. The last several years have seen numerous theoretical analyses of the welfare advantages of different policy rules. Papers include Carlstrom and Fuerst (1995), Ireland (1997), King and Wolman (1996), and Rotemberg and Woodford (1999). These papers posit structural models of the real economy and the monetary transmission mechanism. A principal conclusion of this line of research is that the welfare gain from operating under the first-best rule rather than the second- or even the tenth-best rule is small. For example, in a limited participation model, Carlstrom and Fuerst (1995) report that the welfare gain from switching from a money growth peg (a seemingly disastrous policy in the posited model) to an interest rate peg (the second-best policy) equals only 0.017 percent of the steady-state capital stock. (A limited participation model assumes that only a subset of the economy directly participates in open market operations, and thus only this subset has immediate access to the new cash injected into the economy by the central bank.)

As a consequence of these small welfare numbers, the focus of recent research has shifted to a related question: how to avoid doing harm. Relevant papers include Clarida, Galí, and Gertler (1997), Kerr and King (1996), Benhabib, Schmitt-Grohe, and Uribe (1998), Carlstrom and Fuerst (1998, 2000, and forthcoming), and Christiano and Gust (1999). A central bank that follows a rule in which it responds to endogenous variables may introduce real indeterminacy and sunspot equilibria into an otherwise determinate economy.<sup>1</sup> These sunspot fluctuations are welfare-reducing and potentially quite large. An important focus of this paper is to isolate the conditions sufficient to ensure that the monetary policy rule does not introduce sunspot equilibria into the economy.

The paper's analysis is conducted in the context of a fully articulated general-equilibrium model of a small, open economy. The underlying real model is essentially that of Mendoza (1991). Money is

1. There are actually two types of indeterminacy that may arise. The first is *nominal indeterminacy*: are the initial values of the price level and all other nominal variables pinned down? In our notation this corresponds to the question of whether  $\pi \equiv P/P_{t-1}$  is determined (where  $t$  is the initial time period). Nominal determinacy is a typical occurrence under many interest rate operating procedures, the most celebrated example being an interest rate peg. It is of no consequence in and of itself, but rather is important only if it leads to *real indeterminacy*. By this we mean a situation in which the behavior of one or more real variables is not pinned down by the model. This possibility is of great importance as it immediately implies the existence of *sunspot equilibria*, which in the present environment are necessarily welfare reducing. Sunspots are purely extraneous variables that alter the equilibrium because economic agents believe that they will alter the equilibrium.

introduced by assuming that it is required to facilitate certain transactions. The paper considers both a flexible-price economy in which anticipated inflation effects are paramount, and a limited participation model as in Lucas (1990), Christiano, Eichenbaum, and Evans (1997), and Fuerst (1992), in which unanticipated money supply shocks have real effects.<sup>2</sup> The “smallness” assumption manifests itself in the assumptions of perfect capital markets an exogenous worldwide real rate of interest, and an exogenous real exchange rate. From a theoretical point of view, the “smallness” assumption is particularly attractive, because many of the results that are obscured in a closed economy are more readily apparent here. In addition, certain other shocks (such as to the real exchange rate) are not relevant to a closed economy.

The first two sections of the paper address the issue of real indeterminacy in a small, open economy with flexible prices (section 1) and with limited participation (section 2). A principal conclusion is that to avoid real indeterminacy the central bank must respond aggressively to *past* movements in inflation. It is well known that basing policy responses on market expectations can generate nonuniqueness of equilibria. For this reason, “looking forward” tends to create indeterminacy because monetary policy is driven by an endogenous variable that in turn depends on policy.<sup>3</sup>

A standard argument in the literature is that, to avoid real indeterminacy, the central bank must respond aggressively to either expected inflation (see Clarida, Galí, and Gertler, 1997) or current inflation (see Kerr and King, 1996). These analyses all use reduced-form, sticky-price models, where the underlying structural model is a labor-only economy and money is introduced through a money-in-the-utility function (MIUF) model with a zero cross-partial between consumption and real balances. In sharp contrast, we argue that a necessary and sufficient condition for determinacy is for the monetary authority to react aggressively to past movements in inflation.<sup>4</sup>

2. Open-economy limited participation models include those of Baier (1997), Grilli and Roubini (1992), and Schlagenhauff and Wrase (1995).

3. These questions are of more than academic interest. Several central banks around the world currently use inflation forecasts as an important part of their decisionmaking on policy issues. The recent federal funds rate increases in the United States are evidence that such forecasts are used. Although there were few signs that current inflation was increasing, interest rates were raised at the August and November 1999 meetings of the Federal Open Market Committee because it was feared that higher inflation might well be in the offing.

4. Carlstrom and Fuerst (1998) demonstrate that these results on indeterminacy are robust to a closed-economy model with a more elaborate production technology.

The fundamental differences between our model and the others just described are twofold. As demonstrated in Carlstrom and Fuerst (forthcoming), one major difference comes from the assumption about which money balances enter the utility function. The traditional MIUF assumption is that it is end-of-period balances that enter. But a direct extension of a typical cash-in-advance economy suggests that the money the household has left after leaving the bond market and before entering the goods market is more appropriate. Remarkably, this distinction in timing is critical for questions of determinacy. As argued in Carlstrom and Fuerst (forthcoming), cash-in-advance timing is a more natural choice, and so we adopt it here. We utilize a rigid cash-in-advance constraint, but following the arguments of our earlier paper, the results generalize to an arbitrary MIUF setup with cash-in-advance timing.

The second major difference is in the type of nominal rigidity analyzed. The earlier papers assume a sticky price environment adopted from Calvo (1983). The assumption in that model is that a fixed fraction of firms adjust their prices every period. This implies that, for any arbitrarily long but finite period, not all prices will have adjusted to the levels implied by a flexible-price model. Equivalently, for arbitrary initial conditions, the deterministic dynamics converge to the flexible-price steady state only in the infinite limit. This pricing arrangement is problematic for issues of determinacy. For example, in Clarida, Galí, and Gertler (1997), an equilibrium is indeterminate if perturbations from the equilibrium path lead to explosive inflation dynamics. But surely the Calvo pricing arrangement would not continue to hold along such a path, so that imposing this arrangement along the path seems quite artificial. In this paper we consider an alternative nominal rigidity, namely, the limited participation model mentioned above. But more important, we assume that this nominal rigidity arises for only a finite and definite time period. That is, for arbitrary initial conditions, the model converges to the flexible-price steady state within a *finite* period. This assumption is more appropriate for the analysis of the limiting behavior that is so important for stability analysis.<sup>5</sup>

The final section of the paper turns to the long-run issue of monetary policy: what is the optimal long-run rate of inflation? By positing a Taylor rule in sections 1 and 2, we assume that it is not optimal

5. Carlstrom and Fuerst (2000) demonstrate that these same indeterminacy results hold for a sticky-price model in which prices are sticky for only a finite time period.

for the central bank to follow Friedman's (1969) advice and engineer a long-run deflation that will peg the nominal rate of interest to zero. There is an enormous literature concerning the robustness of Friedman's optimum quantity of money result (see, for example, Woodford, 1990). This paper provides a novel explanation for a positive long-run rate of inflation that is unique to a small, open economy. This "smallness" implies that the country takes the foreign nominal rate of interest as given. To the extent that this foreign rate distorts domestic behavior, there is a rationale for positive domestic nominal interest rates.

### 1. A FLEXIBLE-PRICE MODEL

The economy in the model consists of households, firms, and financial intermediaries, with the latter accepting deposits from households and lending to firms. The firms borrow cash to finance their wage bill. The households consume a single good that is produced abroad, and the firms produce a single good that is sold abroad at the real exchange rate  $e_t$ . The economy is small in that the real exchange rate is taken as given and households' foreign asset accumulation earns a constant real rate of return  $r$ .

Households are infinitely lived, with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - L_t),$$

where  $E_0$  denotes the expectations operator conditional on time-zero information,  $\beta \in (0,1)$  is the personal discount factor,  $c_t$  is consumption at time  $t$ , and  $L_t$  is labor supply at time  $t$ . To purchase consumption goods, households are subject to the following cash-in-advance constraint:

$$P_t c_t \leq M_t + P_t w_t L_t - N_t,$$

where  $P_t$  is the price level,  $M_t$  denotes beginning-of-period cash balances,  $w_t$  denotes the real wage, and  $N_t$  denotes the household's choice of one-period bank deposits. These deposits earn nominal rate  $R_p$  which is paid out at the end of the period. The household's intertemporal budget constraint is given by

$$M_{t+1} = M_t + P_t (w_t L_t + r A_t) - P_t c_t - P_t (A_{t+1} - A_t) + \Pi_t + N_t (R_t - 1).$$

Here  $A_t$  denotes the household's investment in foreign assets that earn the constant real rate of return  $r$ , with  $\beta(1+r) = 1$ . We assume that asset accumulation occurs at the household level and that cash in advance is not needed to finance its purchase.  $\Pi$  denotes the profit flow from firms and financial intermediaries.

Firms in this economy produce an export good using a production function employing domestic labor:

$$y_t = e_t \theta_t f(H_t),$$

where  $\theta_t$  is a measure of aggregate productivity, and  $H_t$  denotes hired labor. One can imagine that  $f$  is constant returns or that land is an additional fixed factor in the production function. In the former case there are no profits to distribute, whereas in the latter case the profit flow to equity owners is simply the rents on land. To finance its wage bill the firm must acquire cash and does so by borrowing short term from the financial intermediary at (gross) rate  $R_t$ .

The intermediary in turn has two sources of cash: cash deposited by households and new cash injected into the economy by the central bank. Hence the loan constraint is

$$P_t w_t H_t \leq N_t + M_t^s (G_t - 1),$$

where  $G_t$  denotes the (gross) money supply growth rate,  $G_t \equiv M_{t+1}^s / M_t^s$ . Note that monetary injections are carried out as lump-sum transfers to the financial intermediary.

We restrict our attention to equilibria with strictly positive nominal interest rates so that the two cash constraints are binding. A recursive competitive equilibrium is given by stationary decision rules that satisfy these two binding cash constraints and the following:

$$E_t \left( \frac{P_t U_{ct+1}}{P_{t+1}} \right) = \beta E_t \left[ (1+r) \frac{P_{t+1} U_{ct+2}}{P_{t+2}} \right], \quad (1)$$

$$\frac{U_{Lt}}{U_{ct}} = \frac{e_t \theta_t f'(L_t)}{R_t}, \quad (2)$$

$$\frac{U_{ct}}{P_t} = \beta E_t \left( R_t \frac{U_{ct+1}}{P_{t+1}} \right), \quad (3)$$

$$c_t + A_{t+1} = e_t \theta_t f(L_t) + (1+r)A_t. \quad (4)$$

The social resource constraint, equation (4), comes from imposing market clearing and netting out firm and intermediary profits from the household's intertemporal budget constraint.

Note that, using equation (3), we can rewrite equation (1) as

$$\frac{U_{ct}}{R_t} = \beta E_t \left[ (1 + r) \frac{U_{ct+1}}{R_{t+1}} \right]. \tag{5}$$

The system given by equations (2), (4), and (5) is isomorphic to Mendoza's (1991) corresponding real business cycle economy except that, because of the cash-in-advance constraint, it is distorted by a consumption tax rate of  $(1 + tc_p) = R_t$ . Because of this implicit consumption tax, the rate of return on foreign assets does not equal the usual consumption Euler equation, equation (5), and the marginal rate of substitution is not equal to the marginal product of labor, equation (2).

This public finance interpretation is key in what follows. For example, Carlstrom and Fuerst's (1995) result that a constant nominal interest rate is preferred to a variable one is a manifestation of the standard result that constant taxes are better than fluctuating ones (holding the mean distortion fixed).<sup>6</sup> However, as demonstrated by Carlstrom and Fuerst (1995) in a calibrated model, these welfare gains are quite small, and so our attention is shifted to issues of indeterminacy.

Below we consider monetary policy rules in which the nominal rate (the consumption tax) is endogenous, responding to movements in the economy. In a real economy, Schmitt-Grohe and Uribe (1997) demonstrate that the endogenous tax rate movements implied by a balanced-budget rule can lead to real indeterminacy. An important issue below is whether interest rate operating procedures can have the same effect.

The small-economy assumption makes stability analysis particularly transparent. Without loss of generality, we now limit the discussion to the deterministic version of the model. As is well known, if the deterministic dynamics are not unique, it is possible to construct sunspot equilibria in the model economy. Substituting equation (3) into equation (1) yields

$$\frac{R_{t+1}}{\pi_{t+1}} = (1 + r). \tag{6}$$

6. Such an interest rate smoothing policy is exactly the typical central bank policy over the seasonal cycle.

As mentioned above, this relationship is not the standard Fisherian relationship. The above nominal interest rate is the rate between  $t + 1$  and  $t + 2$ , *not* between  $t$  and  $t + 1$ . The inflation rate, however, is between  $t$  and  $t + 1$  ( $\pi_{t+1} \equiv P_{t+1}/P_t$ ). The reason for this distortion is that the cash-in-advance constraint is on consumption but not investment. It is this distortion that gives rise to the potential for indeterminacy.

*Proposition 1:* Suppose that monetary policy is given by the forward-looking interest rate rule given by

$$R_t = R_{ss} \left[ \prod_{n=0}^{\infty} \left( \frac{\pi_{t+n+1}}{\pi_{ss}} \right)^{\lambda^n} \right]^{\tau(1-\lambda)}, \tag{7}$$

where  $\tau \geq 0$ ,  $R_{ss} = \pi_{ss}/\beta$ , and  $\lambda \in [0,1)$ .

Then, in the flexible-price model, there is real determinacy if and only if  $0 \leq \tau < 1$ . In any event there is always nominal indeterminacy as  $\pi_t$  is free.

*Proof:* Since equation (6) starts at  $t + 1$ , we scroll equation (7) ahead one period. Taking the logarithms of equation (7), expressing  $R$  and the  $\pi$ s as log deviations, we have

$$\tilde{R}_{t+1} = (1 - \lambda)\tau \sum_{n=0}^{\infty} \lambda^n \tilde{\pi}_{t+n+2}. \tag{8}$$

Exploiting the recursion in equation (8), we have

$$\lambda \tilde{R}_{t+2} = \tilde{R}_{t+1} - (1 - \lambda)\tau \tilde{\pi}_{t+2}. \tag{9}$$

Using equation (6), this then implies that

$$\lambda \tilde{\pi}_{t+2} = \tilde{\pi}_{t+1} - (1 - \lambda)\tau \tilde{\pi}_{t+2}, \tag{10}$$

or

$$\tilde{\pi}_{t+2} = \left[ \frac{1}{\lambda + (1 - \lambda)\tau} \right] \tilde{\pi}_{t+1}. \tag{11}$$

For real determinacy we need this mapping to be explosive. Hence there is real determinacy if and only if  $0 \leq \tau < 1$ . If this is the case,

then  $\pi_{t+j}$  is determined for  $j \geq 1$ . From equation (7), this pins down  $R_{t+j}$  for  $j \geq 0$ , so that equations (1) through (4) uniquely pin down the inflation rate and real behavior. Since equation (11) starts with  $\pi_{t+1}$ , the initial price level,  $\pi_t$ , is free (QED).

An immediate corollary to the above theorem is that an interest rate peg ( $\tau = 0$ ) delivers real determinacy but, like the other rules, has nominal indeterminacy.

Under the policy rule given by equation (8)—the linearized version of equation (7)—the monetary authority responds with an elasticity of  $\tau$  to a geometric weighted average of all future expected inflation rates, where more weight is placed on the near future. A special case of such a rule is that where  $\lambda = 0$  and all the weight is placed on the inflation rate between  $t$  and  $t + 1$ :

$$R_t = R_{ss} \left( \frac{\pi_{t+1}}{\pi_{ss}} \right)^\tau .$$

Under this policy rule, increases in expected inflation increase the nominal rate. But for active policies ( $\tau > 1$ ), these nominal rate increases are also associated with increases in the real rate of interest. Thus we have an implicit consumption tax (the nominal rate) correlated tightly with expected consumption growth (the real rate). The self-fulfilling circle goes something like this. An increase in expected inflation increases investment demand, lowering current consumption. The decline in current consumption increases the real rate of interest; with  $\tau > 1$ , the nominal rate (consumption tax) rises sharply with this real rate movement; this tax movement implies that the initial increase in expected inflation and the decline in current consumption are rational.

The fact that  $\pi_t$  is free for all values of  $\tau$  is just a manifestation of nominal indeterminacy; that is, there is nothing to pin down the initial growth rate of money. This innocent remark has some interesting implications.

*Proposition 2:* Suppose that monetary policy is given by a forward-looking rule that includes the current inflation rate:

$$R_t = R_{ss} \left[ \prod_{n=0}^{\infty} \left( \frac{\pi_{t+n}}{\pi_{ss}} \right)^{\lambda^n} \right]^{\tau(1-\lambda)} , \tag{12}$$

where  $\tau \geq 0$ ,  $R_{ss} = \pi_{ss} / \beta$ , and  $\lambda \in [0, 1)$ .

Then in the flexible-price model there is real indeterminacy for all values of  $\tau \neq 0$ .

*Proof:* The proof proceeds as before, leading to the following difference equation:

$$\tilde{\pi}_{t+2} = \frac{1 - (1 - \lambda)\tau}{\lambda} \tilde{\pi}_{t+1} .$$

Under appropriate conditions this is explosive, so that  $\pi_{t+j}$  is pinned down for  $j \geq 1$ . But even in this case  $\pi_t$  is free and hence so is  $R_t$ . Thus we have real indeterminacy. Once again, if  $\tau = 0$ , we have an interest rate peg and there is no real indeterminacy (QED).

The nominal indeterminacy from before is now real. Before,  $\pi_t$  being free had no real effect, but now, given our interest rate rule,  $R_t$  is free. Since  $R_t$  acts like a tax on consumption, the fact that  $R_t$  is not pinned down implies that real behavior is indeterminate. The reason for this indeterminacy is that our policy rule is responding to endogenous variables. This is why an interest rate peg ( $\tau = 0$ ) is determinate.

This discussion suggests that the central bank should look further backward so that it only responds to exogenous variables. Remarkably, by looking backward the conditions for determinacy are (almost) entirely flipped on their head from when the Taylor rule is forward looking.

*Proposition 3:* Suppose that monetary policy is given by the backward-looking interest rate rule given by

$$R_t = R_{ss} \left[ \prod_{n=0}^{\infty} \left( \frac{\pi_{t-n-1}}{\pi_{ss}} \right)^{\lambda^n} \right]^{\tau(1-\lambda)} , \quad (13)$$

where  $\tau \geq 0$ ,  $R_{ss} = \pi_{ss}/\beta$ , and  $\lambda \in [0, 1)$ .

Then in the flexible-price model there is real determinacy if and only if  $\tau = 0$  or  $\tau > 1$ . In the case of  $\tau > 1$ , there is also nominal determinacy.

*Proof:* Following the strategy pursued in proposition 1, we are led to the following difference equation:

$$\tilde{R}_{t+2} = [\lambda + (1 - \lambda)\tau] \tilde{R}_{t+1} .$$

For real determinacy we thus need  $\tau > 1$ , which then implies that  $\pi_{t+j}$  is pinned down for  $j \geq 0$  (QED).

An interest rate peg is determinate but, like before, has nominal indeterminacy. Notice, however, that if the monetary authority responds aggressively to past inflations ( $\tau > 1$ ), initial inflation and hence the initial money stock are pinned down. This result is a general-equilibrium generalization of McCallum's (1981) earlier result. McCallum argued that because an interest rate peg suffered from nominal indeterminacy, the monetary authority needed a nominal anchor, which could be accomplished by responding to a nominal variable. Proposition 3 confirms this but shows that merely responding to a nominal variable, like past inflation, is not enough. The monetary authority has to aggressively respond to past inflation to ensure nominal determinacy.

For completeness, we consider a backward-looking rule in which the central bank also considers the current inflation rate:

*Proposition 4:* Suppose that monetary policy is given by a backward-looking interest rate rule that also includes the current inflation rate:

$$R_t = R_{ss} \left[ \prod_{n=0}^{\infty} \left( \frac{\pi_{t-n}}{\pi_{ss}} \right)^{\lambda^n} \right]^{\tau(1-\lambda)}, \tag{14}$$

where  $\tau \geq 0$ ,  $R_{ss} = \pi_{ss} / \beta$ , and  $\lambda \in [0, 1)$ .

Then in the flexible-price model there is real determinacy if and only if  $\tau = 0$  or  $1 < \tau < (1 + \lambda)/(1 - \lambda)$ . In the case of  $1 < \tau < (1 + \lambda)/(1 - \lambda)$ , there is also nominal determinacy.

*Proof:* Following the strategy pursued in proposition 3, we are led to the following difference equation:

$$\tilde{R}_{t+1} = \left[ \frac{\lambda}{1 - (1 - \lambda)\tau} \right] \tilde{R}_t.$$

For determinacy, the term in brackets must be outside the unit circle. There are two regions to consider. First, if  $\tau(1 - \lambda) < 1$ , the bracketed term exceeds unity whenever  $\tau > 1$ . If  $\tau(1 - \lambda) > 1$ , we must consider the other side of the unit circle. Now for determinacy we must have

$$\tau < \frac{1 + \lambda}{1 - \lambda}.$$

Combining the two regions, we have the region noted in the proposition (QED).

With an active policy, looking backward can render the economy determinate. Yet the more weight is placed on the distant past (the larger is  $\lambda$ ), the more the bounds for determinacy shrink.

Although at this stage nominal indeterminacy is merely a nuisance, its presence becomes critical in the next section when we consider how the presence of a particular type of nominal rigidity, namely, sluggish portfolios, affects the above results.

## **2. A LIMITED PARTICIPATION MODEL**

A natural criticism of the previous analysis is that it was conducted in a monetary model in which only anticipated inflation has real effects. For example, under most of the policies considered there is nominal indeterminacy in the flexible-price model, implying that an independently and identically distributed shock to the money supply has no effect on any real variables.<sup>7</sup> This section extends the analysis to a more compelling model of monetary nonneutrality in which iid shocks have real effects.

The model we choose to examine is the limited participation model (see, for example, Lucas, 1990, Christiano, Eichenbaum, and Evans, 1996, and Fuerst, 1992). There are at least two reasons for this choice. First, in contrast to sticky-price models, there has been very little work on indeterminacy issues in this type of model.<sup>8</sup> Second, there is a compelling empirical reason to consider such a model. A well-known empirical phenomenon is that, in response to a positive monetary policy innovation (a movement downward in the nominal interest rate), aggregate real behavior displays a hump-shaped time profile, while prices rise slowly to a higher level. A hump-shaped consumption

7. This exception is a backward-looking Taylor rule when  $\tau > 1$ . Otherwise the fact that initial money growth is free implies that iid shocks to money growth will also achieve the interest rate directive.

8. A notable exception is Christiano and Gust (1999). There are two important differences between their analysis and the model presented here. First, they assume that investment purchases are subject to a cash-in-advance constraint, whereas this paper makes the more standard assumption that investment is a credit good. Second, in their model the portfolio rigidity is forever rigid in that the deterministic dynamics converge to the flexible-price model only in the limit. This is analogous to a Calvo (1983) pricing arrangement.

profile implies an increase in the real rate of interest, whereas an upward movement in prices implies that expected inflation cannot fall. But then we are left with a puzzle: how can nominal interest rates fall? The obvious answer is that over some time horizon the standard Fisherian interest rate determination is broken; that is, equation (3) must not hold over some time interval.

This is exactly the assumption made in limited participation models. In particular, these models assume that the (nominal) portfolio choice ( $N_t$ ) is made one period in advance, so that equation (3) is replaced with

$$E_{t-1} \left( \frac{U_{ct}}{P_t} \right) = \beta E_{t-1} \left( R_t \frac{U_{ct+1}}{P_{t+1}} \right).$$

The positive implications of these models are well-known. For example, suppose that

$$U(c_t, L_t) = \ln(c_t) - \frac{L_t^{1+\eta}}{1+\eta}.$$

Then straightforward calculations imply

$$L_t = \left( \frac{N_t + G_t - 1}{G_t} \right)^{\frac{1}{1+\eta}}.$$

With  $N_t$  predetermined, this implies that monetary shocks increase employment and domestic output, while driving down the domestic nominal interest rate. From a policy perspective, the more important observation is that employment does not respond to productivity or real exchange rate shocks, because these shocks drive the nominal interest rate upward. This sluggishness makes a clear case for monetary policy: money growth should be varied to accommodate these shocks. Carlstrom and Fuerst (1995) use this as a starting point to make the case for interest rate pegging in this model. Once again, the welfare gains are small.

Because of this we turn to stability analysis. The attractive feature of the limited participation model is that it breaks the rigid Fisherian interest rate determination. But for the stability properties of interest rate operating procedures, this turns out to be somewhat disastrous since the interest rate is no longer linked to underlying real variables such as consumption growth and expected inflation.

As before, we need only consider the deterministic dynamics. After one period, portfolios can adjust, implying that for stability analysis we must replace equation (3) with its counterpart scrolled forward:

$$\left( \frac{U_{ct+1}}{P_{t+1}} \right) = \beta \left( R_{t+1} \frac{U_{ct+2}}{P_{t+2}} \right). \quad (15)$$

Combining this with the asset accumulation equation (1), we have

$$\frac{R_{t+1}}{\pi_{t+1}} = (1 + r), \quad (16)$$

$$\left( \frac{U_{ct+1}}{\pi_{t+1}} \right) = \left( \frac{U_{ct+2}}{\pi_{t+2}} \right). \quad (17)$$

For stability analysis, the system of equations includes equations (16) and (17), the labor equation (2), and the resource constraint equation (4).

Equation (16) is familiar from before. Under suitable conditions on the interest rate operating procedure it will pin down inflation and interest rate dynamics. The novelty is in equation (17): the limited participation constraint implies that there is little to pin down the initial  $U_{ct}$  and thus the initial asset accumulation decision, even if the initial nominal interest rate is determined.

To have any hope of determinacy we need an extra restriction. The limited participation constraint provides one:  $N_t$  is a predetermined nominal variable. Combining the two binding cash constraints, and using the household's labor choice decision, we have

$$M_t - N_t = P_t \left( c_t - \frac{U_{Lt} L_t}{U_{ct}} \right). \quad (18)$$

Since  $M_t$  and  $N_t$  are predetermined, this provides an additional restriction; in the flexible price model  $N_t$  is free and equation (18) does not restrict real behavior. In particular, if a policy rule pins down the current price level  $P_t$ , then the additional restriction in equation (18) will pin down the initial consumption-asset-accumulation decision. Hence the only way of achieving determinacy is if the interest rate operating procedure pins down the current  $\pi_t \equiv P_t/P_{t-1}$ . This immediately implies the following:

*Proposition 5:* Suppose that monetary policy is given by either a forward-looking rule as in equation (7) or a current/forward-looking

Taylor rule as in equation (12). Then in the limited participation model there is real indeterminacy for all values of  $\tau$ .

It also implies its corollary:

*Proposition 6:* Suppose that monetary policy is given by a backward-looking Taylor rule as in equation (13). Then in the limited participation model there is real determinacy if and only if  $\tau > 1$ . Suppose that monetary policy is given by the current/backward-looking Taylor rule given in equation (14). Then there is real determinacy if and only if  $1 < \tau < (1 + \lambda)/(1 - \lambda)$ .

Notice that a forward-looking Taylor rule and an interest rate peg both suffer from real indeterminacy, because both had nominal indeterminacy in the corresponding flexible-price economy.<sup>9</sup> An active backward-looking Taylor rule, however, pins down the price level. Since portfolios are predetermined in nominal terms, this price-level determination pins down an extra real variable.

### 3. A FLEXIBLE-PRICE MODEL WITH TWO CURRENCIES

This section changes the nature of the analysis from the short run (policy across the business cycle) to the long run (the long-run rate of inflation). Given this focus, we ignore dynamic issues and instead concentrate on steady-state analysis.

In terms of modeling, this section builds on the earlier model by assuming that the home-country agent has preferences over both the imported and the exported good and that the imported good must be paid for with foreign currency accumulated in advance.<sup>10</sup> Hence we have a model with two cash constraints and thus two implicit consumption taxes. The imported good is taxed at the foreign interest rate, whereas the domestically produced good is taxed at the domestic interest rate. In section 1, domestic currency could be used for all transactions, so that there was only one implicit tax—the domestic interest rate. But now there are two taxes, one of which the domestic central bank takes as exogenous. In this environment it may not be optimal for the home country to follow Friedman's dictum and drive the domestic nominal interest rate to zero.

9. Carlstrom and Fuerst (1995) show that an interest rate peg suffers from real indeterminacy in a closed-economy limited participation model.

10. This more complicated modeling environment would not affect the earlier conditions for real determinacy.

Household preferences over the imported good (good 2), the domestically produced good (good 1), and work effort are given by

$$\sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}, L_t) . \quad (19)$$

To finance its consumption purchases, the household uses cash accumulated in advance and thus faces the following cash constraints:

$$P_{1t}c_{1t} \leq M_{1t} + W_tL_t - N_{1t} , \quad (20)$$

$$P_{2t}c_{2t} \leq M_{2t} - N_{2t} , \quad (21)$$

where  $M_{it}$  denotes holdings of the home ( $i = 1$ ) and the foreign ( $i = 2$ ) currency;  $P_{it}$  denotes the nominal price of goods  $i = 1, 2$ ;  $W_t$  denotes the nominal wage expressed in home currency; and  $N_{it}$  denotes bank deposits denominated in the two currencies  $i = 1, 2$ . The intertemporal constraint (expressed in home currency) is thus given by

$$M_{1t+1} + s_t M_{2t+1} = s_t M_{2t} + M_{1t} + W_t L_t - P_{1t} c_{1t} - s_t P_{2t} c_{2t} + \Pi_t \\ + N_{1t} (R_{1t} - 1) + s_t N_{2t} (R_{2t} - 1) + s_t X_t ,$$

where  $R_{it}$  denotes the nominal interest rate denominated in the two currencies  $i = 1, 2$ ;  $s_t$  is the end-of-period nominal exchange rate; and  $X_t$  is a currency transfer from the foreign government. Since this is a small, open economy, we need to exogenously impose an equilibrium condition on domestically owned bank deposits of foreign currency. For simplicity we assume that these are in zero net supply, so that one equilibrium condition imposed below will be  $N_{2t} = 0$ . Similarly, since our focus is on steady-state issues, we abstract from real foreign asset accumulation ( $A_t$ ). This is done without loss of generality.

The first-order conditions to the household's problem include

$$\frac{U_{2t}}{U_{1t}} = \frac{s_t P_{2t} R_{2t}}{P_{1t} R_{1t}} , \quad (22)$$

$$\frac{U_{L_t}}{U_{1t}} = \frac{W_t}{P_{1t}} . \quad (23)$$

As for the firm, it produces good 1 using the production technology from before, and subject to the need to borrow domestic

currency to finance its wage bill. In addition, it imports good 2 at exogenous terms of trade. (As before, the imported good is the numeraire, so that  $e_t$  is the real price of good 1 in terms of good 2.) The firm then sells the two goods at their respective prices  $P_{it}$ . Since the firm does not distribute dividends until the end of the period, it maximizes its end-of-period profits. Hence purchasing power parity is in terms of the end-of-period exchange rate:

$$e_t = \frac{P_{1t}}{s_t P_{2t}} . \tag{24}$$

Interest rate parity is given by

$$\hat{s}_t = \frac{s_t R_{2t}}{R_{1t}} ,$$

where  $\hat{s}_t$  is the beginning-of-period, or spot, exchange rate. The fact that equation (24) does not hold with the spot exchange rate, but instead with the end-of-period exchange rate, is a manifestation of the cash-in-advance distortion.

Substituting the firm's optimization conditions into the household first-order conditions yields

$$\frac{U_{L_t}}{U_{1t}} = \frac{\theta_t f'(L_t)}{R_{1t}} , \tag{25}$$

$$\frac{U_{1t}}{U_{2t}} = \frac{e_t R_{1t}}{R_{2t}} . \tag{26}$$

As noted earlier, this is now a model with two implicit consumption taxes, both of which are manifested in equations (25) and (26). The home-produced good (good 1) is taxed at the domestic nominal rate, whereas the imported good (good 2) is taxed at the foreign rate of interest. For a given foreign interest rate it is clear that the central bank faces a second-best problem. Setting the domestic rate to match the foreign rate may alleviate one distortion, equation (26), but exacerbates another, equation (25).

The social resource constraint comes from imposing the equilibrium decision rules and market-clearing conditions on the household's intertemporal constraint. As previously, we assume that only domestic households hold domestic currency. Hence we need only adjust the earlier resource constraint in equation (4) for domestically held foreign cash balances:

$$\frac{M_{2t+1}^a - M_{2t}^a}{P_{2t}} + e_t c_{1t} + c_{2t} = e_t \theta_t f(L_t) + \frac{X_t}{P_{2t}}, \quad (27)$$

where  $M_{2t}^a$  denotes foreign cash balances per capita held by the representative domestic household (in the earlier model, domestic households held no foreign cash).

In the steady state, domestic labor supply and purchases of the two goods are constant. Since we are uninterested in the wealth effects coming from exogenous foreign price movements, we assume that

$$X_t \equiv (P_{2t+1} - P_{2t})c_2, \quad (28)$$

where  $c_2$  is the steady-state level of the imported good.<sup>11</sup> The household takes this transfer as exogenous so that it does not affect marginal decisionmaking but is only felt at the aggregate resource level. In particular, with the transfer given by equation (28), the binding cash-in-advance constraint implies that foreign money holdings drop out of the resource constraint, so that the steady-state resource constraint is given by

$$e c_1 + c_2 = e \theta f(L). \quad (29)$$

The steady-state welfare problem is to maximize equation (19) subject to equations (25), (26), and (29).

We must utilize numerical methods to make any headway. Hence assume the following functional forms:

$$U(c_1, c_2, L) = \frac{(c_1^{1-\delta} + \mu c_2^{1-\delta})^{\frac{1-\sigma}{1-\delta}}}{1-\sigma} - \frac{\nu L^{1+\eta}}{1+\eta},$$

$$f(L) = L^\alpha.$$

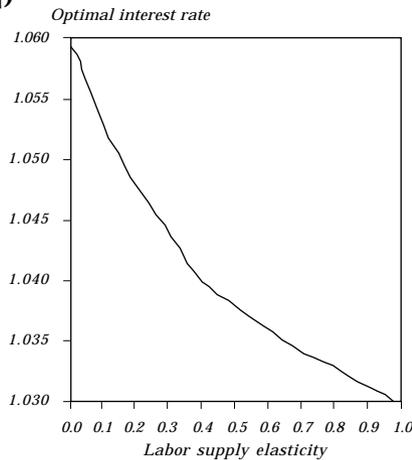
For our benchmark calibration, we set to 1 both the intertemporal elasticity of substitution ( $1/\sigma$ ) and the elasticity of substitution between the two consumption goods ( $1/\delta$ ); we set the labor supply elasticity ( $1/\eta$ ) to 0.333 and the labor share to  $\alpha = 2/3$ . We also normalize  $\theta = 1$ ,  $e = 1$ , and  $\mu = 1$ . The latter implies an import share of  $[c_2/(c_2 + c_1)] = 0.47$ .

11. Thus this assumption is in the spirit of Lucas (1982), who assumes that domestic and foreign households are insured against "seigniorage risk." Without this assumption the analysis would proceed with an inflation tax term in the aggregate resource constraint in equation (29).

The foreign interest rate is set at  $R_2 = 1.06$ . We then choose  $v$  so that, with  $R_1 = 1.06$ , we have  $L = 1/3$ .

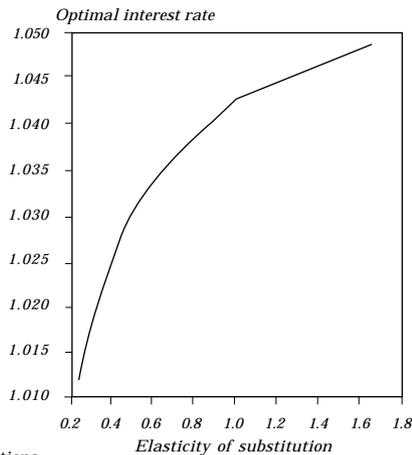
Figures 1 through 3 present the results for differing values of the elasticity of substitution between differing values of the labor supply elasticity ( $1/\eta$ ), the two consumption goods ( $1/\delta$ ), and differing import shares (variations in  $\mu$ ). Across all three figures all other parameters are held constant except the one being analyzed.

**Figure 1. Optimal Interest Rate and Labor Supply Elasticity ( $1/\eta$ )**



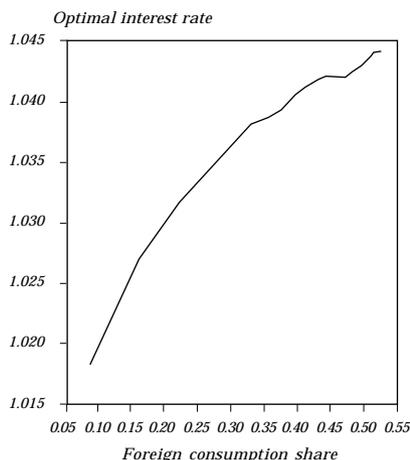
Source: Author's calculations.

**Figure 2. Optimal Interest Rate and Elasticity of Substitution Between  $c_1$  and  $c_2$  ( $1/\delta$ )**



Source: Author's calculations.

**Figure 3. Optimal Interest Rate and Foreign Consumption Share ( $\mu$ )**



Source: Author's calculations.

The figures reveal standard second-best intuition. When there are two distorted margins, the benevolent planner will choose to more heavily distort the margin that is less elastic or less important. The two margins here are the labor-leisure choice in equation (25) and the choice between good 1 and good 2 in equation (26). The first distortion is eliminated with a zero nominal rate (a gross rate of unity), and the second is eliminated by matching the foreign interest rate.

Figures 1 through 3 have the qualitative shape that one would expect. As  $1/\delta$  goes to zero and demands become highly inelastic, the distortionary impact of divergent nominal interest rates declines. Hence the optimal response is to lower the distortion on the labor margin by lowering the domestic interest rate. As labor supply becomes highly inelastic and  $1/\eta$  goes to zero, just the opposite is true, and the optimal response is to more closely match the foreign rate. Figure 3 reveals that as the importance of the foreign sector rises, the cost of this distortion rises. Hence as the foreign share rises, the optimal response is to move closer to the foreign rate. Notice that, except under extreme parameter values, the optimal domestic rate is significantly different from Friedman's zero.

Before closing, it is instructive to compare this steady-state analysis with the corresponding dynamic Ramsey problem (see, for example, Chari, Christiano, and Kehoe, 1996). In the present context the key

issue is the form of the aggregate resource constraint. For the household budget constraint to collapse into the form given by equation (29), one needs to preclude an initial and one-time flight from the foreign currency (a possibility that, by assumption, cannot arise in a steady-state analysis). That is, if the Ramsey planner began with the level of foreign cash balances given by the steady-state problem, he would encourage a one-time drop in these currency holdings by setting the domestic nominal rate to zero. This lower domestic rate would lead the household to decrease its consumption of foreign goods and thus lower its holdings of foreign currency. For all future periods this flight would leave the domestic household at a period-by-period utility level lower than that under the steady-state problem (as consumption of the imported good would be forever lower). To rule out this type of behavior, we must replace equation (28) with the stronger condition

$$X_t \equiv (P_{2t+1}c_{2t+1} - P_{2t}c_{2t}). \quad (30)$$

This transfer scheme imposes an implicit tax on currency flight. Once again the domestic household takes this transfer as exogenous, and thus it does not enter private marginal conditions. However, the Ramsey planner is affected by these transfers, and they lead to an aggregate resource constraint like that in equation (29). Hence, under the assumption made in equation (30), the steady-state analysis and the dynamic Ramsey problem are equivalent.

#### 4. CONCLUSION

This paper has used a standard open-economy model to address two classic questions in monetary policy: What is the appropriate monetary policy across the business cycle? And what is the optimal long-run rate of inflation?

Recent research suggests that an important issue for the first question is what policy restrictions ensure that the central bank's policy rule does not introduce real indeterminacy and sunspot fluctuations into an otherwise stable economy. The message is clear: the central bank should respond aggressively to lagged inflation rates. From the standpoint of indeterminacy issues, a policy that targets current and future expected inflation is disastrous. Responding "passively" to only future expected inflation or targeting the nominal interest rate may

avoid real indeterminacy in a flexible-price model. However, these policies are disastrous when the economy is subject to a nominal rigidity such as that implied by limited participation. In terms of exchange rates, to the extent that a pegged exchange rate implies that the domestic nominal interest rate is given exogenously by the foreign country, a pegged exchange rate is also subject to real indeterminacy and sunspot fluctuations.

As for the second question, concerning the long-run rate of inflation, a small, open economy faces an exogenous foreign interest rate. Almost certainly this rate distorts domestic behavior. Hence the central bank immediately faces a second-best problem. One distortion (a positive foreign rate) can be worse than two (a positive domestic and foreign interest rate). This is a novel reason to stay clear of Friedman's zero nominal rate.

This paper's general-equilibrium or structural approach to policy evaluation has many advantages, including an obvious welfare criterion (lifetime utility of the typical agent) and a clear articulation of what parameters are invariant to policy. General-equilibrium analysis forces one to be specific, and as always the devil is in the details. An example illustrates this point. King and Wolman (1996) analyze a sticky-price general-equilibrium model and conclude that strict price-level targeting is the optimal monetary policy. In contrast, the results of Carlstrom and Fuerst (forthcoming) imply that a plausible alteration in their modeling of money demand implies that an aggressive price-level target is actually destabilizing and introduces sunspot equilibria into the economy. Which conclusion is correct? One must examine the details, all of which a simple reduced-form model obscures.

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