INDEXATION, INFLATIONARY INERTIA, AND THE SACRIFICE COEFFICIENT

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When inflation is chronic, firms develop indexation practices that automatically tie the growth of prices, wages, and other contracts to the performance of some comprehensive price index. The microeconomic advantages of indexation are evident and derive from the immunization of the relative price system against the extravagances of inflation. From a macroeconomic perspective, however, this practice has been criticized for perpetuating the inflationary thrust, accentuating its volatility, and influencing the process of relative price adjustments in the face of real shocks. Those in charge of monetary policy generally oppose indexation under the argument that it raises the costs of reducing or controlling inflation, and its elimination has been considered a prerequisite for ensuring the success of the drastic stabilization plans that have been implemented in countries with a long history of inflation.

This position, however, is not fully shared in the academic literature. Essentially, one segment of the literature indicates that indexation favors the stabilization of output and facilitates the reduction of inflation. The best-known proponent of this position is Friedman (1974), who argues that automatic adjustments reduce the costs of anti-inflationary policies, because they accelerate the speed with which the monetary effects are transmitted to prices and wages and thus reduce the impact on output. Gray (1976) and Fischer (1977) reach the same conclusion, arguing that indexation stabilizes output when monetary shocks predominate, as in an inflationary stabilization plan. This favorable interpretation of indexation is also reflected in empirical literature that uses the coverage of automatic adjustment practices as a measure of wage flexibility that reduces the costs of price stabilization (Ball, 1994b).

An important criticism of this line of argument is that it implicitly assumes that indexation is synchronized and immediate (Simonsen, 1983). The conclusions are substantially modified when these assumptions

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are removed and replaced by lagged, uncoordinated indexation, as shown in the works of Bonomo and García (1994) and Jadresic (1995).[1] The objective of the current paper is to study the relation between price and wage indexation and the costs of reducing inflation in terms of output; this relation is known as the sacrifice coefficient. The analysis complements earlier works by Ball (1994a), Bonomo and García (1994), and Jadresic (1995, 1996a, 1996b).

The emphasis, however, is on the consequences of the frequency of indexation and other characteristics of labor contracts for both the sacrifice coefficient and the output and price trends in an economy that is undergoing an anti-inflationary stabilization program.

A priori arguments indicate that the relation between the sacrifice coefficient and the frequency of indexation is not monotonic: as the frequency of indexation increases, the economy approaches immediate, synchronized adjustment. In this case, the performance of an economy that is responding to changes in monetary policy converges with that of an economy characterized by fully flexible prices, which brings the sacrifice coefficient to zero.

This work examines in depth the relation between the sacrifice coefficient and the frequency of indexation, as well as the consequences for recommendations on monetary policy. The methodology used is to simulate the response of an economy characterized by Calvo-type staggered contracts, which are extended to include automatic adjustment clauses based on past inflation. The renegotiation of these price and wage contracts is unsynchronized, such that there is a constant probability of reopening contracts in any given moment. Intermediate contract openings in which only the nominal level is corrected are also possible, in order to return the contract to its real level prior to renegotiation. The microeconomic explanations for this real rigidity are outside the scope of this paper, but they are assumed to be in line with the literature on menu costs (Mankiw and Romer, 1991). Following Calvo, the supply structure serves as the basis for deriving a very simple model using parameterized, linear, differential equations for the frequency of real renegotiation and nominal adjustments.

The main consequence of automatic contract adjustments is to introduce inertia into the inflationary process, that is, in the first derivative of

1. The literature drawing on Barro and Gordon (1983), which emphasizes the strategic aspects of implementing monetary policy, is inconclusive with regard to the impact of indexation on inflation and the authority's incentives to exploit the inflation-unemployment trade-off.

the price level. This stands in contrast to the original formulation by Calvo (1983) or Taylor (1979), which exhibits inertia only in the price level.

The next section derives the theoretical model of aggregate supply, while subsequent sections analyzes the response of prices and output in the face of three types of inflationary reduction programs. Specifically, section 2 describes a gradual inflationary reduction plan with full credibility and simulates the model for different combinations of parameters, concentrating on the frequency of indexation. Section 3 carries out the same study in the context of a shock plan, and section 4 examines the impact of a gradual plan with partial credibility. Finally, the main conclusions of the paper are presented, together with the implications for the persistence of moderate inflationary processes.

1. The Model

The model, which is derived in continuous time, describes an economy consisting of a continuum of firms in monopolistic competition.[2] The relative price that maximizes each firm's immediate utilities is increasing in the level of aggregate output:

$$p^{*} - P = \mathrm{v}Y \ , \tag{1}$$

where $0 < \mathrm{v} < 1$, Y is the output level, p^{*} is the firm's optimal price, and P is the aggregate price level. Money and monetary policy enter the model through the quantitative equation for money:

$$M - P = Y \ , \tag{2}$$

where M is the money supply.[3] The optimal price in terms of M and P is deduced from equations 1 and 2:

$$p^{*} = \mathrm{v}M + \left(1 - \mathrm{v}\right)P \ . \tag{3}$$

This structural modeling of demand is analogous to that used by Ball (1994a), Bonomo and García (1994), and Jadresic (1991, 1995).

2. With the exception of the supply equation, the model is analogous to that used by Ball (1994a), Bonomo and García (1994), and Jadresic (1995).

3. Alternatively, the model can be extended for the case of an open economy and the real exchange rate can replace the role of real money in the determination of output.

It differs from these works in the specification of aggregate supply, which is an extension of the model outlined in Calvo (1983) and which introduces automatic adjustments. Calvo's model incorporates a formulation in which the duration of the contracts is exponentially distributed, in contrast to the fixed horizon used in the other studies cited. This artifice simplifies the calculation and facilitates the study of the consequences of alternative types of wage contracts on stabilization costs.

Each firm sporadically corrects its price. These adjustments are random events that occur with a constant probability, independently both of previous corrections undertaken by the firm and of current and past corrections undertaken by other firms in the economy. The price adjustments can be either nominal or real. Nominal adjustments correct the price to reflect the degree of inflation that has accumulated since the last price revision. Real adjustments integrally revise the contract, that is, in both its nominal and real components. Nominal adjustments occur with a probability θ per unit of time, and real adjustments occur with probability γ per unit of time. Therefore, in each moment dt, a portion of contracts, $(\gamma + \theta)\mathrm{d}t$, is revised, of which the fraction

$$\lambda = \frac{\gamma}{\gamma + \theta}$$

is fixed on the basis of both actual and future conditions, while the remaining contracts only correct for past inflation.

The firm's effective nominal price is divided into two components:

$$p(\tau,\tau') = z(\tau) + q(\tau,\tau') \ , \tag{4}$$

where $z(\tau)$ is the real premium negotiated in τ and $q(\tau)$ is the nominal component of the contract signed in τ and revised in τ', equal to the prices $P(\tau')$ prevailing at that moment. The firms are indexed in (τ,τ') as corresponds to the moment in which they signed the real and nominal components, respectively.

The firm sets the real premium knowing that the real and nominal revisions are sporadic. The objective is to equalize the expected average price during the life of the contract with the average optimal price that is expected in this same period,[4]

$$z(t) + \int_t^\infty q(t,s)\gamma e^{-\gamma(s-t)}\,\mathrm{d}s = \int_t^\infty p^*(s)\gamma e^{-\gamma(s-t)}\,\mathrm{d}s \ . \tag{5}$$

4. Equation 5 can be interpreted as the optimal result of a quadratic approximation of the firm's true objective function. The discount factor is ignored for simplicity and to maintain coherence with Ball's and Jadresic's derivations.

In addition, the nominal component $q(t,s)$ of the contracts signed in t are defined by the following equation:[5]

$$q(t,s) = e^{-\theta(s,t)} P(t) + \int_t^s \theta e^{-\theta(t',t)} P(t') \mathrm{d}t'. \tag{6}$$

Differentiating z relative to t produces

$$\frac{\mathrm{d}z(t)}{\mathrm{d}t} = \gamma\left[z(t) + P(t) - p^*(t)\right] - \frac{\gamma}{\gamma+\theta}\Pi(t), \tag{7}$$

where $\Pi(t)$ is the immediate inflation rate.

The price level is obtained by aggregating current prices for the different cohorts of firms. It is assumed that the distribution of the cohorts is stationary, such that the distribution in τ (the moment of the last real adjustment) is exponential in $(-\infty,t)$ with parameter γ, and the distribution in τ' (the moment of the last nominal adjustment) is exponential between $(-\infty,t)$ with parameter $\gamma + \theta$. The general price level is then derived as

$$P(t) = \int_{-\infty}^{t} z(\tau)\gamma e^{-\gamma(\tau-t)} \mathrm{d}\tau + \int_{-\infty}^{t} P(\tau')(\gamma+\theta) e^{-(\gamma+\theta)(\tau'-t)} \mathrm{d}\tau'. \tag{8}$$

This equation generalizes Calvo's model (1983) to include adjustments for past inflation ($\theta \neq 0$). As in Calvo (1983) and Taylor (1979), the price level is predetermined, and it is not fully adjusted in response to either expected or unexpected changes in the money supply. This gives rise to the nonneutrality of the money supply and monetary policy. The innovation of this supply equation relative to the earlier models is that inflation, the first derivative of the price level, also exhibits inertia.

The immediate inflation rate is obtained by differentiating the price level equation:

$$\Pi(t) = \gamma z(t) + \theta\left[P(t) - Q(t)\right], \tag{9}$$

where $Q(t)$ is the average price level implicit in the nominal component of the contracts in force, that is, the second integral on the right

5. Equation 6 is obtained from the solution to the differential and border equations: $\frac{\mathrm{d}q(s,t)}{\mathrm{d}s} = \theta\left[p(s) - q(s,t)\right]$ and $q(t,t') = p(t)$.

hand side of equation 8. This equation decomposes inflation into two parts. The first component reflects the premium on the price level in new contracts, $z(t)$. This factor can change instantaneously in response to both observed and anticipated changes in the economy's monetary and nonmonetary conditions. The second component comprises the model's innovation relative to Calvo. The variable $P(t) - Q(t)$ is a predetermined variable that reflects the average size of the nominal corrections stemming from indexation.

Algebraic work generates an alternative form to equation 9, which more clearly reveals the determinants of inflation in the model:

$$\Pi(t) = \underbrace{\gamma \int_0^\infty \gamma e^{-\gamma\tau} \, \mathrm{v} Y(t+\tau)\,\mathrm{d}\tau + \frac{\gamma}{\gamma+\theta}\int_0^\infty \gamma e^{-\gamma\tau}\,\Pi(t+\tau)\,\mathrm{d}\tau}_{expectations}$$

$$+ \underbrace{\frac{\theta}{\theta+\gamma}\int_0^\infty (\theta+\gamma) e^{-(\gamma+\theta)\tau}\,\Pi(t-\tau)\,\mathrm{d}\tau}_{inertia} . \tag{10}$$

Current inflation is determined by expectations with regard to the output gap and future inflation, together with the evolution of lagged inflation. Equation 10 demonstrates that the frequency of indexation, θ, has effects on both expectations and inertia. Greater frequency reduces the weight of future inflation, because automatic indexation keeps this component up to date and increases the weight of lagged inflation, with greater weight on recent observations of the inflation rate. This structural supply equation reveals that empirical inflationary inertia can be generated by both indexation and inertia in the expectations components.

In Calvo's model, as well as in Taylor (1979), this is equivalent to assuming that $\theta = 0$, which implies that the inflation rate is totally flexible and anticipatory, even when the price level exhibits inertia. This flexibility in the inflation rate is not supported empirically, however. This variable generally exhibits a high degree of positive serial autocorrelation, although few empirical studies explicitly express the identifying assumptions that are necessary for distinguishing between structural inertia stemming from indexation and inertia in the inflation fundamentals.

Finally, the variable Q follows the differential equation

$$\frac{\mathrm{d}Q(t)}{\mathrm{d}t} = (\gamma+\theta)\left[P(t) - Q(t)\right]. \tag{11}$$

The system's path is summarized by the three differential, constant coefficient equations 8, 10, and 11, which are written in matrix terms as

$$\frac{\mathrm{d}\,\mathbf{X}(t)}{\mathrm{d}t} = \mathbf{A}\,\mathbf{X}(t) + \mathbf{B}\,\mathrm{E}\left[M(t)\right], \tag{12}$$

where the vector $\mathbf{X}(t) = [z(t), P(t), Q(t)]'$ and the matrices **A** and **B** are 3x3 and 3x1, respectively. The dynamic system defined by **A** has two negative characteristic values, which are convergent, and one positive, which is divergent, such that the dynamic equilibrium has a saddle-path configuration. The variables $P(t)$ and $Q(t)$ are predetermined, and the variable $z(t)$ is adjusted to situate the system on the convergent path.

The dynamic-system solution requires stipulating the performance of the exogenous variable in the system (that is, monetary policy or the money supply). This topic is addressed in the following sections.

2. Gradual Stabilization with Full Credibility

2.1 Gradual Stabilization

This section studies an inflationary stabilization experiment that is analogous to that proposed by Ball (1994a), but extended to include indexation of contracts. Given the assumption that $t \leq 0$, money grows at a constant rate equal to π, and the firms expect that this situation will be maintained forever.

$$M(t) = \pi t \text{ and} \tag{13a}$$

$$\frac{\mathrm{d}\,M(t)}{\mathrm{d}t} = \pi\ , \tag{13b}$$

where $t < 0$.

It is assumed that this situation has been maintained for a long time, such that the economy is in a stationary state defined by the conditions

$$\frac{\mathrm{d}\,z(t)}{\mathrm{d}t} = 0\ , \tag{14a}$$

$$\frac{\mathrm{d}\,Q(t)}{\mathrm{d}t} = \pi\ , \text{ and} \tag{14b}$$

$$\frac{\mathrm{d}\,p(t)}{\mathrm{d}t} = \pi\ , \tag{14c}$$

These three conditions give rise to the conclusions that in the long run, the price level should equalize the money supply, the inflation rate is π, and the real premium, z, is constant:

$$\bar{z}(t) = \frac{\pi}{\theta + \gamma} \,, \tag{15a}$$

$$\bar{p}(t) = \pi t \,, \quad \text{and} \tag{15b}$$

$$\overline{Q}(t) = \bar{p}(t) - \frac{\pi}{\theta + \gamma} \,, \tag{15c}$$

where $t < 0$.

If $t = 0$, however, the central bank announces an unexpected plan for gradual inflationary reduction, exponentially bringing the issue rate to zero with velocity μ:

$$\frac{\mathrm{d}M'(t)}{\mathrm{d}t} = \pi e^{-\mu t} \,, \tag{16}$$

where $t \geq 0$ and M' is the announcement of the money growth rate.[6] The dynamic-system solution is described in the appendix.

2.2 Choice of Parameters

The model is derived in terms of final prices, but the empirical counterpart to the parameters γ and θ is calibrated according to the typical characteristics of wage contracts in Chile. In this case, equation 8 is reinterpreted to represent the average level of nominal wages in the economy, and firms are assumed to set their prices with a constant margin over the unitary labor costs.

Jadresic (1991) uses data on collectively negotiated contracts and econometric estimates of the aggregate performance of the wage index. He concludes that the typical wage contract in the Chilean economy stipulates semiannual automatic adjustments and biannual renegotiation. If these values are taken as a measure of the respective distributions, then $\gamma = 0.5$ and $\gamma + \theta = 2.0$, and thus the fraction

6. Ball (1994a) and Bonomo and García (1994) examine the case in which the issue rate is linearly brought to zero. This modification does not change the qualitative conclusions obtained in the present paper.

that takes future economic conditions into account when renegotiating their contracts is $\lambda = 0.25$.[7]

However, other moments in the empirical distribution of adjustments are not satisfactorily adjusted through the exponential function imposed by the theoretical model. Jadresic (1995) indicates that the frequency of adjustment is very concentrated in the semiannual time frame, at least within the collectively negotiated contracts. In 1990, nearly 10 percent featured adjustment clauses between one and three months, 80 percent between four and six months, and 10 percent over six months. This distribution contrasts with that generated by the exponential function with $\gamma = 0.5$ and $\gamma + \theta = 2.0$, in which 38 percent of workers have adjustments less than or equal to three months and 62 percent have adjustments less than or equal to six months. The tails of the exponential distribution are wider at both extremes. This suggests that the impact of announcements on monetary policy are transmitted more rapidly to prices, because a greater fraction of workers adjust their prices in the short term. At the same time, however, the real effects are delayed longer, because the tail of the distribution is longer.

The parameter v reflects the complementarity of the price decisions. Ball (1994a, 1994b) and Bonomo and García (1994) use $v = 1.0$ and $v = 0.25$, whereas Jadresic (1991, 1995) only considers $v = 1.0$. For the purpose of comparison with the results in these articles, v is set at 0.25 in the tables and figures presented in this paper; the discussion signals quantitative and qualitative differences between this case and $v = 1.0$.

2.3 Output, Money, and Inflation

In the base exercise, the parameters are set at $\gamma = 0.5$ and $\gamma + \theta = 2.0$. Inflation starts at 6 percent, and the money growth rate falls at the velocity of $\mu = 0.46$. The average life of the stabilization program—that is, the time it takes the money growth rate to reach 3 percent—is eight quarters. The model defined by equation 12 determines the price and output trends, which are used to generate some indicators that summarize the results of the stabilization plan.

Figure 1 plots the trends in money, output, and inflation. For $t = 0$, which represents the moment in which the policy is announced and the beginning of the monetary adjustment process, the figure shows

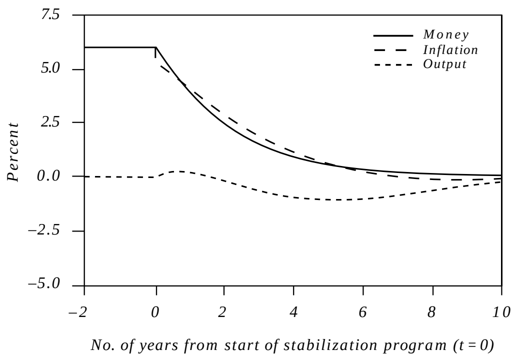
7. Alternatively, if this fraction is interpreted as the median, then $\gamma = 0.35$ and $\gamma + \theta = 1.39$.

an immediate, moderate drop in the inflation rate, from 6.0 to 5.3 percent. The firms and workers that renegotiate prices anticipate the gradual expansion of the disinflation process and incorporate it into their price decisions. This contributes to immediately reducing the real premium, *z*, relative to its historical level, which slows the pace of inflation despite the fact that the money issue rate initially stays at its historic level of π = 6.0 percent. Consequently, the economy enters a slightly expansionary phase, with real money balances and production growing above the long-term rate. The inflation rate continues to fall, but more slowly than the growth rate of money, because there is a holdover of inflation in those prices that need to recover their real historic level and also because the economy is expanding.

The expansionary process comes to an end after two quarters, when the accumulated output gap reaches just 0.2 percent. From this moment on, the inflation rate exceeds the money growth rate, and real balances and, therefore, output begin to fall. In the seventh quarter, the economy enters a persistent, though not very deep, recession. The economy hits bottom in the fifth year, with a negative output gap of –1.0 percent, and then gradually and variably begins to recover its long-term level.

The accumulated losses in the contractionary period exceed the gains observed in the expansionary period, such that the process of inflationary stabilization gives rise to a net sacrifice. The sacrifice coefficient, *S*,

Figure 1. Trends in Output, Money, and Inflation under Gradual Stabilization and Full Credibility[a]



Source: Author's calculations.
a. Model simulation of the base exercise (θ = 1.5 and γ = 0.5), with gradual stabilization velocity (μ = 0.46) and ν = 0.25.

is defined as the accumulation through time of output losses caused by each point of permanent inflation reduction:

$$S = -\int_{t}^{\infty} \frac{y\,\mathrm{d}t}{\pi} \cdot$$

In the base exercise, this equation yields $S = 0.92$.[8]

Inflation falls slowly, but the trend is relatively in line with the money issue rate. It takes nearly seventeen quarters for inflation to reach 1 percent; in the case of the money issue rate, it takes almost fifteen quarters.

Considering that the model is highly stylized, it is surprising that the sacrifice coefficient predicted for this experiment by the model is realistic and falls in the center of the range typically observed in empirical studies on stabilization experiences. Ball (1994b) examines a large sample of disinflation processes in industrialized countries for the period 1960-91. The observed values for S vary between 0 and 3.6, with a sample average of 0.8 (annual data) and 1.4 (quarterly data).

2.4 Frequency of Wage Adjustments

The effect of indexation on the inflationary dynamics of the economy and on the costs of stabilization are evaluated in table 1 and figure 2, which show, respectively, the sacrifice coefficient and the trends in inflation and output for different combinations of the frequency of indexation (θ) and real adjustment (γ). As the table indicates, for an economy without indexation ($\gamma = 0.5$ and $\theta = 0.01$), the model predicts a negative sacrifice coefficient ($S = -5.30$), that is, a net output gain associated with stabilization. The corresponding panel in figure 2 shows the output and inflation trends for a subset of the result in table 1. For $t = 0$, inflation falls immediately and more drastically than in the former example, from 6.0 to 2.2 percent. Its later convergence is slower, however, taking almost twenty-one quarters to reach 1 percent. The initial drop in inflation is larger because the nominal component of the renovated contracts has a much longer effective duration (as does the real component), which causes the premium, z, to encompass a larger fraction of the expected reduction in the money issue rate. The difference between the inflation rate and the money issue rate strongly expands the economy, peaking at the end of eleven quarters with a positive gap of 3.9 percent. The convergence with long-term equilibrium is monotonic, such that the economy never enters a recessive phase.

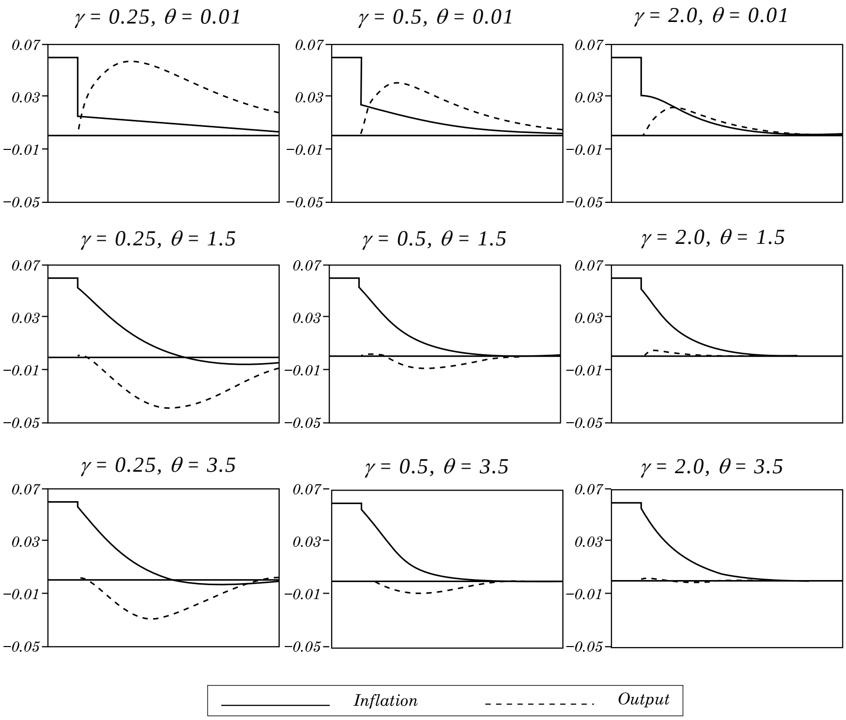
8. The qualitative results of the model under study are independent of the initial inflation rate, while the quantitative results are directly proportional to it. Thus the coefficient S is independent of the initial inflation level.

Table 1. The Sacrifice Coefficient under Gradual Stabilization and Full Credibility as a Function of the Frequency of Nominal Indexation and Real Adjustments[a]

Frequency of nominal indexation (θ)	*Frequency of real adjustments (γ)*				
	0.25	*0.50*	*1.00*	*2.00*	*10.00*
0.0	–11.30	–5.30	–2.04	–0.68	–0.04
0.5	6.64	0.01	–0.75	–0.43	–0.03
1.5	5.04	0.92	–0.11	–0.20	–0.03
3.5	3.13	0.76	0.08	–0.06	–0.02
10.0	1.46	0.40	0.09	0.00	–0.01

Source: Author's calculations.
a. The sacrifice coefficient, *S*, was derived for the indicated combinations of γ and θ based on an adjustment velocity in the money growth rate of $\mu = 0.46$, with $\nu = 0.25$.

Figure 2. Output and Inflation Trends and the Frequency of Nominal and Real Adjustments[a]



Source: Author's calculations.
a. Model simulation with gradual stabilization velocity ($\mu = 0.46$) and $\nu = 0.25$ for different values of the frequency of nominal (θ) and real (γ) adjustments.

This counterintuitive result is characteristic of models featuring staggered prices and gradual stabilization. Given gradual stabilization and full credibility, firms and wage earners anticipate the success of the stabilization program and incorporate in advance the impact of the reduction of the money issue rate on prices. This induces an initial expansion of output, because prices fall more rapidly than the money stock. Ball uses a Taylor-type model to demonstrate that neither absolute nor net output losses are generated by a linear reduction of the issue rate with a duration greater than or equal to 0.68 times the length of the contracts.

This theoretical result directly contradicts both a long macroeconomic tradition and most of the available empirical evidence. However, introducing indexation into the aggregate supply reverses this result. The automatic adjustments create inertia in the inflationary process, which severely restricts the conditions of credibility and gradual implementation that are necessary for triggering a net expansion in the economy, rather than a net sacrifice coefficient, in response to the reduction of inflation. Bonomo and García (1994) conclude that by introducing nominal adjustments over half the life of the contract, the minimum duration of the linear stabilization process that avoids the recessive cycle is just under three times the length of the contract. Indexation also affects the timing of the announcement of the stabilization program. In the model without indexation, the initiation of a moderate stabilization process should be announced in advance by at least one and a half times the average contract length in order to avoid a recession. In the model with indexation, the minimum period increases to four and a half times. The authors conclude that even when automatic adjustments increase the frequency of the nominal price adjustments, indexation increases the costs of the disinflation process.

The pernicious effect of indexation on stabilization costs is corroborated in this paper, but this does not mean that greater indexation, in the sense of more frequent adjustments for accumulated inflation, will necessarily be damaging for economic performance. The effect of the frequency of indexation on stabilization costs is not monotonic. The costs of stabilization rise when the frequency of indexation increases from zero to positive, but for some value of θ, the marginal impact becomes negative again to the extent that the economy recovers nominal neutrality. Essentially, when the adjustment is almost instantaneous, the economy recovers its neutrality in the face of nominal policies, and the costs of stabilization (and the sacrifice coefficient) approach zero. This result is consistent with the findings of Friedman (1974), Gray (1976), and Fischer (1977).

For $\gamma = 0.5$, increasing indexation from a semiannual to a quarterly frequency reduces the accumulated net output losses from 5.5 to 4.6 percent, and the sacrifice coefficient falls from 0.92 to 0.76 (see table 1). The trend is plotted in figure 2. The initial impact on inflation is slightly lower, but inflation then falls more rapidly in conjunction with the money issue rate. The pattern is similar to that found in the base exercise, but the size and duration of the cycle's expansionary and contractionary phases are smaller. When the frequency increases to a monthly rate, the costs of stabilization are reduced by half ($S = 0.40$). The stabilization costs also fall if the frequency is reduced from semiannual to annual, and the sacrifice coefficient reaches just 0.01. Thus within the structures examined, and given the typical biannual structure of price contracts, semiannual adjustment has the highest associated stabilization costs, since both increases and decreases in the frequency of indexation reduce the costs.

Changes in the frequency of real adjustments, γ, also have a nonmonotonic impact on stabilization costs. Infrequent adjustments tend to generate a positive expansionary effect, but as the frequency increases, the economy tends to recover its nominal neutrality.

The relation between the frequency of nominal and real adjustments, however, should be one of substitution. As described above, the typical contract in the Chilean economy has a two-year horizon with semiannual adjustments. In the U.S. economy, the typical contract is shorter, usually lasting one year, and it does not contemplate intermediate indexation; staggered contracts have an average duration of three years (Taylor, 1998). There is thus a substitution between the frequency of indexation and the frequency of full renegotiation. In this light, it is relevant to reexamine the analysis presented in table 1, compensating for the changes in the frequency of indexation with changes in the frequency of real adjustment. Table 2 shows the sacrifice coefficient associated with combinations of the total frequency of adjustments, $\gamma + \theta$, and the proportion,

$$\lambda = \frac{\gamma}{\gamma + \theta} ,$$

which represents the fraction of prices that are renegotiated in each moment with attention to the future evolution of the economy. The results point to an inverse (and monotonic) relation between these two parameters and the sacrifice coefficient. As the frequency of real or nominal adjustments increases, for any given value of λ, the sacrifice coefficient approaches zero, and as the proportion of prices that are fixed with attention to future prices and the future output gap rises, the sacrifice coefficient again approaches zero.

Table 2. The Sacrifice Coefficient under Gradual Stabilization and Full Credibility as a Function of the Frequency of Adjustments[a]

Real adjustments as fraction of total (λ)	*Frequency of total adjustments* ($\gamma + \theta$)				
	0.5	*1.0*	*2.0*	*4.0*	*10.0*
0.10	113.80	40.00	7.09	1.22	0.09
0.25	35.30	6.46	0.92	0.07	–0.02
0.50	4.78	0.01	–0.31	–0.14	–0.03
0.75	–2.66	–1.49	–0.58	–0.19	–0.03
0.90	–4.70	–1.90	–0.65	–0.20	–0.04

Source: Author's calculations.
a. The sacrifice coefficient, S, was derived for the indicated combinations of $\gamma + \theta$ and λ based on an adjustment velocity in the money growth rate of $\mu = 0.46$, with $\nu = 0.25$.

3. Immediate Stabilization with Full Credibility

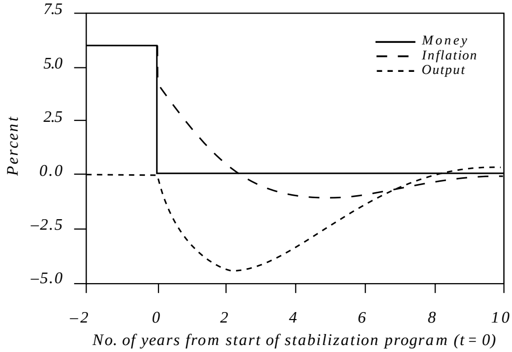
The speed at which the stabilization program is implemented affects the output and inflation trends, as well as the net costs of the program. The faster the reduction in the money growth rate, the shorter the duration and narrower the scope of the initial boom and the greater the depth of the later recession—and thus the greater the total costs of the stabilization process. This section analyzes the case of a sudden, drastic stabilization program (see figure 3). The issue rate moves immediately from π to zero, and equation 16 is replaced by

$$\frac{\mathrm{d}M'(t)}{\mathrm{d}t} = 0 . \tag{17}$$

where $t \geq 0$. This equation corresponds to the extreme case in which μ tends toward infinity.

Table 3 shows the inflation and output trends for the case of immediate stabilization with the parameters of the base exercise ($\gamma = 0.5$ and $\theta = 1.5$). For $t = 0$, inflation initially falls to 4.5 percent and then continues falling until it reaches 1 percent after a year and a half. Because the inflation rate exceeds the issue rate, the economy immediately enters a recession, which bottoms out after nine quarters with a gap of nearly 4.5 percent. The economy then slowly recovers its long-term level, and minor cyclical fluctuations are produced. The total cost of the stabilization process is equivalent to 18 percent of output (that is, S = 2.0), as compared to the lower cost of 6 percent described in the previous section.

Figure 3. Trends in Output, Money, and Inflation under Immediate Stabilization and Full Credibility[a]



Source: Author's calculations.
a. Model simulation of the base exercise (θ = 1.5 and γ = 0.5), with immediate stabilization velocity and ν = 0.25.

A comparison of table 3 and table 1 generates two observations. First, relative to gradual stabilization, the cost of immediate stabilization in terms of output increases for all combinations of the parameters. The sacrifice coefficient is positive in all cases. Second, the qualitative standard of the relation between the frequency of indexation and the sacrifice coefficient remains nonmonotonic. In comparison with the base example (γ = 0.5 and θ = 1.5), increasing the frequency of indexation from semiannual to quarterly adjustments reduces the sacrifice coefficient from 3.0 to 2.2, while reducing the frequency from semiannual to annual adjustments increases the sacrifice coefficient to 3.25. The total elimination of indexation, however, reduces the sacrifice coefficient to just 0.21. Thus a partial reduction of the indexation frequency does not necessarily reduce the costs of stabilization, although its total elimination does.

Table 3. The Sacrifice Coefficient under Immediate Stabilization and Full Credibility[a]

Frequency of nominal indexation (θ)	*Frequency of real adjustments (γ)*				
	0.25	*0.50*	*1.00*	*2.00*	*10.00*
0.0	1.51	0.21	0.03	0.00	0.00
0.5	13.20	3.25	0.66	0.11	0.00
1.5	9.25	3.00	0.83	0.19	0.00
3.5	6.10	2.18	0.72	0.21	0.01
10.0	3.25	1.24	0.46	0.16	0.01

Source: Author's calculations.
a. The sacrifice coefficient, *S*, was derived for the indicated combinations of γ and θ based on an adjustment velocity in the money growth rate of μ = 10,000, with ν = 0.25.

4. Stabilization with Partial Credibility

Thus far, the analysis has assumed full credibility. The announcement of a stabilization program is unexpected, but it is fully incorporated into the expectations and decisions of the price formation. This section investigates the consequences of relaxing this assumption, introducing the possibility of abandoning the announced plan before its completion. It is assumed that the monetary authorities begin to implement the money path defined below (equation 18), but in each moment there is a probability h that the stabilization effort will be stopped at that point. If the central bank abandons the plan, then expectations are that the issue rate will remain forever at the level prevailing at that time. Consequently, if the plan is abandoned in τ, the issue rate follows the path,

$$\frac{\mathrm{d}M'(t)}{\mathrm{d}t} = \begin{cases} \pi e^{-\mu t}, & \text{where} \quad 0 \le t \le \tau \\ \pi e^{-\mu \tau}, & \text{where} \quad t > \tau . \end{cases} \tag{18}$$

The probability h measures the program's credibility; a higher value for h indicates a lower credibility.

4.1 Expected Trend with Partial Credibility

The effective trend for the system depends on the stochastic realization of abandoning the program. Given the system's linearity, however, the expected system trend can be calculated on the basis of the expected money trend defined in equation 18. This can be written as

$$\mathrm{E}\left[m(t)\right] = \frac{h\pi t}{h+\mu} + \frac{\mu\pi\left(1-e^{-(h+\mu)t}\right)}{(h+\mu)^2} . \tag{19}$$

The dynamic system should verify the following limit conditions:

$$p(0) = 0 , \tag{20a}$$

$$Q(0) = -\frac{\pi}{\theta+\gamma} , \quad \text{and} \tag{20b}$$

$$\lim_{\tau\to\infty} \mathrm{E}\left[z(\tau)\right] = \frac{h}{h+\mu}\,\frac{\pi}{\theta+\gamma} . \tag{20c}$$

Table 4 shows the expected sacrifice coefficient associated with the different frequency parameters for the case of gradual stabilization with partial credibility.[9] The parameter h is set arbitrarily at 0.69, which is equivalent to a 0.5 probability of the program's being abandoned within a year. In all cases, the expected cost of the stabilization effort is higher than for the corresponding case with full credibility presented in table 1. The expectation that the program will be abandoned early delays the incorporation of the expected money trend into the evolution of recontracted prices. This reduces the length and breadth of the initial expansion, as well as the length and depth of the contractionary phase in those cases featuring recession. This observation confirms the results achieved by Ball (1994a, b) and Bonomo and García (1994) in the context of models based on staggered wages with a fixed horizon.

As in the previous cases, the expected stabilization costs remain nonmonotonic relative to the frequency of indexation. Essentially, table 4 qualitatively replicates the results in table 1, but at a higher cost level. Increasing the frequency of indexation from semiannual to quarterly and from quarterly to monthly reduces the expected sacrifice coefficient. However, lowering the frequency of indexation to annual adjustments or to zero also reduces the sacrifice coefficient.

A reduction in the credibility of the stabilization program (expressed in an increase in the credibility parameter, h) increases the costs associated with every case in table 4, but it does not alter the nature of the qualitative results. When the program's credibility is reduced by half (h = 1.38), the sacrifice coefficient rises from 1.8 to 2.2 in the base example. Increasing the frequency from semiannual to quarterly indexation reduces the coefficient to 1.6, while the move to annual indexation reduces it to 2.0.

Table 4. The Sacrifice Coefficient under Gradual Stabilization and Partial Credibility[a]

Frequency of nominal indexation (θ)	*Frequency of real adjustments* (γ)				
	0.25	*0.50*	*1.00*	*2.00*	*10.00*
0.0	–4.4	–2.6	–1.2	–0.5	0.0
0.5	9.9	1.5	–0.2	–0.3	0.0
1.5	7.0	1.8	0.2	0.0	0.0
3.5	4.4	1.3	0.3	0.0	0.0
10.0	2.2	0.7	0.2	0.0	0.0

Source: Author's calculations.

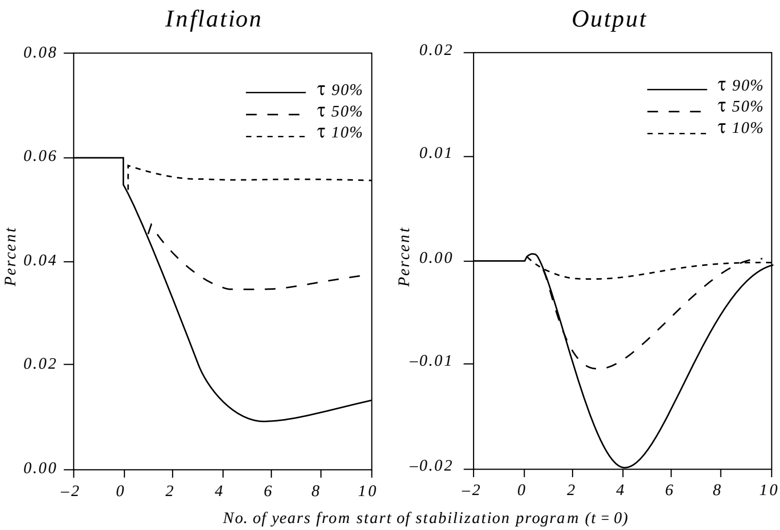
a. The sacrifice coefficient, S, was derived for the indicated combinations of γ and θ based on an adjustment velocity in the money issue rate of μ = 0.46, with ν = 0.25 and a credibility parameter of h = 0.69.

9. In this case, the sacrifice coefficient is defined as $\mathrm{E}(S) = -\frac{h+\mu}{\mu\pi}\int_t^\infty \mathrm{E}y\,dt$.

4.2 Effective Realizations with Partial Credibility

In addition to calculating the expected inflation and output trends, the model can be used to obtain their evolution for effective realizations of τ, which is the date at which the stabilization program is abandoned. The expected money and price trends are calculated in each moment for $t < \tau$, conditional on the implementation of the program up to that point, and the implications for the price and output trends are derived for that moment. The expected money and price trends are then recalculated for τ, conditional on the program's abandonment at that point, and the implications for the output trend are derived from τ forward. Figures 4 and 5 show the inflation and output trends for three different abandonment dates and different frequencies of indexation. The abandonment dates are calculated to represent the tenth, fiftieth, and ninetieth percentiles of the probability distribution of τ; they therefore provide a reference for the distribution of the results for each configuration of the parameters. Figure 4 illustrates the case of quarterly indexation, and figure 5 the case of monthly indexation.

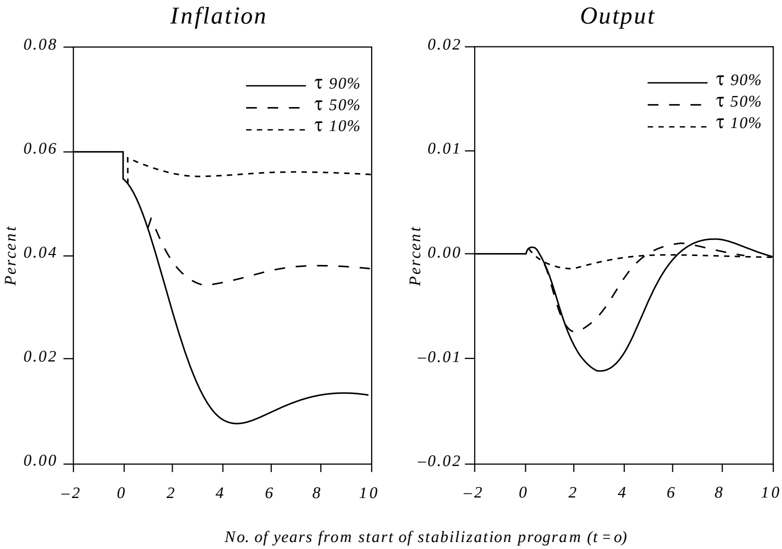
Figure 4. Trends in Output, Money, and Inflation under Gradual Stabilization, Partial Credibility, and Semiannual Indexation[a]



Source: Author's calculations.

a. Model simulation of the base exercise ($\theta = 1.5$ and $\gamma = 0.5$), with gradual stabilization velocity ($\mu = 0.46$), partial credibility ($h = 0.69$), and $\nu = 0.25$. The program is abandoned in $\tau = 10$ percent, $\tau = 50$ percent, and $\tau = 90$ percent.

Figure 5. Trends in Output, Money, and Inflation under Gradual Stabilization, Partial Credibility, and Monthly Indexation[a]



Source: Author's calculations.
a. Model simulation of the base exercise (θ = 11.5 and γ = 0.5), with gradual stabilization velocity (μ = 0.46), partial credibility (*h* = 0.69), and ν = 0.25. The program is abandoned in τ = 10 percent, τ = 50 percent, and τ = 90 percent.

A comparison of the depth and breadth of the economic cycle for the different degrees of indexation reveals that more frequent indexation reduces the volatility of output. When indexation is monthly and the stabilization program is extended beyond expectations (τ = 90 percent), this produces a recession that reaches its low point in the course of the third year, with a maximum gap slightly greater than 1 percent, and the economy accumulates a cost of 3.2 percentage points of output (*S* = 0.7). When the indexation is semiannual (the base exercise), the total cost increases to 9.9 percentage points of output (*S* = 1.9). In the absence of automatic indexation, the initial expansionary effect predominates and the economy receives a net benefit.

These results fully support the intuition outlined in Gray (1976) and Fischer (1977). In the face of monetary shocks such as the implementation of a stabilization program, greater indexation reduces the volatility of output and increases the volatility of inflation.

5. Conclusions

The analysis presented above gives rise to two basic conclusions. First, price and wage indexation increases the costs of reducing inflation. In the model of price (or wage) formation characterized by rational expectations and a degree of adjustment inflexibility, indexation introduces inertia into the inflationary process and increases the sacrifice coefficient. For the base model, given semiannual average adjustment and biannual contracts, the sacrifice coefficient fluctuates between 0.9 and 3.0, depending on the type stabilization program (gradual versus drastic) and its credibility.

Second, the relation between the frequency of adjustment and the sacrifice coefficient is nonmonotonic. The costs of stabilization increase when the frequency of indexation moves from zero to positive, but for some value of θ the marginal impact returns to negative as the economy recovers its nominal neutrality. Therefore, decreasing the frequency of indexation in the economy, which usually happens when inflation is low, does not necessarily reduce the sacrifice coefficient, and increasing the frequency of adjustment is not necessarily damaging. In the base model, the semiannual adjustment structure constitutes a minimum, such that any movement—whether toward more or less frequent adjustments—reduces the cost of stabilizing inflation.

This last observation helps explain the persistence of moderate inflationary processes of 20 to 40 percent. The sacrifice coefficient tends to increase for intermediate levels of indexation such as annual, semiannual, or quarterly, but it drops to practically zero when adjustments are made monthly. If this adjustment frequency responds positively to the inflation level, then the economy may find itself caught in a trap of moderate inflation. For low levels of average inflation, individuals and firms do not index their contracts. This keeps the costs of the stabilization plan at relatively low levels, and the benefits are felt almost immediately. This holds when the plan enjoys full credibility and is designed under a strategy of gradual implementation. As the average inflation level increases and individuals and firms begin to use automatic adjustment practices to protect themselves from inflation, the costs of the stabilization program rise significantly and the program's impact on inflation is delayed. The effects are multiplied when the plan's credibility is questionable. These conditions reduce the net benefits associated with the plan, or at least the short-term benefits observed by the authorities, such that inflation tends to take root in the economy while there are no political incentives to reduce it. When both the level of inflation and the frequency of indexation rise, the benefits of reducing inflation increase. At some point the stabilization costs begin to fall, until the authorities once again have incentives for reducing or eliminating inflation.

APPENDIX

The Model Solution with Gradual Stabilization and Full Credibility

The dynamic system formed by equation 12 is written as

$$\frac{\mathrm{d}\mathbf{X}(t)}{\mathrm{d}t} = \mathbf{A}\ \mathbf{X}(t) + \mathbf{B}\ \mathrm{E}\left[M(t)\right] .$$

The homogeneous part of the system has the following general solution for $\overline{\mathbf{X}}(t)$:

$$\overline{\mathbf{X}}(t) = C_1\, \mathrm{v}_1\, e^{\lambda_1 t} + C_2\, \mathrm{v}_2\, e^{\lambda_2 t} + C_3\, \mathrm{v}_3\, e^{\lambda_3 t} ,$$

where λ_i and v_i are the characteristic values and vectors associated with the rectangular matrix **A** and where C_1, C_2, and C_3 are constants determined on the basis of the limit conditions that should satisfy the system, namely,

$$p(0) = 0 ,$$

$$Q(0) = -\frac{\pi}{\theta + \lambda} , \text{ and}$$

$$\lim_{\tau \to \infty} z(\tau) = 0 .$$

One of the equation's characteristic values is negative; it can therefore be eliminated to ensure that the system converges with its stationary state.

The general solution for the dynamic system formed by equation 12 takes the form

$$\mathbf{X}(t) = \overline{\mathbf{X}}(t) + K_1 + K_2\, e^{-\mu t} ,$$

where $t \geq 0$ and the constants K_1 and K_2 are determined by the indeterminate coefficient method. The solution for the model featuring immediate stabilization corresponds to the limit of the above solution when $\mu \to \infty$.

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