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Changing Inflation Dynamics, Evolving Monetary Policy

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This paper surveys and modestly extends the theory of menu-cost models of the behavior of the aggregate price level after large-cost shocks. It does so in the context of an economy with a high underlying rate of inflation. It concentrates on the effect of large permanent and unexpected increases in the nominal price of inputs on the price level at different horizons. We use a simple theoretical model where increases in nominal cost will increase aggregate prices one for one in the long run. We study how the nominal rigidities implied by a menu cost distribute the increases in the price level between the impact effect immediately after the cost shock and the subsequent price adjustment until the price catches up with its long-run increase. In other words, we study the passthrough of large-cost shocks at different horizons. We pay particular attention to the role of the underlying inflation rate as well as to the role of the size of the cost shock, since both elements are important to determine the dynamics of aggregate prices.

Our interest in that question comes both from interest in testing aspects of price-setting theories and from a practical monetary policy point of view. On the theoretical side, the differential effect of large versus small shocks is the hallmark difference between menu-cost models and time-dependent models of price adjustment. Hence, the characterization of the model's behavior is important to be able to

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discern between theories with observed experiences. On the monetary policy side, we are interested in this particular question because of the recent experience in Argentina, an economy where inflation has been quite high for international standards in the last decade, and where due to changes in macroeconomic policies in the last four years there have been large-cost shocks. In particular, there have been large changes in exchange rates as well as extremely large changes in the price of regulated prices, mainly inputs related to energy. In this context we ask the question of whether the response to these large-cost shocks is close to one of an economy with fully flexible prices or to one with time-dependent price setting, such as the Calvo model.

We give a short narrative of instances where, in a single month, there have been very large-cost changes in the inputs for goods that make up the core CPI for Argentina between 2012 and 2018. We also use a comprehensive, never used in the price adjustment literature, micro-data set underlying the construction of the core CPI for the city of Buenos Aires to compute the objects analogous to the ones we describe in the theory. From the comparison between statistics in the model and in the data, we conclude that either full price flexibility or time-dependent rules are quite counterfactual. Nevertheless, in the cases of very large positive cost shocks in an economy with an underlying high inflation, the passthrough of the shock to prices is quite fast. In the most extreme case, when the exchange rate jumped almost 25% in one day (from 31.94 pesos per dollar on August 29th to 39.60 on August 30th), the fraction of firms changing prices, which is 24% in normal times, rose to almost 60% between August and September 2018.

We believe that through this narrative approach we can make a plausible case that regulated prices can be thought of as exogenous changes in cost, and that the same will be true for some of the large changes in exchange rates. The change in regulated prices followed a period of many years during which, despite the fact that inflation was very high for international standards, regulated prices were frozen in nominal terms. The situation became untenable from the fiscal point of view, costing at least 3% of GDP, and also due to the distortions that this policy generated. The normalization of prices occurred in steps due to the very large increases that it implied, as well as due to challenges in the courts. To get an idea of the magnitude of the change in relative prices, during the sample period, the price of natural gas relative to the core CPI was multiplied by a factor of five and the relative price of electricity by a factor of three. On the exchange-rate changes there were different reasons for the observed large changes. In 2014 there was a devaluation within a regime with severe capital controls and dual exchange rates. At the end of 2015 and beginning of 2016, there was a large devaluation, when the multiple exchange-rate regime was abandoned essentially overnight. The sharp depreciation of the exchange rate throughout April–August 2018 is probably the result of a mix of a reaction to a change in economic policies (perhaps larger than what policymakers had anticipated) and exogenous shocks within the framework of a dirty floating exchange-rate regime. In our model, cost shocks are once-and-for-all unexpected changes, which may not be completely accurate for some of the episodes.[1]

We use the microdata underlying the construction of the consumer price index (CPI) in the city of Buenos Aires to compare the predictions of the model with actual observations after several of the large-cost increases that occurred between 2014 and 2018. Some readers may be aware that the National Statistical Agency (*Instituto Nacional de Estadística y Censos, Indec*) produced price indices that are widely regarded as underestimating the inflation rate between 2008 and 2015.[2] The price index of the city of Buenos Aires has the advantage that it was independent from the National Statistical Agency and produced figures very similar to the privately produced price indices.

We propose a theoretical model where firms have both idiosyncratic as well as aggregate changes in costs. We assume that firms are monopolistic competitors and that they face a fixed menu cost for changing prices. The solution to the firms' price-setting problem gives rise to a classical sS rule for price changes, which postulates price increases as well as price decreases. The optimal decision rule, as well as the response of the aggregate price level, depends crucially on the ratio of the inflation rate to the variance of the idiosyncratic shocks. While idiosyncratic shocks make the analysis more complicated, we think that they are essential to the answer of the problem for two reasons. First, they are required to reproduce the large fraction of price decreases that are observed even when inflation rate is above

1. For instance, the normalization of the exchange-rate system and removal of capital controls is a policy that was to a large extent announced by the two main parties before the elections at the end of 2015. Indeed there is an unsettled debate in the Argentine economic circles on whether the effect of the likely increase in the exchange rate that the unification of the exchange-rate markets will entail was anticipated and included in price changes months before it happened. We discuss these episodes in more detail in section 1.

2. See, for instance, Cavallo (2013).

25% per year. Second, the behavior of the passthrough depends on the magnitude of these shocks relative to the level of inflation. In the theoretical section, we compare the effect of cost increases in three cases: a very low inflation rate, which is the case of most economies; high inflation rates, of the order of Argentina during this period (say, 25% per year); and very high inflation. The degree of passthrough depends on both the size of the steady-state inflation rate and the size of the shocks. It turns out that, even for inflation rates as large as 25% per year, still one can see clear effects of price stickiness. Yet, for large-cost increases of the magnitude that occurred in Argentina, the passthrough occurs in an extremely short time. In particular, there is a very large impact effect, with most of the adjustment occurring at the time of the shock, and a very short half-life, smaller than two months, for the remaining adjustment. Indeed this matches what we see during the months of large-cost increases in Argentina: a very sharp increase in the fraction and in the size of price increases, almost no change in the fraction and in the size of price decreases, and a jump on the inflation rate close to the size of the cost increase.

It is well known that cost shocks explain a large fraction of the variance of inflation in standard estimates of medium size New Keynesian models. Nevertheless, these costs are typically a residual in the standard specification. On the other hand, there is a literature that tries to use identified cost shocks to evaluate different price-setting mechanism. Our paper adds to the literature that studies the effect of large-cost shocks in price-setting models with menu costs of price adjustment. Early examples of this literature are Gagnon (2009) and Gagnon and others (2013). These papers consider the experience of Mexico in the mid-90s, when there was both a large step devaluation (about 40%) and changes in the VAT (from 10% to 15%). Another similar recent exercise is the one in Karadi and Reiff (2019), which uses changes in the VAT in Hungary. In both cases, a version of a menu-cost model is used to interpret the microdata underlying the construction of CPI and to compare the predictions of this class of models with the data. A related, yet different evidence, is the study of the exchange-rate passthrough to export and import prices in customs data in Bonadio and others (2016), comparing small changes in the Swiss franc's value with the large change that occurred when the Swiss National Bank abandoned its peg to the euro in January 2015. Finally, Álvarez, Lippi, and Passadore (2016) estimate panel regressions of the short-term passthrough of exchange-rate changes to consumer prices that include non-linear terms in the size of the exchange-rate changes.

Differently from the previous studies, these panel regressions do not use microprice statistics. Relative to Gagnon (2009), Gagnon and others (2013), and Karadi and Reiff (2019), and motivated by the levels of inflation in Argentina during the period of interest, this paper has a more systematic treatment of the role of the running (or steady-state) inflation and of the size of the cost shocks on the level of short-term passthrough, and on the overall speed of adjustment. Finally, relative to Álvarez and others (2019) we study a different period of time for Argentina, but more importantly, in this paper we concentrate on the effect that large unexpected-cost shocks have on price dynamics, as opposed to the effect of different steady-state inflation levels, which is the main point of Álvarez and others (2019).

The remainder of the paper is organized as follows. In section 1 we provide a brief narrative of macroeconomic events in Argentina in the period 2012–2018 as they pertain to nominal cost shocks and inflation. The theoretical analysis is in section 2. The firm's problem is described in subsection 2.1; subsection 2.2 describes the steady-state distribution of price markups over nominal marginal costs; subsection 2.3 explains how inflation affects optimal decision rules; subsection 2.4 derives analytically expressions for the impulse responses of consumer prices to cost shocks; and subsection 2.5 contains numerical simulations of how cost shocks affect consumer price dynamics for small and for large-cost shocks, and for three different inflation rates (low, high, and extremely high). Finally, section 3 compares the predictions of the model with the evidence emerging from city of Buenos Aires consumer price microdata.

1. Nominal Cost Shocks and Inflation, Argentina 2012-2018

In this section we provide some background on the evolution of inflation and the nominal value of some key inputs during our sample period, July 2012 to December 2018. We first comment on the monetary policy framework and exchange-rate developments, and later on regulated price policies for energy inputs.

We divide the analysis of the monetary policy framework in three periods: the dual exchange-rate regime prior to December 2015, the inflation-targeting regime between March 2016 and December 2017, and the abandonment of the inflation-targeting regime throughout 2018.

During the first period, the monetary policy framework was a dual exchange-rate regime. The Central Bank fixed the Argentine peso price of the U.S. dollar for transactions related to international trade as well as for some limited financial ones. Capital controls were binding and a shadow exchange-rate market that carried a premium over the official one developed. We believe that this was caused by the high rate of money growth due to the monetary financing of deficits. Part of the monetary financing of deficits was sterilized with Central Bank debt that reached close to 6% of GDP in March 2016. The average rate of core inflation between July 2012 and December 2015 in the city of Buenos Aires was 31% (with a median of 28%).[3] The official exchange rate crawled at a median rate of 17% (annualized median monthly rate of devaluation). Between November 2013 and March 2014, a succession of jumps in the exchange rate resulted in a cumulative increase of 32% in the exchange rate. Core CPI prices in those months rose by 15%.

On December 15th, 2015, exchange-rate controls were removed and the exchange rate started to float with limited central bank intervention. The unification of the foreign-exchange market entailed a depreciation of the peso of over 50% between December 2015 and February 2016.[4] In March 2016 the Central Bank adopted an interest rate peg as its policy instrument in the context of an incipient inflation-targeting regime, which was formally adopted in September. The national inflation targets were 12–17% for 2017, 10% ± 2% for 2018, and 5% ± 1.5% for 2019. The actual core inflation rates in the city of Buenos Aires were 35% in 2016, 24% in 2017, and 43% in 2018. Wages increased by 34% in 2016, 24% in 2017, and 21% in the first nine months of 2018 (prices increased by 30% in this period).[5]

Starting in December of 2017, there was a gradual abandonment of the inflation-targeting framework. On December 28th, 2017, the inflation target for 2018 was raised from 10% to 15%, and in January the Central Bank lowered interest rates by 150 basis points from 28.75% to 27.25%. The market perception was that this interest rate move was motivated by political pressure. A speculative run on the Central Bank's debt unraveled between April and September 2018. In the period between April and September (end of period), the Central Bank paid (did not rollover) 529 billion pesos of short-term debt (40% of its debt and 53% of the monetary base in April), and the monetary

3. Average and median of annualized monthly inflation.

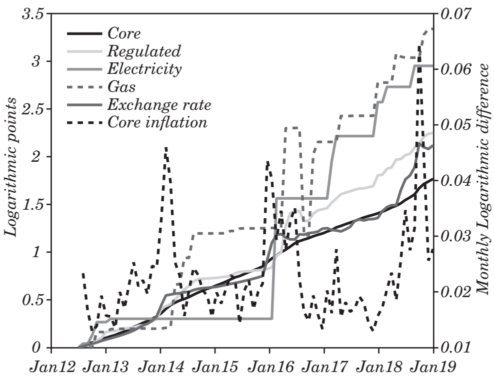
4. See table 1 and figure 1.

5. Core inflation for the city of Buenos Aires. Wages are from *Ministerio de Trabajo, Empleo y Seguridad Social* (2018). [Ministry of Labor, Employment and Social Security]

base increased by 250 billion pesos in the same period.[6,7] This led to a sharp depreciation of the Argentine peso. The peso-dollar exchange rate quivered around 17 pesos per dollar in the period July–December 2017, then around 20 pesos between late January and the end of April, and reached 41 pesos per dollar in late September. The evolution of the (log) exchange rate is depicted in figure 1.

Public utilities prices were also an important source of large nominal cost shocks during our sample period. The nominal price of regulated goods such as electrical power, natural gas, water, and public transportation was practically frozen for a decade up to 2014. With the price level rising, the relative price of regulated goods, especially energy items, steadily eroded. The gap between the nominal marginal cost of these regulated goods and their sale prices was covered with government subsidies that in 2015 are conservatively estimated around 3% of GDP. A small attempt at normalizing these prices resulted in large nominal increases in the price of natural gas during 2014. A more ambitious normalization started in 2016.

Figure 1. Cost Shocks and Inflation



Source: Authors' calculations.
Note: Prices on the left axis are the natural logarithm of the price index published by the city of Buenos Aires normalized to one at the initial date. The exchange rate is the natural logarithm of the monthly average published by the Central Bank. Core inflation is the monthly log difference of the core price level (rest), which excludes seasonal and regulated goods and services.

6. See Bassetto and Phelan (2015) for a related theoretical model.

7. Establishing the reasons behind this run is difficult and goes beyond the purpose of this paper. Several explanations have been proposed in local economic circles, including the importance of current account deficits, the effect of negative productivity shocks on agriculture (a very large drought), the size and maturity of the Central Bank debt, changes in local taxation of capital flows, the fear of fiscal dominance in the near future, and the adverse international financial circumstances, just to cite a few.

Figure 1 provides an overview of the evolution of the nominal variables described in the preceding paragraphs. All prices are from the city of Buenos Aires CPI.[8] The solid lines depict the natural logarithm of the core CPI, regulated prices in the CPI, the price of electricity, the price of natural gas, and the exchange rate. Core inflation excludes seasonal products and regulated prices. All five are normalized to their July 2012 value.

The black solid line is the price level for core goods and services while the dotted black line represents its rate of change (in log differences). The dark grey line represents the exchange rate expressed as pesos per dollar. There are three major devaluations. During the dual exchange-rate period, the exchange rate exhibits a rate of growth below that of core prices except for the period of the devaluation around January of 2014. There is a second sharp depreciation of the peso between December 2015 and February 2016 after the removal of capital controls. The last episode of depreciation of the peso, about 80%, occurred between April and September of 2018.

Regulated consumer prices in the city of Buenos Aires are represented by the light grey line in figure 1. As it was the case for the exchange rate, prior to December 2015 these prices grow at a slower pace than core prices and there are two important relative price corrections, one in 2014 and another in 2016. The main drivers of regulated prices are the prices of energy, especially electricity, and gas. The middle grey line represents the price of electricity. It is a step function with jumps of 253% in February 2016, 92% in January–March 2017, 68% in December–February 2018 and 25% in August 2018. The price of natural gas follows a similar pattern.[9,10]

We summarize the behavior of nominal cost shocks in a proxy variable, which we refer to as cost proxy of cost shock. We assume that consumer goods are produced with labor, tradable inputs, and regulated goods. In our model we assume that there is a consolidated sector that produces and retails consumption goods. It purchases an

8. We work with data for the city of Buenos Aires as the reliability of national statistics between 2007 and 2016 has been severely questioned. For example, on February 2013, the International Monetary Fund issued a declaration of censure against Argentina in connection with the inaccuracy of CPI data from the National Statistical Agency, the Indec. Also see Cavallo (2013) for a comparison of national statistics and online prices across several Latin American countries, including Argentina.

9. See table 1.

10. The price of natural gas fell 68% in August 2016 because the Supreme Court stayed the April price increase on procedural grounds. The price increases were resumed after the Executive remedied the judicial objection to the previous price increase.

aggregate input in a flexible price competitive market and sells its output to consumers in a monopolistically competitive market. To construct our intermediate input measure, we assume following Jones (2011) that the share of intermediate inputs in the production of the final consumption goods is 50%, and that the remaining is a labor share of 50%. The weights of energy and tradable intermediate goods are 10% and 40%, respectively. Hence, our measure of cost shocks is obtained by computing first a geometric price index for this aggregate input as $p_R^{0.1} E^{0.4} W^{0.5}$, where p_R denotes regulated prices, E is the peso/U.S. dollar exchange rate, and W are nominal wages.

Figure 2 depicts the cost shock proxy variable, wholesale prices and core inflation.[11] The proxy for nominal cost shocks tracks wholesale prices closely with a correlation of 0.92. Observe that spikes in nominal cost shock inflation are associated with spikes in core inflation.

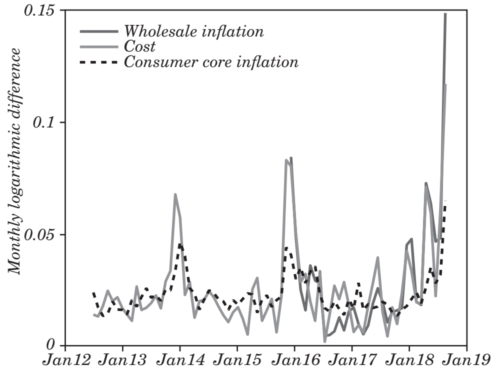
Table 1. Large Nominal Cost Shocks

	Elect.	*Natural gas*	*Exchange rate*		*Elec.*	*Natural gas*	*Exchange rate*
Oct-12	9			Feb-17	48		
Nov-12	15	18		Mar-17	30		
Jan-14			12	Apr-17		31	
Feb-14			11	Jul-17			7
Apr-14		57		Dec-17	43	42	
Jun-14		32		Jan-18			8
Aug-14		29		Feb-18	17		
Dec-15			19	Apr-18		36	
Jan-16			19	May-18			17
Feb-16	253		8	Jun-18			12
Apr-16		184		Aug-18	25		9
Jul-16			5	Sep-18			28
Aug-16		–68		Oct-18		27	
Oct-16		134		Nov-18		6	
Nov-16		14					

Source: Authors' calculations.
Note: We report the percentage change between prices in month t and t–1. Price data for electricity and natural gas is from the Consumer Price Index for the city of Buenos Aires. Exchange rate is the change in the monthly average exchange rate.

11. There is no official reliable data on wholesale prices prior to December 2015.

Figure 2. Cost Shocks and Inflation



Source: Authors' calculations.
Note: Prices on the left axis are the natural logarithm of the price index published by the city of Buenos Aires normalized to one at the initial date. The exchange rate is the natural logarithm of the monthly average published by the Central Bank. Core inflation is the monthly log difference of the core price level (rest), which excludes seasonal and regulated goods and services.

In the next section we present a theoretical model of the pricing decision of a retail firm in order to study the speed and the magnitude of the passthrough of these cost shocks to consumer prices.

2. Passthrough of Cost Shocks to Prices: Theory

In this section we study the nominal passthrough to consumer prices of the nominal cost shocks in a menu-cost model of price adjustment. We consider the problem of a monopolistically competitive firm that faces idiosyncratic demand and productivity shocks in an environment in which the cost of an aggregate input grows at a constant inflation rate. We study how the aggregate consumer price level reacts to an unexpected jump in nominal marginal costs in this environment.

We first describe the firm's price-setting problem in subsection 2.1 and then look at the cross sectional distribution of prices in subsection 2.2. We then proceed to describe how steady-state inflation affects decision rules in subsection 2.3, which refers to Álvarez and others (2019). Finally, in subsection 2.4, we report the impulse response of price distributions to cost shocks and, in subsection 2.5, we compare the reaction of prices to small and large-cost shocks and find interesting nonlinear effects.

2.1 Price-Setting Problem for the Firm

We consider an economy where firms' marginal nominal costs have a common and an idiosyncratic component. The common component is given by a nominal cost, which we will assume that, after an initial value is realized, will grow at a deterministic constant rate. The (log of the) idiosyncratic component follows driftless Brownian motion, with innovation variance σ^2. Firms face a downward sloping demand, act as monopolistic competitors, and must pay a fixed menu cost to adjust their price. The firm's marginal (and average) cost will then be $W(t)x(t)$, where $W(t)$ is the nominal cost of the aggregate input and $x(t)$ is an idiosyncratic shock. We assume that $x(t) = exp(\sigma B(t))$, where $B(t)$ is a standard Brownian motion, independent across firms. We will start the firms at a steady state, where W_t has been growing at a constant inflation rate π, so that $W(t_0+T) = W(t_0)e^{T\pi}$.

We will use g to describe the logarithmic deviation of the firm's current markup relative to the one that maximizes instantaneous profits. We refer to this variable as the "price gap". The price gap is positive, $g > 0$, if the price of the product is relatively high or if the firm's cost is relatively low. We will assume that the optimal markup is independent of the level of the costs and of the demand, as it will be the case with an iso-elastic demand function and constant marginal cost. Note that during a period where the firm does not change prices, its markup changes only because its cost changes, i.e., $dg(t) = -d\log W(t) - d\log x(t)$. For instance, at steady state, we have that $dg = -\pi dt - \sigma dB$ during the period at which the price of the good does not change. At the times when the firm decides to pay the fixed cost and change prices, $g(t)$ changes discretely and in equal proportion (equal in log points) to the change in price.

The firm problem can be summarized by the Bellman equation:

$$V(g) = \max_{\tau} \mathbb{E}\left[\int_0^\tau e^{-\rho t} F\left(g\left(t\right)\right)dt + e^{-\rho\tau}\left[\psi + \max_{\tilde{g}}\ V\left(\tilde{g}\right)\right] \middle| g(0) = g\right] \qquad (1)$$

subject to

$$dg(t) = -\pi dt - \sigma dB(t) \text{ for all } 0 \le t \le \tau \text{ and } g(0) = g. \qquad (2)$$

In this problem ψ is the fixed cost, ρ is the (real) discount rate, π is the inflation rate (of the aggregate input), σ is the idiosyncratic volatility of the cost shocks, and $F(\cdot)$ is the instantaneous profit function, written as a function of the price gap. The expectation in the objective function is with respect to the cost shocks, the values of the path for $B(t)$. Mathematically speaking, the object of choice τ are *stopping times*. A *stopping time* indicates the time and circumstances under which prices will be adjusted. In this kind of problem, the optimal rule is that τ occurs the first time that $g(t)$ is outside a range of inaction. After paying the cost and deciding the optimal price we find the optimal return point as $g^{*} = arg\max_{\tilde{g}} V(\tilde{g})$, or $V(g^{*}) = \max_{\tilde{g}} V(\tilde{g})$. In appendix B we write the ordinary differential equation (o.d.e.) and boundary conditions which simultaneously determine the function $V(\cdot)$ and the values of $\underline{g}, g^{*}$ and $\overline{g}$. In appendix C we formulate and compute a discrete time and discrete state-space problem that approximates the continuous one.

The optimal policy for the firm is an sS rule described by three numbers $\underline{g}, g^{*}$ and $\overline{g}$. We refer to $\underline{g} < \overline{g}$ as the boundaries of the range of inaction, and to $g^{*} \in (\underline{g}, \overline{g})$ as the optimal return point. The inaction region is thus described by the interval $[\underline{g}, \overline{g}]$. If the price gap g is in the inaction region, the price of the firm stays constant and the price gap changes with the cost changes—with opposite sign. If the price gap is lower than $\underline{g}$, so that the markup is very low, then the firm will pay the fixed cost ψ and increase prices so that, right after the price change, the price gap will be g^{*}. Likewise, if the price gap is higher or equal than $\overline{g}$, so that the markup is very high, the firm will pay the fixed cost ψ and decrease its price, so that, right after the price change, the price gap becomes g^{*}.

Note that we are writing the profit function F as a function of the price gaps exclusively. In general, even at steady state, it should be a function of both the price and the cost. Below, we explain the conditions under which this simplification can be obtained. We choose units so that F is measured in either terms of units of the aggregate input, or as deviations of maximized steady-state profits—our preferred choice. Of course, the fixed cost ψ has to be measured in the same units as F. The optimal decision rule depends only on the ratio ψ/B, since the value function is homogeneous of degree one in ψ and B. Intuitively, the fixed cost matters only through the relative advantages of changing prices, captured by the curvature of F, rather than on each of them separately. Furthermore, the discount factor ρ is a real interest rate measuring the intertemporal price of the aggregate input. The inflation rate π is also the nominal change in the price of the aggregate input. Since ρ, π and σ^2 are rates per time unit, the optimal decision rule depends, apart from on ψ/B, only on the ratios $\{\rho/\sigma^2, \pi/\sigma^2\}$.

Derivation of the profit function F. We have written the profit function F having the price gap g as its only argument. This is a simplification which provides lots of tractability. In Álvarez and others (2019), we work without this simplifying assumption and obtain essentially the same results.[12] We describe here the assumptions so that we can use the simplified version. Let $Q(p/W)z$ be the quantity demanded as a function of p/W, the ratio of the nominal price of the good p and the nominal price of the generic input W. In steady state we can write this relative price or the relative price with respect to some aggregate good without loss of generality. It also turns out that in the set-up described by Golosov and Lucas (2007), which is in no way pathological, this is a consequence of the general equilibrium structure. The variable z is a multiplicative shifter of the demand, which we use for two different illustrations. The nominal marginal cost is xW. We will proceed by steps. First we will derive the profit $\tilde{F}$ with (g, z, x) as arguments. Then we will add assumptions to eliminate (x, z) from it.

Let's use m for the log of the optimal markup, i.e., let the nominal price that maximize instantaneous profits be: $P^* = e^m xW$. Thus g is defined as:

$$g \equiv \log\left(\frac{\frac{P}{Wx}}{\frac{P^*}{Wx}}\right) = \log\left(\frac{\frac{P}{Wx}}{M}\right) = \log\left(\frac{P}{Wx}\right) - m \text{ or } P = Wxe^{g+m}. \tag{3}$$

Now we are ready to write the profit function. The units of profits will be first in terms of the real value of the aggregate input. Profits in nominal terms are $[P - xW]Q(P/W, z)$, i.e., nominal markup times quantity. By dividing this expression by W, we obtain profits in time t units of the aggregate input.

$$\tilde{F}(g,x,z) = \left[\frac{P}{xW} - 1\right] Q\left(\frac{P}{Wx}x\right) zx = \left[e^{g+m} - 1\right] Q\left(e^{g+m}x\right) zx \tag{4}$$

Furthermore, *assuming that Q is iso-elastic*, with elasticity equal to η, so that $Q(P/W) = A(P/W)^{\eta} z$ for some constant A, then the optimal markup M, or its log m, will be constant and equal to $m = \log(\eta/(\eta-1))$. In this case profits will be:

$$\tilde{F}(g,x,z) = \left[e^{g+m} - 1\right] e^{-\eta(g+m)} A x^{1-\eta} z.$$

12. See the section on the model with random walk shocks and CES demands, which we refer to Kehoe and Midrigan (2015) version of the Golosov and Lucas (2007)'s model.

It should be clear that, by definition, F is maximized when $g = 0$.

Adding the *extra assumption* that the demand shifter satisfies $z = x^{\eta-1}$, then we obtain that $\tilde{F}$ does not depend on (x, z). To be honest, this is a strange assumption; it requires the shock that increases cost to simultaneously push the demand up, so that the maximized profit remains the same for any value of x. On the other hand, it simplifies the algebra a lot! As mentioned above, in Álvarez and others (2019) we work out both versions of the models and find very small differences.

An alternative is to use a *second order approximation* of the function $\tilde{F}(g, x, z)$ and to retain only the leading terms on g. In particular, we use a second order approximation of $\tilde{F}(g, x, z)$ around $g=0$, $x=\overline{x}$ and $z=\overline{z}$. Using that $g = 0$ maximizes profits, and the multiplicative separable nature of the profit function into three terms, ignoring the terms that are higher than second order and the terms not involving g, we have

$$F(g) \equiv -\frac{(\eta-1)\eta}{2} g^2 = -Bg^2 \text{ so that } B \equiv \eta(\eta-1)/2 \qquad (5)$$

where we are measuring profits relative to the maximized profit at $x=\overline{x}$ and $z=\overline{z}$, which equals $\tilde{F}(0,\overline{x}) = \left(\frac{\eta}{\eta-1}\right)^{-\eta}\left[\frac{A\overline{x}^{1-\eta}\overline{z}}{\eta-1}\right]$. Of course, we can combine these assumptions too.

Lack of First-Order Strategic Complementarity. Finally, the result in equation (5) is useful because it states that in the model we focus on, there is no *first-order strategic complementarity*. This is because we can summarize the behavior of the rest of the firms in z. This will be the case in the standard New Keynesian model, where there will be at least two effects on the profit function coming from aggregate consumption: one is that with CES demand for each firm, higher output of each of the other firms shifts the demand up, and the other, in opposite sign, that higher output decreases the Arrow-Debreu price of the good in the current period. But as we have seen, these effects—captured by z—are of third order in the profit function in the current set-up.[13] Interestingly, this means that we can use as an accurate approximation the firm's decision rules characterized by $\{\underline{g}, g^*, \overline{g}\}$ even if the rest of the economy is not at a steady state, as long as the firm expects constant growth rate of the aggregate input

13. The general equilibrium version in Golosov and Lucas (2007) also makes the value of the nominal aggregate input, labor in their case, depend only on the path of nominal money.

prices. We will rely heavily on this property to characterize the impulse response of prices to a one-time common shock to their cost.

2.2 Steady-State Distribution of Price Gaps and Price Changes

We let $f(g)$ be the steady-state density of the distribution of price gaps. This density has support $[\underline{g}, \overline{g}]$. It solves a simple ordinary differential equation, balancing the flows in and out when a price change occurs. Its shape depends on the parameters $\{\underline{g}, g^*, \overline{g}, \pi/\sigma^2\}$. Of course, $\underline{g}, g^*, \overline{g}$ depend on all the parameters that define the steady-state problem of the firm described above, namely $\{\psi/B, \rho/\sigma^2, \pi/\sigma^2\}$ or, using the simplification described in appendix B, it is described by just $\{\psi/B, \pi/\sigma^2\}$. The equations that determine the steady-state density given the decision rules and parameters are:

$$0 = \pi f'(g) + \frac{\sigma^2}{2} f''(g) \text{ for all } g \in \left[\underline{g}, g^*) \cup (g^*, \overline{g}\right]$$

with boundary conditions:

$$0 = f(\underline{g}) = f(\overline{g}) \text{ and } \int_{\underline{g}}^{\overline{g}} f(g)\, dg = 1 . \tag{6}$$

The first line is the Kolmogorov forward equation, and the second line has the three relevant boundary conditions. The solution is given by the sum of two exponential functions. The boundary conditions on the extreme of the inaction regions indicate that there is zero density in those points. Intuitively, this is because they are exit points, so it is "hard" to accumulate density near them. This result will be important for some of the results below.

The shape of the density f depends on the inflation rate relative to the idiosyncratic variance π/σ^2, both through their direct effect as seen in equation (6), and indirectly through their effect on $\underline{g}$, $g^*$and $\overline{g}$. For finite π/σ^2, the distribution has zero density in its two boundaries, it is strictly increasing in $(\underline{g}, g^*)$, non-differentiable at g^*, and strictly decreasing in$(g^*, \overline{g})$. In particular for $\pi/\sigma^2 = 0$, the distribution is *symmetric* around $g^* = 0$, and has a tent-shape, with $f''(g)=0$. As π/σ^2 increases, the value of g^* becomes positive, and the shape of f becomes concave from $[\underline{g}, g^*)$ and convex between $(g^*, \overline{g}]$. As $\pi/\sigma^2 \to \infty$, the distribution converges to a uniform distribution between $[\underline{g}, g^*]$ and to a zero density everywhere else, as in the model of Sheshinski and Weiss (1979), which has $\sigma^2 = 0$ and finite π.

The change in the shape of the invariant distribution f reflects, in a very intuitive way, the different strength of the idiosyncratic shocks (measured by σ^2), which are symmetric, and the effect of inflation (measured by π), which is asymmetric. As inflation increases, the price gaps g naturally tend to pile up in the left side, since the cost increase in expected value as the nominal price remains fixed. Indeed as $\pi/\sigma^2 \to \infty$, this effect is so strong that price gaps essentially march deterministically from g^* to $\underline{g}$, and hence the distribution is uniform, as stated above. Lastly, the drastic change in shape of f around $g^*$is also intuitive, since after a price is changed, the new price of the product is set so that $g = g^*$, which explains why the mass is highest at this point and why it is not differentiable, since the behavior of f around this point is governed by mass coming from the boundaries of the range of inaction.

In steady state the size of price changes is given by a very simple formula of the thresholds of the sS rule. Price increases are given by $g^* - \underline{g}$, since the firm increases its price when the markup is very low, i.e., the first time g reaches $\underline{g}$. Likewise, price decreases occur when the markup has reached the value $\overline{g}$ and are of size $\overline{g} - g^*$. Denoting the size price increases by Δ_p^+ and the size price decreases by Δ_p^-, we have that: $\Delta_p^+ = g^* - \underline{g}$ and $\Delta_p^- = \overline{g} - g^*$.

We denote the average number of price changes per unit of time by λ_a. This can be easily computed as the reciprocal of the time between price changes, by the fundamental theorem of renewal theory. The expected time until a price change $T(g)$ for a firm with current price gap g solves the following Kolmogorov backward equation:

$0 = -\pi T'(g) + \frac{\sigma^2}{2} T''(g)$ for all $g \in [\underline{g}, \overline{g}]$with boundary conditions: $0 = T(\underline{g}) = T(\overline{g})$.

The boundary conditions are quite natural, at either $\underline{g}$ or $\overline{g}$ there will be a price change, and hence the expected time to reach them is zero! Since right after a price change $g = g^*$, then the expected time until the next price change is $T(g^*)$, and hence the frequency of price changes $\lambda_a = 1/T(g^*)$. Using a similar procedure we can find the frequency of price increases λ_a^+ and the frequency of price decreases λ_a^-. For instance, letting $T^+(g)$ be the expected time until a price increase, it is easy to see that it satisfies the same Kolmogorov backward equation than T. The difference is in the boundary conditions, which are $T^+(\overline{g}) = T^+(g^*)$, since a price decrease will occur at $\overline{g}$ but we need to keep counting; we also have $T^+(\underline{g}) = 0$, since a price increase occurs at $\underline{g}$. The frequency of price increases is thus: $\lambda_a^+ = 1/T^+(g^*)$. A similar argument holds for the expected time until a price decrease T^-, i.e., it solves the same o.d.e. than T with boundary conditions $T^-(\underline{g}) = T^-(g^*)$ and $T^-(\overline{g}) = 0$. The frequency of price decreases is thus: $\lambda_a^- = 1/T^-(g^*)$.

We note that the length of the range of inaction equals the sum of the average size of price increases plus the average size of price decreases: $\overline{g} - \underline{g} = \Delta_p^+ + \Delta_p^-$. This observation will become useful because these average sizes can be measured, and the length of the range of inaction is important to understand when a cost shock is large. Finally, the second moment of price changes is given by $E\left[\Delta_p^2\right] = \frac{\lambda_a^+}{\lambda_a}(\Delta_p^+)^2 + \frac{\lambda_a^-}{\lambda_a}(\Delta_p^-)^2$.

In appendix C we give a simple alternative numerical procedure by using a discrete time and discrete state version of the model to compute the steady-state distribution and the frequency of price changes per unit of time.

2.3 Optimal Decision Rules and Inflation

This subsection analyzes how optimal pricing decisions vary with the rate of normalized inflation, π/σ, keeping the normalized fixed cost, ψ/B, constant.[14]

For $\pi/\sigma \approx 0$, then the decision rules are approximately symmetric, with $g^* = 0$ and $\overline{g} = -\underline{g}$.[15] This implies that around zero inflation, the price increases and price decreases have approximately the same size $\Delta_p^+ = \Delta_p^-$. Moreover, the frequency of price increases and price decreases is the same around zero inflation, so $\lambda_a^+ = \lambda_p^-$.

For higher inflation rates relative to idiosyncratic volatility π/σ^2, the optimal return markup g^* becomes larger. This is because due to inflation, during inaction markups decrease in expected value, and thus this expected effect, is compensated by starting with a higher markup. In Álvarez and others (2019) we show that for small inflation the main adjustment is *not* in the frequency of price changes λ_a, but

14. Appendix B shows that, for a given fixed cost relative to the curvature of profits ψ/B, the optimal decision rules depend on the normalized level of inflation π/σ^2.

15. We say *approximately* because under the quadratic approximation at exactly $\pi/\sigma^2 = 0$ the decision rules are symmetric.

instead in the difference between the frequency of price increases and decreases, i.e., the derivative of $\lambda_a(\pi)$ with respect to inflation is zero at $\pi/\sigma^2 = 0$, but the derivative of $\lambda_a^+(\pi) - \lambda_a^-(\pi)$ is strictly positive. We also show analytically that it accounts for 90% of the change in inflation at low inflation; the other 10% is explained by changes in $\Delta_p^+ - \Delta_p^-$. Summarizing, as we move from zero steady-state inflation (where frequency and size of price increases and decreases are symmetric) to positive steady-state inflation, the model predicts that the frequency of price increases $\lambda_a^+(\pi)$ will be higher than the one of decreases $\lambda_a^-(\pi)$, and that the size of price increases and decreases will be approximately the same, i.e., $\Delta_p^+ \approx \Delta_p^-$. Nevertheless, while we show that the average size is similar, we also show that as we move from zero steady-state inflation, $\Delta_p^+ > \Delta_p^-$.[16]

For very large inflation, $\lambda_a^+(\pi) \to \lambda_a(\pi)$ and $\lambda_a^-(\pi) \to 0$, as $\pi \to \infty$, so most price changes are increases, and the model converges to Sheshinski and Weiss (1979), in the sense made precise in Álvarez and others (2019).

2.4 Impulse Response of the Price Level to Unexpected Cost Shocks

In this subsection we characterize the impulse response of the aggregate (log of the) price level to a once-and-for-all increase in the nominal price of the aggregate input of size δ, measured in logs. We will consider different inflation levels π and different sizes of the shock δ.

We start with an economy that is in the steady-state distribution of price gaps. This economy is characterized by parameters $\{\psi/B, \rho, \sigma^2\}$ and π. Firms will face a once-and-for-all jump on the nominal price of the aggregate input and believe that, after this jump, the price of the inputs will rise at the same inflation rate π as before the jump. We are interested in computing the effect on the (log of the) aggregate price level of the goods produced by these firms. We will distinguish between the impact effect on the price level, i.e., the effect on the moment that the unexpected jump occurs, and the subsequent effects which lead the price level to adjust up to the full amount δ of the shock.

We can think of the price level just before the shock as the limit: $\bar{P} \equiv \lim_{t \uparrow 0} P(t;\pi)$. Likewise, we can let the value of the (log of the) price

16. For a precise statement, see Propositions 1 and 3 in Álvarez and others (2019). Numerically, we find the changes given by these derivatives at zero inflation to be accurate even up to inflation rates of 30% per year.

of the aggregate input just before the shock be $\overline{W} \equiv \lim_{t \uparrow 0} W(t)$. Of course, for the price of the aggregate input we have that the jump is $\delta = \lim_{t \downarrow 0} W(t) - \lim_{t \uparrow 0} W(t) \equiv \lim_{t \downarrow 0} W(t) - \overline{W}$. As mentioned above, we assume that $d\log W(t)/dt = \pi$ for $t \neq 0$. Throughout the exercise, the parameters $\{B, \psi, \rho, \sigma^2\}$ and the invariant distribution implied by the optimal decision rules, $\{\underline{g}, g^*, \overline{g}\}$ are fixed.

We will denote the price level t periods after the shock $P(t; \delta, \pi)$ for an economy that is hit by the shock of size δ when the inflation rate of the aggregate input before and after the cost shock is π. We will let the price level right before the shock to be denoted by $\overline{P}$. We distinguish between the impact effect, which we denote by $\Theta(\delta, \pi)$, and the subsequent rate of change of the (log of the) price level, denoted by $\theta(t; \delta, \pi)$. Thus

$$P(t;\delta,\pi) = \overline{P} + \Theta(\delta,\pi) + \int_0^t \theta(s;\delta,\pi)ds \tag{7}$$

Whenever it is clear, we omit δ and π from the expression for P, Θ and θ. Also, while P is the log of the aggregate price level, whenever it is clear, we will refer to it as just the price level.

Since we are measuring P in logs, then $\theta(s; \delta, \pi)$ is the inflation rate of the CPI s periods after the shock δ has occurred in an economy with steady-state inflation rate π. In particular, after the impact effect at time $t = 0$, the term $\theta(s; \delta, \pi)ds$ yields the contribution to the average (log) price of the firms that are adjusting prices at times between s and $s + ds$. This contribution is equal to $\theta(s; \delta, \pi) = [\Delta_p^+ \lambda_a^+(s) - \Delta_p^- \lambda_a^-(s)]$ where Δ_p^+ and Δ_p^- are the same as in steady state, but the frequency of price increases and decreases, $\lambda_a^+(s)$ and $\lambda_a^-(s)$, are time-varying. The reason these frequencies are time varying is that the density of the distribution of firms indexed by their price gap g, denoted also by $f(g,t)$, is time varying. This density changes through time because the cost shock δ and the price changes that occur right after the cost shock have displaced it from its steady state. Thus, even following the same time invariant decision rules as in steady state, it takes time for the density to return to its steady-state level described by equation (6). See the appendix C for the discrete time and discrete state space analog computation of the path of the distribution and of λ's, or see Álvarez and Lippi (2019) for the continuous time characterization of the impulse response by using an eigenvalue-eigenfunction decomposition.

We will compare the path of the log of the aggregate price level $P(t;\delta,\pi)$ against the path for the price level of the aggregate input. Recall that the aggregate input grows at rate π before and after the shock, and jumps by δ log points at time $t = 0$. In the long-run prices will increase by as much as the shock to the aggregate input, so that

$$\delta = \lim_{t\to\infty}\left[P\left(t;\delta,\pi\right) - \pi t\right] - \bar{P} = \lim_{t\to\infty}\left[W(t) - \pi t\right] - \bar{W} \tag{8}$$

Impact effect. We define the impact effect Θ as the jump in the price level at $t = 0$, i.e.,

$$\Theta\left(\delta,\pi\right) = \lim_{t\downarrow 0} P\left(t;\delta,\pi\right) - \bar{P} \tag{9}$$

To be clear, if prices are fully flexible, we will have $P(t) = W(t) + \mu$ for some constant μ at all times. With menu costs, after the jump in the aggregate input prices, we expect that $\Theta \leq \delta$ and that over time the price level $P(t)$ catches up with the increases in the path of $W(t)$. Later we show that this is true for values of δ and π/σ^2 that are not too large.

The impact effect is simple to compute following this two-step procedure. First we shift the distribution of price gaps from the steady state to the one right after the shock but before the prices adjust, so the new density is $f(g + \delta)$ with support $[\underline{g} - \delta, \bar{g} - \delta]$. This is so because with the common increase in cost, the price gap of each firm decreases by δ. Second, using the lack of first-order strategic complementarity, all the firms that end up with price gaps g below $\underline{g}$ will increase their prices from their new value for g to $\bar{g}^*$. Thus:

$$\Theta\left(\delta,\pi\right) = \int_{\underline{g}-\delta}^{\underline{g}} \left(g^* - g\right) f\left(g+\delta\right) dg \tag{10}$$

This yields the following derivatives:

$$\frac{\partial}{\partial\delta}\Theta\left(\delta,\pi\right) = \left(g^* - \underline{g} + \delta\right) f(\underline{g}) + \int_{\underline{g}-\delta}^{\underline{g}} \left(g^* - g\right) f'\left(g+\delta\right) dg$$

$$\frac{\partial^2}{\partial\delta^2}\Theta\left(\delta,\pi\right) = f\left(\underline{g}\right) + \left(g^* - \underline{g} + \delta\right) f'(\underline{g}) + \int_{\underline{g}-\delta}^{\underline{g}} \left(g^* - g\right) f''\left(g+\delta\right) dg\,.$$

Evaluating these expressions at $\delta = 0$, and using that, by definition $\Theta(0,\pi) = 0$ and that at the exit points of the invariant density we have $f(\underline{g})$, we obtain the following expansion of Θ on δ:

$$\Theta\left(\delta,\pi\right) = \frac{1}{2}\Delta_p^+ f'\left(\underline{g}\right)\delta^2 + o(\delta^2) \tag{11}$$

As argued elsewhere,[17] if the shock δ is small, then the impact effect Θ is very small. Mathematically speaking, Θ is of second order in δ. We can also see that the leading coefficient of δ increases with inflation since, as explained above, as π/σ^2 increases, the density becomes more concave in the lower segment, until in the limit $f'(\underline{g}) \to \infty$ as $\pi/\sigma^2 \to \infty$. Thus the impact effect is of smaller order than δ, but the coefficient of δ^2 increases with inflation. Whether this is an important effect for the level of inflation rates for the period of Argentina under consideration is an important issue that we will discuss below.

Two extreme examples help to organize ideas: First, consider the case where $\pi/\sigma^2 = 0$, then $f'(g) = 1/(\Delta_p^+)^2$ is constant, and thus for $\delta \leq \Delta_p^+$, equation (10) becomes

$$\Theta(\delta,\pi) = \frac{1}{(\Delta_p^+)^2}\int_{\underline{g}-\delta}^{\underline{g}} \left(g^* - g\right)\left(g - \underline{g} + \delta\right)dg = \frac{\delta^2}{2}\frac{1}{\Delta_p^+}\left(1 + \frac{1}{3}\frac{\delta}{\Delta_p^+}\right).$$

As in the general case, for small δ, the value of Θ is very small. Yet when δ is large, say in the order of magnitude of Δ_p^+, the impact effect can be large. For instance, if $\delta = \Delta_p^+$ we have $\Theta(\delta, \pi) = 2/3\delta$, which is smaller than δ, but of the same order of magnitude than δ. Second, consider the other extreme case, where $\pi/\sigma^2 \to \infty$. In this case f converges to a uniform distribution between $[\underline{g}, g^*]$ and thus equation (10) becomes

$$\Theta(\delta,\pi) = \frac{1}{\Delta_p^+}\int_{\underline{g}-\delta}^{\underline{g}} \left(g^* - g\right)dg = \delta\left(1 + \frac{\delta}{2}\frac{1}{\Delta_p^+}\right),$$

which is of order δ. Note that for small δ, this gives the same answer as the case of full price flexibility, i.e., $\Theta \approx \delta$, and indeed it converges to a version of Caplin and Spulber (1987)'s neutrality case. Interestingly, when δ is not infinitesimal, then $\Theta > \delta$. In this case, since $P(t) - \overline{P} - \pi t \to \delta$ as $t \to \infty$, there must be an overshooting in the short run, and thus prices should have an eco and oscillate as they converge to their path. Indeed if $\delta = \Delta_p^+$ we have $\Theta(\delta, \pi) = 3/2\delta > \delta$.

17. See Álvarez and Lippi (2014), Álvarez, Le Bihan, and Lippi (2016), and Álvarez, Lippi, and Passadore (2016).

The stark difference between the cases with $\pi/\sigma^2 \approx 0$ and $\pi/\sigma^2 \to \infty$ calls for an evaluation in the case of Argentina during the period of interest where inflation rate is quite high, but far away from hyper-inflationary levels, say on the order of 25% per year when we exclude the peaks. Is this closer to the $\pi/\sigma^2 \to \infty$ limit, or is it closer to the $\pi/\sigma^2 \to 0$ limit? Additionally, is the size of the cost changes for δ for this period in Argentina large enough that we have to go beyond the approximation in equation (11)? Note that a relevant theoretical comparison is how large δ is relative to Δ_p^+. Motivated by these considerations, we will evaluate the relevant expressions for calibrated parameters values and compare them with the "observed" impact effects.

Initial slope of the impulse response. We now characterize the initial inflation rate, just after the impact effect. For this we consider a very short-time interval right after the shock, which we denote by Δ. On the one hand, we note that immediately after the shock, there is no density near the upper bound $\overline{g}$. On the other hand, there is a strictly positive density at the lower-bound $\underline{g}$. Recall that price increases will occur for the firms in the lower bound of the inaction region, which gets an idiosyncratic increase in cost, and hence a decrease in g. Using the assumption of the Brownian motion for the idiosyncratic shocks, it can be shown that about half of the firms at the lower bound will increase prices in the very small interval of time following the aggregate cost shock. This means that, for a very short interval immediately after the impact effect, there is an extremely large number of price increases and almost no price decreases.

In particular, after the shift due to the common cost shock, the density is zero in the upper interval $g \in [\overline{g} - \delta, \overline{g}]$. Thus, $\lambda_a^-(\Delta) \to 0$ as $\Delta \to 0$. On the other hand, after the impact effect, and differently from what happens at steady state, there is a positive density at $g = \underline{g}$. This density is equal to $0 < f(\underline{g} + \delta) < f'(\underline{g})\delta$, where the inequality holds due to the concavity of the steady-state density f in $[\underline{g}, g^*]$. Consider the discrete-time discrete-state approximation developed in appendix C, where each step of the process for g and of the discretized steady-state distribution are of size $\sqrt{\Delta}\sigma$. The number of firms changing prices per unit of time $\lambda_a^+(\Delta)$ equals the density at the boundary times the step size $\sqrt{\Delta}\sigma$ times the probability that those firms have an increase in cost, denoted by p_d, and divided by the length of the time period Δ. For a diffusion, as $\Delta \to 0$ then $p_d \to 1/2$. Thus the fraction of firms changing prices per unit of time is $f(\underline{g} + \delta)\sigma/(2\sqrt{\Delta})$. This implies that $\lambda_a^+(\Delta) \to \infty$ as $\Delta \to 0$. We have then $\theta(\Delta) \to \lambda_a^+$

$(\Delta)\Delta_p^+ \to \infty$ as $\Delta \to 0$. Yet, $\theta(\Delta)\Delta \to 0$, as $\Delta \to 0$, so the integral for $P(t)$ is still well defined. An alternative more general way to show that the slope of the impulse response is infinite is in Álvarez and Lippi (2019), who use an eigenvalue-eigenfunction decomposition of the relevant linear operator.

2.5 Comparative Static of Cost Shocks

In this subsection we compare the effect of a small and a large-cost shock, say $\delta = 0.01$ and $\delta = 0.1$ for three economies: a low inflation one $\pi = 0.025$, a large inflation one, $\pi = 0.25$ and one close to the hyperinflationary range, $\pi = 2.5$. Recall that cost shocks are measured in logs, so we are trying 1% and 10% once-and-for-all shocks. Inflation rates are measured as annually continuously compounded (c.c.), so we are trying 2.5%, 25% and 250%, but in the last two cases recall that continuously compounded and annually compounded can be meaningfully different.[18]

Calibration. We use the same parameters for all cases. The parameters for the firm problem are chosen so that at $\pi = 0.25$, i.e., 25% annual continuously compounded inflation, the steady-state statistics resemble the same statistics in Argentina for the period under study. We use $\eta = 7$, which has a markup of just above 15% and a fixed cost of $\psi = 0.012$ yearly frictionless profits. We use an annual discount rate $\rho = 0.04$ and an annual volatility of idiosyncratic shock of $\sigma = 0.20$.

With these parameters the model implies $\Delta_p^+ = 0.12$ and $\Delta_p^- = 0.097$. The average number of price changes per year are $\lambda_a^+ = 2.88$ and $\lambda_a^- = 0.89$.[19] These figures are similar to the averages for Argentina, when we omit periods of abnormal cost increases. The size of cost changes in the data is around 10%–11%.[20] The annual number of price changes measured as the fraction of outlets changing prices in a "normal" month times 12 is $\lambda_a^+ \approx 0.24 \times 12 = 2.88$ and $\lambda_a^- \approx 0.073 \times 12 = 0.88$.[21]

We will discuss the effect of small ($\delta = 0.01$ or 1%) and large ($\delta = 0.10$ or 10%) cost shocks for each of the three continuously

18. Continuously compounded yearly inflation at rate π implies that the ratio of prices (or cost) at the end of the year relative to prices at the beginning of the year is e^{π}. For example, with $\pi = 0.25$, this ratio is $e^{0.25} \approx 0.28$, and for $\pi = 2.5$, this ratio is $e^{2.5} \approx 12.2$!

19. We use a time period $\Delta = 1/365/12$, so two hours.

20. See figure 8.

21. See figure 7.

compounded annualized inflation rates we consider: $\pi = 0.025$, $\pi = 0.25$ and $\pi = 2.5$. The plots corresponding to each of the inflation rates are in figures 3, 4, and 5, respectively. The small shock is of a size we consider in the upper bound of what a normal monetary shock is. The large-cost shocks are similar of the type of shocks we argue occurred in Argentina during the period under consideration, which we view as extremely large. The three inflation rates correspond to: (i) a "common" inflation rate for a developed economy ($\pi = 0.025$ or 2.5% c.c. per year), (ii) a high inflation rate which is about the average running inflation during the period of study for Argentina when we exclude the spikes we associate to the jumps in cost ($\pi = 0.25$ or 25% c.c. per year), and (iii) a very large inflation on the hyperinflation range of almost 23% monthly compounded inflation ($\pi = 2.5$ or 250% c.c. per year).[22] For each of the inflation rates, we present three panels of plots: the first panel with the path of the log of the aggregate CPI level and with the path of the log of the price of the aggregate input, the second panel with the density of the invariant distribution right before and right after the cost shock, and the third panel with the path for the (monthly moving average of the) frequency of price increases and price decreases. Each panel displays two cases, corresponding to the small-cost change (1%) on the left side of the panel, and to the large-cost change (10%) on the right side of the panel. In total there are nine subplots for each of the three inflation rates.

Some general comments on the objects of figures 3, 4, and 5 are in the same order.

First, in the top panel for each figure, we have the path of the log of the nominal price for the CPI and of the path of the log of the nominal price of the aggregate input. We normalize the price of the nominal input so that, at the time of the cost shock, both the core CPI and the price of the aggregate input are equal. The time of the cost shock is labeled $t = 1$, and time is measured in years for the first and third panels. Second, in the middle panel for each figure we have the invariant distribution for the price gaps at steady state and its version right after the cost shock, but before the price changes. In the horizontal axis we have indicated the values of $\underline{g}, g^*, \overline{g}$. The price gap is measured in log point deviations, so that $\overline{g} = 0.1$ represents 10% log point difference between the static maximizing markup and

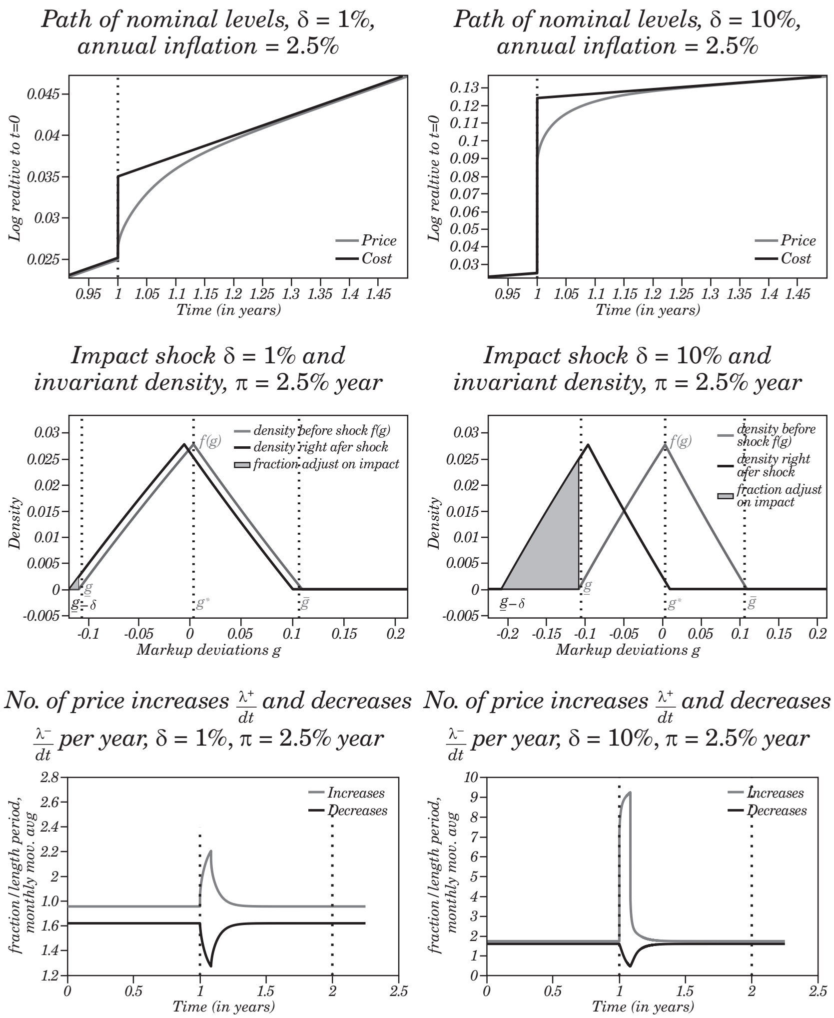
22. To be clear, for $\pi = 2.5$ annual continuously compounded rate, we have that the monthly compounded rate is $(e^{\frac{2.5}{12}} - 1)\times 100 \approx 23$.

the markup corresponding to that value of g. We have shaded in grey the distribution of price gaps right after the cost shocks. This shaded area measures the fraction of firms (or products) that change prices on impact, i.e., at the time of the cost shock. Third, the bottom panel displays the average number of price changes per year. This is defined as follows: we take the fraction of firms (or products) that change prices per model period and then we divide it by the length of the model period. In the plot we display a centered monthly moving average of this number.[23] We take a monthly moving average for comparability with the Argentine data, which uses monthly frequency, and also to smooth out the large jump. Note that at the time of the increase of the cost, these frequencies increase smoothly for about a month due to the fact that we use a centered moving average.

We end this section with a brief discussion on how the pattern of the different statistics in these figures illustrates the analytical properties derived above. First we discuss the difference between the impact effect of the jump in the cost of the aggregate input on the price level and the rate of converge of the price to the cost. Let's first concentrate on the case of low ($\pi = 0.025$) and high inflation ($\pi = 0.25$), i.e., figures 3 and 4, respectively. Even though the inflation rate that roughly corresponds to Argentina is the large case (π = 25% c.c. per year), the pattern for the passthrough is similar in figures 3 and 4. For both inflation rates, the instantaneous passthrough is larger for the large-cost shock (as can be seen by comparing the left and right subplots). Nevertheless, as expected, in the case of high inflation (π = 25%), the passthrough is higher and the convergence is faster, i.e., the half-life of the shock with high inflation is one half relative to the low inflation one. The convergence rates in the figures with large shocks are so high, with half-lives below two months, that we are very close to full price flexibility.

23. The centered moving average at time t takes an average of half of $(1/\Delta)/12$ model periods before the date t, and half of $(1/\Delta)/12$ model periods after the date t, where Δ is the length—measured in years—of the model period. See section C for details on the computations.

Figure 3. Passthrough of Nominal Cost Shocks for Low Inflation (π = 2.5% c.c.)



Source: Authors' calculations.

The case of very large inflation of figure 5 with a continuously compounded inflation π = 250% per year is different. As anticipated in the theoretical section, large shocks in an inflationary economy induce an overshooting of the price level on impact as shown in figure 5b.

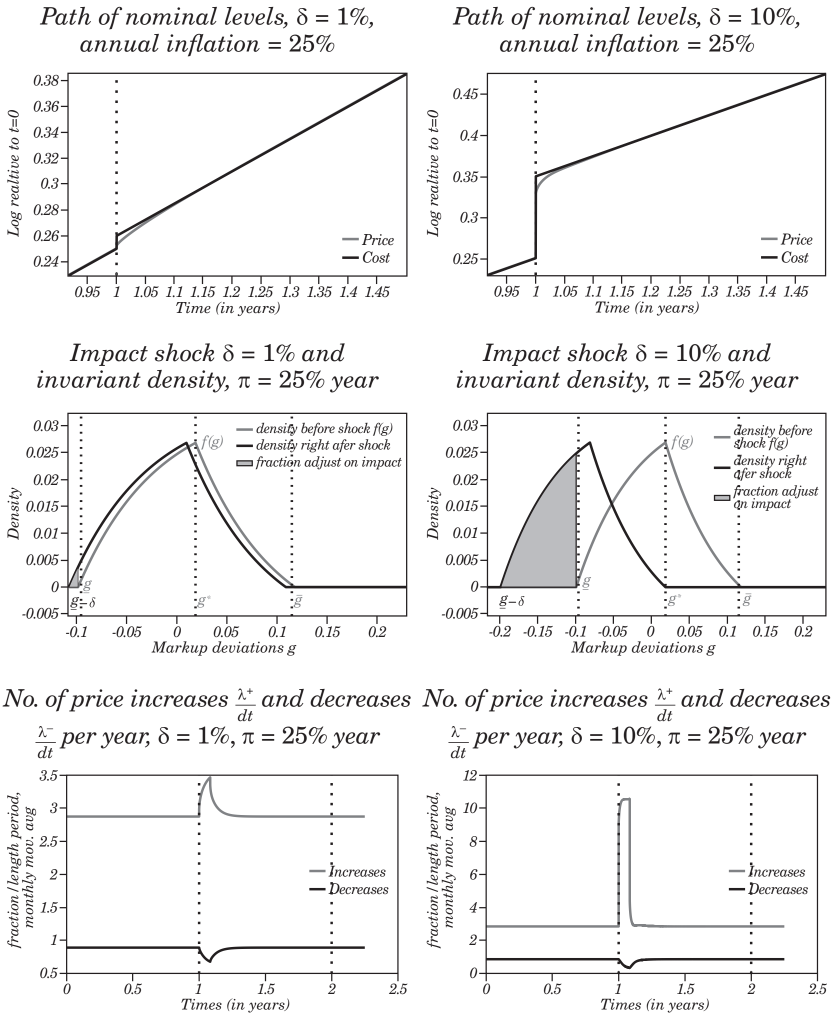
As inflation is very high, firms that adjust prices (and pay the menu cost) find it optimal to save on future menu costs by raising prices by more than the cost shock. The case of a small shock depicted in figure 5a instead, is similar to Caplin and Spulber (1987), also as expected.

Now we turn to the middle panel of each of the three figures, which in itself is useful to understand the instantaneous passthrough of the top panel just discussed. Note that for low inflation ($\pi = 2.5\%$ c.c. per year), the invariant distribution is almost a tent-map, as it should be for exactly zero inflation. Indeed, it is theoretically known that the effect of steady-state inflation around zero inflation is very small. For high inflation ($\pi = 25\%$ c.c. per year), the invariant distribution is convex-concave, as explained in the theoretical section above. Also as explained in the theoretical section, by comparing the small ($\delta = 0.01$) and large-cost shocks ($\delta = 0.1$) corresponding to the left and right panels respectively, it is seen that the number of firms (or products) that change prices (i.e., the size of the grey-shaded area) increases more than proportionally as the shock increases from 1% to 10%. Alternatively, we can see that the approximation that the impact effect on prices is proportional to the square of the cost shock δ is accurate for this range of shocks. Moreover, the size of the grey-shaded area for the 10% shock in the low inflation rate $\pi = 2.5\%$ case is smaller than in the high inflation rate $\pi = 25\%$ case, due to the convex-concave nature of the invariant distribution for the higher inflation rates. Again, the case of very large inflation ($\pi = 250\%$ c.c. per year) is different.

The share of firms changing prices on impact is much closer to be proportional to δ than in the cases of lower inflation, as can be seen in figure 5d.

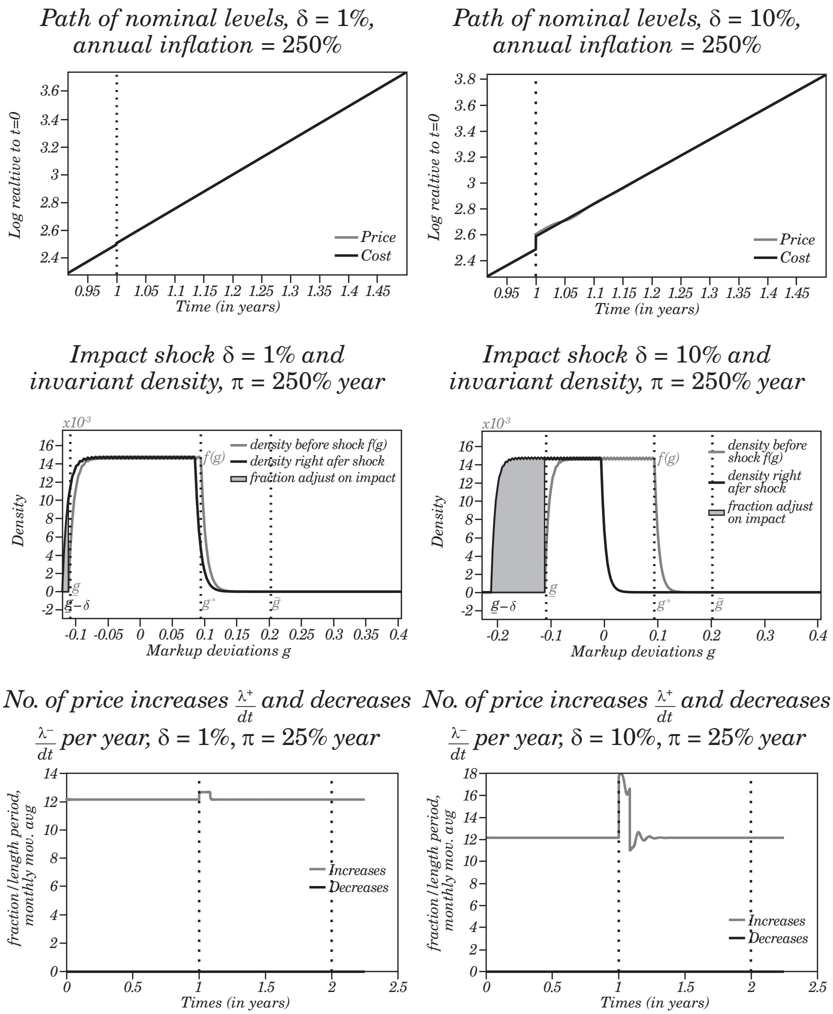
Lastly we turn to the behavior of the frequency of price increases and decreases, the bottom panel of the figures for each inflation rate. First, note that for low inflation ($\pi = 2.5\%$ c.c. per year in figure 3) while the rise in the frequency of price increases is moderately larger than the decline in the frequency of price decreases for small-cost shocks ($\delta = 0.01$ in the left panel), this difference is much larger for the case of high-cost changes ($\delta = 0.10$ in the right panel). The pattern is similar in the case of high inflation ($\pi = 25\%$ c.c. per year in figure 4), except that the differences between increases and decreases are a bit more stark. Instead, again, the situation for very large inflation ($\pi = 250\%$ c.c. per year, or figure 5) is different. Since there are almost no price decreases, there is no detectable change in them. On the expected number of price decreases, the behavior is very different between small-cost shocks ($\delta = 0.01$ in the left panel) and large-cost shocks (δ = 0.1 in the right panel). For small-cost shocks, we have the one-time blip that is characteristic of the mechanism in Caplin and Spulber (1987). Instead, as explained in the theory above, for large-cost shocks, there is overshooting which leads to a subsequent echo effect, which is seen in the damped oscillations in the path of the frequency of price increases.

Figure 4. Passthrough of Nominal Cost Shocks for High Inflation (π = 25% c.c.)



Source: Authors' calculations.

Figure 5. Passthrough of Nominal Cost Shocks for Very High Inflation (π = 250% c.c.)



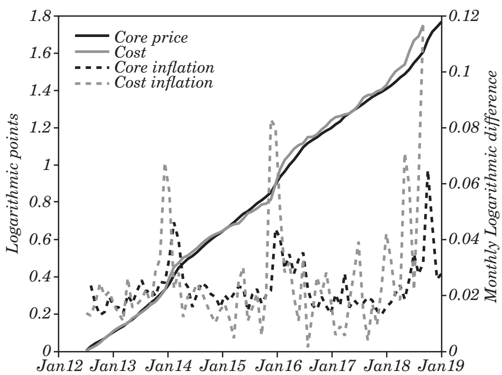
Source: Authors' calculations.

3. Argentina's Evidence on Large-cost Shocks and Price Dynamics

In the previous section we studied how firms facing menu costs of price adjustments react to unexpected shocks to nominal marginal costs. We then studied the aggregate behavior of prices, paying particular attention to the effect of these shocks on the average price level, on the size of price changes, and on the frequency of price adjustment, both on impact and over time. In this section we look at how prices in the city of Buenos Aires reacted to the large nominal shocks described in section 1 and draw conclusions on the applicability of the model.

Figure 6 provides an overview of the behavior of our proxy of the nominal cost and of core prices. We prefer to look at core inflation because, as it excludes seasonal and regulated goods, this measure of prices avoids the mechanical direct impact that changes in regulated prices have on the CPI.

Figure 6. Nominal Costs and Pricve Levels



Sources: Prices are from for the city of Buenos Aires Statistical Office. Exchange rates are from the Central Bank, and wages are from *Ministerio de Trabajo, Empleo y Seguridad Social* (2018) [Ministry of Labor, Employment and Social Security].

Note: Cost is a weighted geometric average of regulated prices, the exchange rate and wages with weights 0.1, 0.4 and 0.5, respectively.

There are three large jumps in costs: one in early 2014, a second one in early 2016, and a third one in May–September 2018. The first one is mainly due to a 23% devaluation that took place in the second half of January. As our measure of costs is based on monthly averages, our cost proxy jumps in December 2013 and in January 2014. The 40% weight of tradable goods in our cost measure implies that the jump in cost is slightly above 9%, roughly in synch with the size of price changes in the data[24] and in the simulated examples in subsection 2.5. Figure 6 also shows that there is a spike in inflation associated to each spike in nominal costs. Between November and February 2013, the cost proxy increased by 12%, while the price level increased by 8%. The second shock took place in the first half of 2016. It consisted of a sequence of cost shocks stemming from the impact of the removal of capital controls on the exchange rate and from the change in the relative price of regulated energy prices as shown in table 1. The impact effect relative to the size of the shock is smaller than in the first shock. The persistence of the shocks is reflected in the persistence of the high inflation. Finally, in the first quarter of 2018, there are nominal shocks of about 4% related to regulated prices and, starting in May, there are two exchange-rate lead spikes in nominal costs with peaks of 7% in May and 12% in September.

Several issues prevent us from using the data underlying figure 6 to estimate the impulse response of core prices to cost shocks analyzed in subsection 2.4. (i) At the time a shock hits, prices might still be adjusting to previous shocks. (ii) The cost shock might have been partially anticipated.[25] (iii) An aggregate shock might change the relative price between consumer goods and wholesale goods. (iv) There might be other aggregate cost or productivity shocks not captured by our cost proxy. Nevertheless, we can check if the frequency and if the size of price changes in the data are consistent with the theoretical analysis in 3, illustrated in figures 3 to 5.

Figures 7 and 8 show the frequency and the size of price changes. The figures are based on the data underlying the city of Buenos Aires core consumer price index (IPCBA-rest). The city collects approximately 70,000 prices per month for 628 goods and services.[26]

24. See figure 8.

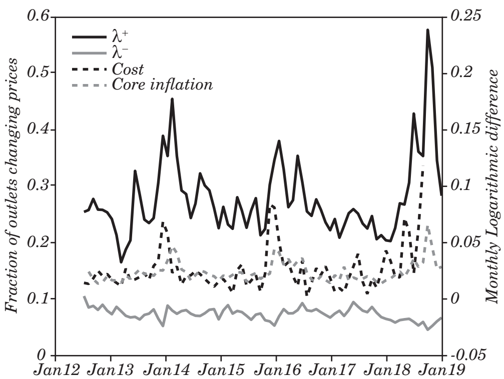
25. This issue is especially relevant for the devaluation of December 2015. See for example Neumeyer (2015) and Levy-Yeyati (2015).

26. The instructions to the enumerators and other methodological issues are described in *Dirección General de Estadística y Censos* (2013).

The frequency of price changes is computed as the fraction of prices that either increased or decreased between two consecutive observations within the goods and services included in the core measure of inflation. The size of price changes is the geometric equally weighted average of the absolute value of price increases/decreases. The methodology for computing these statistics is described in Álvarez and others (2019), where we discuss the property of this simple estimator and perform robustness checks.

Figure 7 shows the fraction of outlets changing price each month, our proxy for costs, and the core inflation level. Observe first that the average level of the fraction of price increases in "normal" times is 0.24 and the one for price decreases is 0.073. The magnitude of the frequency of price decreases is interesting because it indicates that, for the levels of underlying inflation during 2012–18 in Argentina, the benchmark menu-cost model of Sheshinski and Weiss (1979) is unlikely to be the appropriate one—something that we will also see as we examine other statistics. It rules out the case of $\pi/\sigma \rightarrow \infty$ in figure 5, pointing out the importance of idiosyncratic shocks that induce some firms to lower prices even when costs are rising at a cruising speed of 2% per month.

Figure 7. Cost Shocks and the Frequency of Price Changes



Sources: Prices are from the city of Buenos Aires Statistical Office. Exchange rates are from the Central Bank. Wages are from *Ministerio de Trabajo, Empleo y Seguridad Social* (2018) [Ministry of Labor, Employment and Social Security]. Note: λ+ is the fraction of outlets increasing prices each month. λ− is the analogue for outlets decreasing prices. Cost is a weighted geometric average of regulated prices, the exchange rate and wages with weights 0.1, 0.4 and 0.5, respectively. The dotted line is the core inflation for the city of Buenos Aires (IPCBA - rest).

Second, the three cost shocks identified in our narrative in 2014, 2016, and 2018 are large in the sense that their size is similar to the size of the price changes in normal times. In these three cases, the reaction of the frequencies of price changes is consistent with the predictions of the model for the case of a large shock with high inflation, illustrated in figure 4f. The shock at the beginning of 2014 is short-lived and of a magnitude similar to the $\delta = 0.1$ in the simulated example. As suggested by the theory, the data shows a contemporaneous spike of the fraction of outlets raising prices to 0.45. The transitory decrease in the fraction of outlets lowering prices seems to be in the data, but it is hard to distinguish it from noise. The second episode corresponds to the sequence of cost shocks that took place in the first half of 2016.

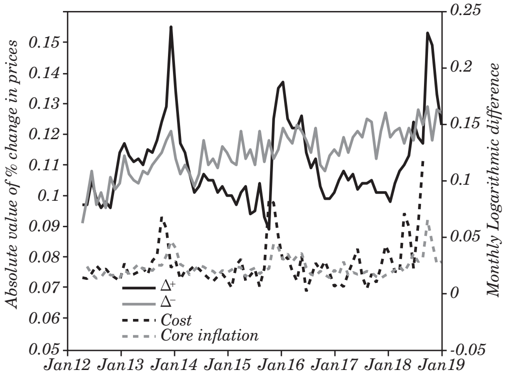
These shocks also induced an increase in the fraction of outlets raising prices that is less pronounced and more persistent than the one in 2014. The frequency of price increases peaks at 0.38 in January 2016 and at 0.35 in May 2016. There seems to be a fall in the fraction of outlets lowering prices in the period leading to the devaluation but, again, it is hard to distinguish it from noise in the data. Finally, the fraction of outlets raising prices rises throughout 2018 reacts to the cost shocks, with peaks of 0.43 in June and 0.57 in September, coinciding with important jumps in the exchange rate.

Figure 8 describes the absolute size of price changes conditional on a price change taking place. In “normal” times, the absolute size of positive and negative price changes is similar, around 10–11%. This is consistent with the theory. However, the fact that the absolute value of price decreases is sometimes higher than the one for price increases after 2014 is inconsistent with the theory.[27]

The reaction of the size of price changes to the cost shocks also gives empirical support to the menu-cost model. In the theory, when there is a large-cost shock, many firms end up with a markup that is outside the inaction range, as shown is figure 4d. This implies that the magnitude of price increases has to grow in the presence of large unanticipated shocks. The magnitude of price decreases, on the other hand, should not change as a result of increases in nominal marginal costs, as no firm’s markups are pushed above the upper band, $\overline{g}$. These two predictions of the theory are observed in the data. For the three shocks, we observe significant increases in the magnitude of price increases in figure 8, which reach 1.5 times “normal” values.

27. These differences may just be noise.

Figure 8. Cost Shocks and the Size of Price Changes



Sources: Prices are from the city of Buenos Aires Statistical Office. Exchange rates are from the Central Bank. Wages are from *Ministerio de Trabajo, Empleo y Seguridad Social* (2018) [Ministry of Labor, Employment and Social Security]. Note: Δ+ is the average percentage change in prices across outlets conditional on a price increase. Δ– is the analogue for price decreases. Cost is a weighted geometric average of regulated prices, the exchange rate and wages with weights 0.1, 0.4 and 0.5, respectively. The dotted line is the core inflation for the city of Buenos Aires (IPCBA - rest).

Also, the magnitude of price decreases does not show abnormal patterns around the time of cost shocks.

We conclude this section by saying that the behavior of the frequency and of the size of price changes supports the passthrough of costs shocks to prices predicted by the menu-cost model of price adjustment. In "normal" times with small shocks, the frequency and the size of price changes are consistent with the simulations for $\pi = 0.25$ and $\delta = 0.01$ in subsection 2.5, thus supporting the choice of parameter values for ψ/B and π/σ used in the simulations. The frequency of price decreases of 0.073 and the fact that the size of price increases and decreases imply that idiosyncratic firm shocks are important. The expected duration of prices, computed as $(\lambda^+ + \lambda^-)^{-1}$, is 3.2 months, thus supporting the importance of price rigidities. For the episodes that we interpret as large unanticipated cost shocks, the frequency and the size of price adjustments react as the theory predicts with large increases in the size and in the frequency of price increases, and with no discernible effects on these variables for the case of price decreases. This leads us to conjecture that the fast passthrough of costs to prices, predicted by the theory but hard to estimate in our dataset, is likely to be present in the data.

References

Álvarez, F., M. González-Rozada, A. Neumeyer, and M. Beraja. 2019. "From Hyperinflation to Stable Prices: Argentina's Evidence on Menu-Cost Models." *Quarterly Journal of Economics* 134(1): 452–505.

Álvarez, F. and F. Lippi. 2014. "Price Setting with Menu Costs for Multi-Product Firms." *Econométrica* 82(1): 89–135.

———. 2019. *The Analytic Theory of a Monetary Shock*. Tech. rep.: The University of Chicago.

Álvarez, F., F. Lippi, and J. Passadore. 2016. "Are State- and Time-Dependent Models Really Different?" *NBER Macroeconomics Annual 2016*, volume 31.

Álvarez, F., H. Le Bihan, and F. Lippi. 2016. "The Real Effects of Monetary Shocks in Sticky Price Models: A Sufficient Statistic Approach." *American Economic Review* 106 (10): 2817–51.

Bassetto, M. and C. Phelan. 2015. "Speculative Runs on Interest Rate Pegs." *Journal of Monetary Economics* 73(C): 99–114.

Bonadio, B., A. Fischer, and P. Sauré. 2016. "The Speed of Exchange Rate Pass-through." Globalization Institute Working Papers 282, Federal Reserve Bank of Dallas.

Caplin, A.S. and D.F. Spulber. 1987. "Menu Costs and the Neutrality of Money." *Quarterly Journal of Economics* 102(4): 703–25.

Cavallo, A. 2013. "Online and Official Price Indexes: Measuring Argentina's Inflation." *Journal of Monetary Economics* 60(2): 152–65.

Dirección General de Estadística y Censos, CABA. 2013. *Índice de Precios al Consumidor de la Ciudad de Buenos Aires. Principales Aspectos Metodológicos.*

Gagnon, E. 2009. "Price Setting during Low and High Inflation: Evidence from Mexico." *Quarterly Journal of Economics* 124(3): 1221–63.

Gagnon, E., D. López-Salido, and N. Vincent. 2013. "Individual Price Adjustment along the Extensive Margin." In *NBER Macroeconomics Annual 2012, volume 27*, edited by D. Acemoglu, J. Parker, and M. Woodford. Chicago, IL: University of Chicago Press.

Golosov, M. and R.E. Jr. Lucas. 2007. "Menu Costs and Phillips Curves." *Journal of Political Economy* 115: 171–99.

Jones, C. 2011. "Intermediate Goods and Weak Links in the Theory of Economic Development." *American Economic Journal: Macroeconomics* 3(2): 1–28.

Karadi, P. and A. Reiff. 2019. "Menu Costs, Aggregate Fluctuations, and Large Shocks." *American Economic Journal: Macroeconomics* 11(3): 111-46.

Kehoe, P. and V. Midrigan. 2015. "Prices Are Sticky After All." *Journal of Monetary Economics* 75: 35–53.

Levy-Yeyati, E. 2015. "¿De qué hablamos cuando hablamos de dólar e inflación?" *Blog Foco Económico* URL http://focoeconomico.org.

Ministerio de Trabajo, Empleo y Seguridad Social. 2018. *Boletín de Remuneraciones de los Trabajadores Registrados*.

Neumeyer, A. 2015. "El dólar y los precios: Anticipando el fin del cepo." *Blog Foco Económico* URL http://focoeconomico.org.

Sheshinski, E. and Y. Weiss. 1979. "Demand for Fixed Factors, Inflation, and Adjustment Costs." *Review of Economic Studies* 46(1): 31–45.

APPENDIX A

Derivation of the Profit Function

Define

$$\tilde{F}(g,x,z)=\Pi(g)Ax^{1-\eta}z \text{ where } \Pi(g)=\left[e^{g+m}-1\right]e^{-\eta(g+m)}.$$

Thus

$$\Pi'(g)=e^{(1-\eta)(g+m)}(1-\eta)+\eta e^{-\eta(g+m)}$$

and evaluating it at $g = 0$

$$\Pi'(0)=e^{-\eta m}\left[e^{m}(1-\eta)+\eta\right]\Rightarrow e^{m}=\frac{\eta}{\eta-1}.$$

Likewise

$$\Pi''(g)=e^{(1-\eta)(g+m)}(1-\eta)^{2}-\eta^{2}e^{-\eta(g+m)}$$

and evaluating it at $g = 0$

$$\Pi''(0)=e^{(1-\eta)m}(1-\eta)^{2}-\eta^{2}e^{-\eta m}=\left(\frac{\eta}{\eta-1}\right)^{1-\eta}(1-\eta)^{2}-\eta^{2}\left(\frac{\eta}{\eta-1}\right)^{-\eta}$$

$$=\left(\frac{\eta}{\eta-1}\right)^{-\eta}\left[\left(\frac{\eta}{\eta-1}\right)(1-\eta)^{2}-\eta^{2}\right]=\left(\frac{\eta}{\eta-1}\right)^{-\eta}\left[\eta(\eta-1)-\eta^{2}\right]$$

$$=-\left(\frac{\eta}{\eta-1}\right)^{-\eta}\eta=-\left(\frac{\eta-1}{\eta}\right)^{\eta}\eta.$$

The level of the optimized value of Π is:

$$\Pi(0)=[e^{m}-1]e^{-\eta m}=e^{(1-\eta)m}-e^{-\eta m}=\left(\frac{\eta}{\eta-1}\right)^{1-\eta}-\left(\frac{\eta}{\eta-1}\right)^{-\eta}$$

$$=\left(\frac{\eta}{\eta-1}\right)^{-\eta}\left[\frac{\eta}{\eta-1}-1\right]=\left(\frac{\eta}{\eta-1}\right)^{-\eta}\left(\frac{1}{\eta-1}\right).$$

A second order expansion of $\tilde{F}$ around $(0, \overline{x}, \overline{z})$ gives:

$$\tilde{F}(g,x,z) = \Pi'(0)A\overline{x}^{1-\eta}g + \Pi(0)(1-\eta)A\overline{x}^{-\eta}\overline{z}(x-\overline{x}) + \Pi(0)A\overline{x}^{1-\eta}(z-\overline{z})$$

$$+\frac{1}{2}\Pi''(0)A\ \overline{x}^{1-\eta}\overline{z}g^2 + \frac{1}{2}\ \Pi(0)A(1-\eta)(-\eta)\overline{x}^{-\eta-1}\overline{z}(x-\overline{x})^2 + 0\frac{1}{2}(z-\overline{z})^2$$

$$+\Pi'(0)(1-\eta)A\overline{x}^{-\eta}\overline{z}(x-\overline{x})g + \Pi'(0)A\overline{x}^{1-\eta}0(z-\overline{z})g$$

$$+\Pi(0)A\overline{x}^{1-\eta}0(z-\overline{z})(x-\overline{x}) + o\left(\|x-\overline{x},g\|^2\right).$$

Since Π'(0) = 0, we have:

$$\tilde{F}(g,x,z) = \Pi(0)(1-\eta)A\overline{x}^{-\eta}(x-\overline{x}) + \frac{1}{2}\Pi''(0)A\ \overline{x}^{1-\eta}g^2$$
$$+\Pi(0)A\overline{x}^{1-\eta}(z-\overline{z})$$

$$+\frac{1}{2}\ \Pi(0)A(1-\eta)(-\eta)\overline{x}^{-\eta-1}(x-\overline{x})^2 + o\left(\|x-\overline{x},g\|^2\right).$$

Finally, ignoring the terms that are smaller than second order, or that do not involve g, and dividing by $\tilde{F}(0, \overline{x}, \overline{z})$, the maximum profit at $\overline{x}$, we have:

$$F(g) = -\frac{1}{2}\eta(\eta-1)g^2 \equiv Bg^2.$$

Appendix B

Value Function and Optimal Decision Rules

The value function $V(\cdot)$ and the optimal decision rules $\{\underline{g}, g^*, \overline{g}\}$ solve the following system of o.d.e. and boundary conditions:

$$\rho V(g) = F(g) - \pi V'(g) + \frac{\sigma^2}{2} V''(g) \text{ for } g \in [\underline{g}, \overline{g}]$$

as well as value matching and smooth pasting at the boundary of the range of inaction as well as optimality of g^*:

$$V(\underline{g}) = V(g^*) - \psi, V(\overline{g}) = V(g^*) - \psi, \text{and } V'(\underline{g}) = V'(\overline{g}) = V'(g^*) = 0$$

As explained in Álvarez and others (2019), the decision rules depend on ψ/B, π/σ^2 and ρ/σ^2. One can further simplify the problem by considering the case in which $\rho \to 0$, the so called "ergodic" control case. Indeed the solutions are very insensitive to ρ when it takes small values—one can show that the derivative of them with respect to ρ is zero when evaluated at $\rho = 0$. The appendix in Álvarez and others (2019) describes the limit that defines the ergodic control case and characterizes its solutions. One can also compare the solutions obtained numerically with the method described in appendix C, which requires $\rho > 0$; they are almost identical. Hence, to simplify, we will parametrize the solutions by two ratios: ψ/B and π/σ^2.

Appendix C

Numerical Solution of Value Function and Simulated Statistics

An alternative way to compute both the value function and the optimal policies, as well as the effect of cost shocks of the price level, is to discretize time as well as the state space. We consider there a discrete time and discrete state version of the problem. We let Δ be the length of the time period. We will let $\sqrt{\Delta}\sigma$ be the distance between any two points in the state space. For this purpose, we represent the Brownian motion during the range of inaction as:

$$g(t+\Delta)-g(t)=\begin{cases} +\sigma\sqrt{\Delta} & \textit{with probability } p_u = \frac{1}{2}\left(1-\frac{\pi\sqrt{\Delta}}{\sigma}\right) \\ -\sigma\sqrt{\Delta} & \textit{with probability } p_d = \frac{1}{2}\left(1+\frac{\pi\sqrt{\Delta}}{\sigma}\right) \end{cases}$$

so that $\mathbb{E}[g(t+\Delta)-g(t)] = -\pi\Delta$ and $\mathbb{E}[g(t+\Delta)-g(t)]^2 = \Delta\sigma^2$. Thus we can let the state space be $G=\{k\sqrt{\Delta}\sigma\}_{k=-M}^{k=M}$ with typical element $g_k \in G$. The value function is just a vector $V \in \mathbb{R}^{(2M+1)}$. The discretized version solves the following Bellman equation:

$$V_k = \max\left\{\psi + \max_j V_j, \Delta F\left(g_k\right) + \frac{1}{1+\Delta\rho}[p_u V_{\max\{k+1,K\}} + p_d V_{\min\{k-1,-K\}}]\right\}$$

for $k = -K, -K+1, \ldots, K-1, K$. The choice of K should be large enough so that the range on inaction is well in the interior of G. The choice of Δ should be small enough so that it approximates well the continuous time limit. The solution of this problem will give the thresholds $\underline{g}$ and $\overline{g}$, as well as $k^* = \text{argmax}_k\{V_k\}$ so that $g^* = g_{k^*}$.

We let f be a $2K+1$ positive vector containing the fraction of firms with different price gaps. For instance, f_k will be the fraction of firms with price gap g_k. The law of motion for the vector with the distribution of firms $f(t+1)$ can be represented by a square stochastic matrix L of $2K+1 \times 2K+1$ dimensions, so $f(t+\Delta) = Lf(t)$, where we use $f(t)$ for the column vector of $2K+1$ values of the fractions of firms $f_k(t)$ at time t with price gap g_k. The matrix L can be thought as the sum of two matrices. The first matrix has zeros in all the entries, except in the entries next to the diagonal where it has either p_u or p_d, keeping tract of the mass of firms within the inaction region. The second matrix is also sparse—it has zeros and ones, with ones indicating that the firms that are outside the range of inaction will transit to optimal return point k^* with probability one. Recall that we have: $f_j(t + \Delta) = \sum_{k=-K,K} L_{j.k} f_k(t)$. Let $\underline{k}$, $k^*$and $\overline{k}$ be the indices of the elements of f that correspond to $\underline{g}$, $g^*$and $\overline{g}$, respectively.

The elements $L_{i,j}$ are zero, except in the following cases.

1. For $k = -K, \ldots, \underline{k} - 1)$, then $L_{k^*,k} = 1$,
2. For $k = \overline{k} + 1, \ldots, K$, then $L_{k^*,k} = 1$,
3. For $k = \underline{k} + 1, \ldots, \overline{k} - 1$, then $L_{k,k+1} = p_d$ and $L_{k,k-1} = p_u$,
4. $L_{\underline{k},\underline{k}+1} = p_d$ and $L_{\overline{k},\overline{k}-1} = p_u$, and
5. $L_{k^*,\underline{k}} = p_d$ and $L_{k^*,\overline{k}} = p_u$.

As it is well known, the invariant distribution can be obtained by computing the powers of L^T for a large value of T. Alternatively, the invariant distribution is the eigenvector associated with the eigenvalue equal to one of matrix L.

The fraction of price changes per period of length Δ is given a $\lambda_a^+(t) = \frac{1}{\Delta} l^+ f(t)$, where l^+ is a vector that adds the components of the vector $f(t)$ with values below $\underline{g}$. In particular, the row vector l^+ has:

1. $l_k^+ = 1$ for $k = -K, \ldots, \underline{k} - 1$,
2. $l_{\underline{k}}^+ = p_d$, and
3. $l_k^+ = 0$ for $k = \overline{k} + 1, \ldots, K$.

We can define $\lambda_a^-(t) = \frac{1}{\Delta} l^- f(t)$ in an analogous way. The row vector l^- has:

1. $l_k^- = 1$ for $k = \overline{k} + 1, \ldots, K$,
2. $l_{\overline{k}}^- = p_u$, and
3. $l_k^- = 0$ for $k = -K, \ldots, \overline{k} - 1$.

Thus, the fraction of firms that change prices between t and $t + \Delta$ per unit of time is $\lambda_a(t) = \lambda_a^+(t) + \lambda_a^-(t)$. Note that the definition of λ_a, λ_a^+ and λ_a^-, divides the fraction of firms that adjust prices by Δ so that it is comparable, at least for small Δ, to the continuous time expressions. Hence, if we multiply the expressions for $\lambda_a(t)$, $\lambda_a^+(t)$, or $\lambda_a^-(t)$ by Δ, we will obtain fraction of firms that change prices during the interval t and $t + \Delta$.

Discrete-time – discrete-state impulse response. We can write the equivalent of the impulse response on prices using the matrix L.

We start with $\bar{f}$ given by the invariant distribution implied by L. Then we let $\hat{f}$ $(0; \delta)$ be the $2K+1$ vector of the displaced initial conditions. This is a shift of the mass to the left by an amount equal δ. This vector is given by:

$$\hat{f}(0;\delta) = \bar{f}_{k+v} \text{ for } k = -K,\ldots,K - v \text{ and } \hat{f}_k(0;\delta) = 0 \text{ otherwise,} \tag{12}$$

and where $v = \delta / \left[\sigma\sqrt{\Delta}\right]$ which, for simplicity, we assume to be an integer. Now we can compute the impact effect Θ:

$$\Theta = \sum_{k=-K}^{\underline{k}-1}\left(g_{k^*} - g_k\right)\hat{f}_k\left(0;\delta\right) - \sum_{k=\bar{k}+1}^{K}\left(g_k - g_{k^*}\right)\hat{f}_k\left(0;\delta\right). \tag{13}$$

Then we compute the sequence of distribution of firms indexed by their price gap as:

$f(\Delta) = [L]^j \hat{f}$ $(0; \delta)$ for $j = 1, 2,...$

With these elements we compute the rate of increase in prices θ's:

$$\theta\left(j\Delta\right) = \Delta_p^+\lambda_a^+\left(j\Delta\right) - \Delta_p^-\lambda_a^-\left(j\Delta\right) \text{for } j = 1,2,\ldots \tag{14}$$

where

$$\lambda_a^+\left(j\Delta\right) = \frac{1}{\Delta} l^+ f\left(j\Delta\right) \quad \text{for } j = 1,2,\ldots, \tag{15}$$

and

$$\lambda_a^-\left(j\Delta\right) = \frac{1}{\Delta} l^- f\left(j\Delta\right) \quad \text{for } j = 1,2,\ldots \tag{16}$$

The analog of the continuous time impulse response is:

$$P\left(j\Delta\right) = \bar{P} + \pi\Delta + \Theta + \sum_{k=1}^{j}\theta\left(k\Delta\right)\Delta \quad \text{for} \quad j = 1,2,\ldots$$

where $\bar{P}$ is the price level in steady state just before the cost shock, or $\bar{P} = P(-\Delta)$.

To simplify the exposition we have not taken into account the regular idiosyncratic shocks that also occur during the period between $t = 0$ and $t = \Delta$. To correct for this effect we have included $\pi\Delta$ to the initial value of $\bar{P}$, which is the correct value, since we are starting with the invariant distribution. This has the interpretation that the shock δ occurs at the end the discrete period, but before the next period. As $\Delta \to 0$, these effects can be neglected.