# Revisiting Overborrowing and its Policy Implications

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Economies with imperfect financial market access may experience crises that cause significant economic dislocation. These crises are characterized by the sudden stop of domestic or international credit flows and they are associated with large declines in consumption, output, relative prices, and asset prices.[1]

An important question for emerging-market economies is whether, in normal times when access to financial markets is unconstrained and

1. The recent global crisis arising from the U.S. subprime mortgage market is the most vivid example of a financial sudden stop, but the long sequence of emerging market crashes since the mid-1990s is an equally important illustration of how disruptive financial "sudden stops" can be.

plentiful, excessive borrowing affects the likelihood and the severity of these crises. This question is important because the policy implications of alternative answers are very different. If there is excessive or inefficient borrowing in good times (that is, "overborrowing"), policy should be geared primarily toward addressing the ex ante inefficiency that causes it; for example, by imposing a tax on capital flows or other forms of capital controls and prudential regulations to reduce the incentives to borrow excessively.[2] In this case, policy should focus less on mitigating the consequences of a crisis when one occurs, and more on strengthening the ex ante incentives to borrow efficiently in good times. In contrast, if there is no overborrowing in good times, policy should focus primarily on designing efficient ex post intervention mechanisms in bad times (such as nationally or multilaterally financed bailouts), to minimize the costs of the inevitable crises associated with imperfect access to financial markets.[3] We emphasize here that, as Benigno and others (2009) discuss, there is an important link between ex ante and ex post policies: indeed, full knowledge of ex post policies might modify agents' behavior in normal times and hence the required ex ante intervention.

A rapidly growing literature has examined this issue. In early contributions, Fernández-Arias and Lombardo (1998) and Uribe (2007) examined the possibility of overborrowing in economies subject to exogenous (either individual or aggregate) debt limits. More recently, Lorenzoni (2008) and Korinek (2010) have explored the possibility of overborrowing qualitatively in models in which the debt limit is endogenous. Uribe (2007) and Bianchi (2009) examined the issue quantitatively with contrasting results. While Uribe (2007) finds no overborrowing, Bianchi (2009) finds that overborrowing is quantitatively relevant and has significant welfare implications. In endowment economies, Korinek (2010) and Bianchi (2009) suggest that only macro-prudential policies have scope to prevent and mitigate crises. In contrast, based on a model with production similar to the one used in this paper, Benigno and others (forthcoming) find underborrowing in their baseline model and conclude that both ex ante and ex post policy interventions are needed to achieve constrained efficiency.[4]

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2. See, for instance, the recent introduction of a tax on international portfolio flows by Brazil, or Chile's earlier experience with capital controls on foreign inflows.

3. See Caballero (2010) for a detailed discussion of alternative modalities of ex post interventions.

4. Benigno and others (2009) find that it is optimal (in Ramsey's sense) to intervene ex post, once a sudden stop actually occurs.

This paper analyzes quantitatively the extent to which there is overborrowing in a business cycle model for emerging market economies. We investigate overborrowing in a small, open-economy model with production and imperfect access to international capital markets, as in Benigno and others (forthcoming). Our occasionally binding credit constraint is embedded in a standard two-sector (tradable and non-tradable goods) small open economy in which financial markets are not only incomplete but also imperfect, as in Mendoza (2002). For simplicity's sake, in this model production occurs only in the non-tradable sector of the economy. The asset menu is restricted to a single-period, risk-free bond paying off the exogenously given foreign interest rate. In addition to asset market incompleteness, we assume that access to foreign financing is constrained to a fraction of households' total income. Thus, foreign borrowing is denominated in units of the tradable good but is leveraged on income generated at different relative prices (that is, the relative price of a non-tradable good). The specification of the borrowing constraint thus captures "liability dollarization," a key feature of emerging market capital structure (for example, Krugman, 1999; Aghion, Bacchetta, and Banerjee, 2004).[5] As is well known, however, pecuniary externalities like the one at work in our model can arise in much more general circumstances: namely, whenever a relative price enters the specification of a financial friction in a multiple good economy (see Arnott, Greenwald, and Stiglitz, 1994 for a detailed discussion and a survey of the theoretical literature).

Two defining features of this environment are common in most of the related literature. First, the international borrowing constraint binds only occasionally: the crisis, defined as the event in which the constraint binds, is an endogenous event that depends on agents' decisions, the policy regime, and the state of the economy. Second, in this environment the scope for policy intervention arises from the existence of a pecuniary externality stemming from individual agents failing to internalize the aggregate impact of their borrowing decisions on the relative price of non-tradable goods. This in turn affects the value of collateral.[6]

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5. The latest wave of crises in emerging Europe and corporate sector problems in Mexico and Brazil in the fourth quarter of 2008 represent striking evidence of the importance of this kind of feature.

6. Benigno and others (2009), among others, show that the competitive equilibrium allocation of this economy is not constrained-efficient in the sense of Kehoe and Levine (1993). Benigno and others (2009) also discuss how efficiency can be restored with a distortionary tax on non-tradable consumption in a deterministic two-period version of the model used here. Implementation issues are not discussed further in this paper.

To investigate overborrowing quantitatively we compare the competitive equilibrium (CE) with the constrained efficient allocation chosen by a welfare-maximizing social planner (SP), and solve using global solution methods. That is, we solve for decision rules for all endogenous variables across both states of the world, when the constraint binds and when it does not. This approach assumes that behavior distant from crisis periods is based on full knowledge of what the equilibrium will be when the economy enters the crisis state. This solution method, while computationally costly, is critical for understanding the interaction between different states of the world.[7]

We find that overborrowing is a quantitative matter: it depends on both the model specification and the values for model parameters. Specifically, in our production model, CE and SP allocations diverge when the constraint binds and when it does not, with under- or overborrowing in normal times (that is, when the constraint does not bind) depending on the parametrization of the economy. In the baseline calibration, we find underborrowing in normal times. In an alternative calibration, with more impatient agents and more volatile shocks, we find overborrowing in normal times. In both cases, however, in times of crisis (that is, when the constraint binds), there is inefficient underborrowing. That is, in crisis, agents in CE always consume less tradable goods than in the SP allocation.

In general, the main difference between CE and SP allocations is that the social planner takes into account the effects of his or her consumption choices on aggregate prices, and thus on the value of collateral (the literature refers to this as a "pecuniary externality"). The implications of this pecuniary externality depend on the structure of the economy. In general, even in normal times, the possibility that the constraint might bind in the future increases the current marginal utility of tradable consumption (that is, increases the private marginal value of saving). But the *social* marginal value of saving (from the perspective of the social planner) is higher than the *private* value (from the perspective of individual agents), because of the pecuniary externality effect. All else being equal, this mechanism involves higher saving in the SP allocation compared to the CE allocation, and generates overborrowing in the endowment economies studied by Bianchi (2009) and Korinek (2010).

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7. The technical challenge in solving such a model is that the constraint binds only occasionally and changes location in the state space of the model, depending on the realization of both the exogenous and the endogenous state variables.

But in a production economy an opposite force arises. The relatively higher marginal utility of tradable consumption from the social planner's perspective generates a higher *social* marginal benefit of supplying one more unit of labor compared to the *private* one in normal times. Relatively higher production and consumption of non-tradable goods can then lead to relatively higher borrowing and tradable consumption in the SP compared to the CE, and thus generates the possibility of underborrowing.

The relative strength of these two effects depends on the parametrization of the economy: for example, the second channel dominates the first in our baseline calibration, but we find that the first channel dominates the second when agents are more impatient and shocks are more volatile, thus inducing overborrowing rather than underborrowing. Overborrowing always arises in the endowment economies we study, because the second effect is not present. Also, in the endowment case, the planner cannot manipulate the value of collateral when the constraint binds, as he or she cannot alter the production possibilities of the economy: thus CE and SP allocations must always coincide once the crisis occurs in an endowment economy.[8]

From a qualitative point of view, our findings suggest that only for ex post interventions is there a clear cut rationale to address the economic dislocation associated with the sudden stop. These findings also suggest that the design of economy-wide, ex ante intervention policies is not robust: indeed, different structures of the economy or different calibrations of the same economy may require different interventions, depending on the presence of either under- or overborrowing.

We then measure quantitatively the gap between CE and SP allocations. To do so, we determine the percentage of consumption that agents are willing to forgo to move from one allocation to the other, in every state and for every date. We find that in production economies, the overall welfare gains from implementing the SP allocation are one order of magnitude larger than in endowment economies. In addition, welfare gains are always larger near crisis times than in normal ones, in both production and endowment economies.

In terms of policy implications, our findings are consistent with the position that nationally or multilaterally financed bailouts are

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8. The equivalence between SP and CE allocations arises in states of the world in which the crisis occurs (that is, the constraint is binding) for both allocations.

important to help mitigate the effects of crises. In contrast, our analysis suggests that the case for economy-wide, macro-prudential policy intervention tools, such as taxes on capital flows and capital controls (as opposed to interventions specifically targeting the financial system), is very weak.

There are important caveats to these policy conclusions. Moral hazard, time-consistency considerations, and the economic cost of distortions are not present in the class of models analyzed in this paper. As a result, the case for ex post (ex ante) policy intervention may be over (under) stated by our analysis. Considering moral hazard would weaken the case for ex post interventions. In addition, Chari and Kehoe (2009) show that the lack of credibility of efficient ex post intervention policies call for an ex ante prudential intervention geared toward containing the excesses induced by the time-inconsistency of the optimal ex post intervention. This would further strengthen the case for ex ante interventions.

Nonetheless, while it is well known that bailouts can induce moral hazard, it is less well understood that prudential regulations and capital controls can hamper long-run growth. Nikolov (2009), for instance, studies the private choice of leverage in a model with heterogeneous firm productivity, based on a stochastic version of Kiyotaki and Moore (1997). He finds that mandating tighter, economy-wide leverage ratios than those chosen by private agents in a competitive equilibrium does reduce aggregate volatility, but at the cost of lowering average growth, with welfare-reducing consequences. As a result, in his model, the aggregate leverage ratio of the competitive equilibrium is constrained-efficient. This further weakens the case for ex ante interventions.[9]

The rest of the paper is organized as follows. Section 1 discusses the pecuniary externality that may give rise to under- or overborrowing. Section 2 describes the model we use. Section 3 discusses its parametrization and solution. Section 4 illustrates the model's working and basic properties, and reports our main quantitative results, comparing CE and SP equilibria using alternative model specifications and parameter values. Section 5 discusses the policy implications, while section 6 concludes.

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9. This limitation does not apply to the policy analysis of Benigno and others (2009), in which the Ramsey planner explicitly trades off the benefits of intervening either ex ante or ex post against the efficiency costs of doing so with a distortionary tax on non-tradable consumption. In contrast, all contributions in the existing literature just compare competitive allocations with socially planned ones, discussing implementation issues without accounting for any implementation cost.

# 1. OVERBORROWING AND PECUNIARY EXTERNALITIES

Before turning to the presentation of the model, we discuss the source of the externality that may give rise to over- or underborrowing and hence scope for policy intervention. Overborrowing has been discussed extensively in the literature so our discussion of the pecuniary externality that may give rise to it takes the form of a review of the relevant literature.

In an early contribution, Fernández-Arias and Lombardo (1998) investigate analytically whether an economy with an aggregate debt limit tends to overborrow relative to an economy in which the debt limit is imposed at the level of the individual agent. They find that agents fail to internalize the debt limit, and the economy overborrows. Uribe (2007) investigates overborrowing quantitatively and finds that the amount borrowed is independent of foreign lenders basing their decisions on individual as opposed to aggregate variables.

The models used in these early analyses are similar. The key difference between the two environments is that in Uribe (2007), when the constraint is binding, the domestic interest rate adjusts and induces agents to internalize the credit limit, while Fernández-Arias and Lombardo (1998) assume that the domestic interest rate is equal to the world interest rate and agents fail to internalize the debt ceiling in their deterministic model. Both papers, however, share two common ingredients. First, the debt ceiling is exogenously specified.[10] Second, this is a one-good economy, in which the pecuniary externality that is our focus cannot arise (see Benigno and others, 2009, section 2, for more details).

Later work has considered richer environments in which there are multiple goods and the borrowing limit is endogenous. In these environments, the interaction between the borrowing constraint and the dependence of the borrowing limit on a relative price generates a pecuniary externality that is not internalized in the competitive equilibrium allocation and might give rise to constrained-inefficient borrowing. The social planner, on the other hand, takes into account the way in which this relative price is determined in the competitive allocation when choosing an optimal plan and accordingly selects a constrained-efficient amount of borrowing (again, see Benigno and

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10. Uribe (2007) considers one extension in which the constraint is endogenous in the sense explained in the previous section. In this case, he finds small amounts of overborrowing.

others, 2009, for more details). For instance, in a closed economy model, Lorenzoni (2008) shows that entrepreneurs do not take into account the effects of asset prices on the amount that they can borrow, so that in the competitive equilibrium, under certain specific assumptions, financial contracts lead to excessive borrowing. Korinek (2010) and Bianchi (2009) carried out similar analyses in a small open economy similar to our baseline model, but without production, in which the amount that individuals can borrow depends on the income generated in both sectors of the economy and their relative price. Both authors concluded that there was overborrowing, qualitatively (Korinek, 2010) and quantitatively, with potentially significant welfare consequences (Bianchi, 2009). The policy implication of these analyses was the recommendation of economy-wide prudential taxation on capital flows to bring the competitive allocation of the economy into line with that chosen by the social planner for efficiency.

In related work, in his stochastic version of the Kiyotaki and Moore (1997) model, Nikolov (2009) finds that, when the leverage ratio is a choice variable, these pecuniary externalities do not necessarily induce sizable divergence between the CE and the SP. This is because, interestingly, in Nikolov's (2009) model, there is not only production but also firm heterogeneity. Thus, in this environment, there is a trade-off between the lower volatility and the lower average growth associated with mandating a lower aggregate leverage ratio than that privately chosen in the CE of the economy. So mandating lower regulatory leverage ratios may impose significant efficiency costs that, in this setup, are welfare reducing.

## 2. THE MODEL

The model that we propose is a simplified version of the one used by Benigno and others (forthcoming). This is a simple two-sector (tradable and non-tradable) small open production economy, in which financial markets are not only incomplete but also imperfect, as in Mendoza (2002), and in which production occurs only in the non-tradable sector.

### 2.1 Households

There is a continuum of households $j \in [0,1]$ that maximize the utility function

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \left[ \beta^t \frac{1}{1-\rho} \left( C_{j,t} - \frac{H_{j,t}^{\delta}}{\delta} \right)^{1-\rho} \right], \tag{1}$$

where $C_j$ denotes the individual consumption basket and $H_j$ the individual supply of labor. For simplicity, we omit the $j$ subscript for the remainder of this section, but it is understood that all choices are made at the individual level. The elasticity of labor supply is $\delta$, while $\rho$ is the coefficient of relative risk aversion. In equation (1), the preference specification follows from Greenwood, Hercowitz, and Huffman (1988): in the context of a one-good economy this specification eliminates the wealth effect from the labor supply choice. Here, in a multi-good economy, the sectoral allocation of consumption will affect the labor supply decision through relative prices. The consumption basket, $C_t$, is a composite of tradable and non-tradable goods:

$$C_t \equiv \left[ \omega^{\frac{1}{\kappa}} (C_t^T)^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}}, \tag{2}$$

where the parameter $\kappa$ is the elasticity of intratemporal substitution between consumption of tradable and non-tradable goods, while $\omega$ is the relative weight of the two goods in the consumption basket.

We normalize the price of tradable goods to 1. The relative price of the non-tradable good is represented by $P^N$. The aggregate price index is then given by

$$P_t = \left[ \omega + (1-\omega)(P_t^N)^{1-\kappa} \right]^{\frac{1}{1-\kappa}},$$

with a one-to-one link between the aggregate price index, $P$, and the relative price, $P^N$. Households maximize utility subject to their budget constraint, which is expressed in units of tradable consumption. The constraint each household faces is

$$C_t^T + P_t^N C_t^N = \pi_t + W_t H_t - B_{t+1} + (1+i)B_t, \tag{3}$$

where $W_t$ is the wage in units of tradable goods, $B_{t+1}$ denotes the net foreign asset position at the end of period $t$ with gross real return $1+i$. Households receive profits, $\pi_t$, from owning the representative firm. Their labor income is given by $W_t H_t$.

International financial markets are incomplete and access to them is also imperfect. The asset menu includes only a one-period bond denominated in units of tradable consumption. In addition, we assume that the amount that each individual can borrow internationally is limited by a fraction of his current total income:

$$B_{t+1} \geq -\frac{1-\phi}{\phi}(\pi_t + W_t H_t). \tag{4}$$

This constraint captures the effects of liability dollarization, since foreign borrowing is denominated in units of tradables, while the income that can be pledged as collateral is also generated in the non-tradable sector. This constraint is also endogenous as it depends on the current realization of profits and wage income. We don't explicitly derive the credit constraint as the outcome of an optimal contract between lenders and borrowers. However, we can interpret this constraint as the outcome of a lender-borrower interaction, in which the lender will not permit borrowing beyond a certain limit.[11] This limit depends on the parameter $\phi$, which measures the tightness of the borrowing constraint and depends on current gross income that could be used as a proxy of future income.[12]

Households maximize equation (1) subject to (3) and (4), by choosing $C_t^T$, $C_t^N$, $B_{t+1}$, and $H_t$. The first-order conditions of this problem are the following:

$$C^T : \left(C_{j,t} - \frac{H_{j,t}^\delta}{\delta}\right)^{-\rho} \omega^{\frac{1}{\kappa}}(C_t^T)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_t, \tag{5}$$

$$C^N : \left(C_{j,t} - \frac{H_{j,t}^\delta}{\delta}\right)^{-\rho} (1-\omega)^{\frac{1}{\kappa}}(C_t^N)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_t P_t^N, \tag{6}$$

$$B_{t+1} : \mu_t = \lambda_t + \beta(1+i)E_t(\mu_{t+1}), \tag{7}$$

11. As emphasized by Mendoza (2002), this form of liquidity constraint shares some features, namely the endogeneity of the risk premium, which would be the outcome of the interaction between a risk-averse borrower and a risk-neutral lender in a contracting framework, as in Eaton and Gersovitz (1981). It is also consistent with anecdotal evidence on lending criteria and guidelines used in mortgage and consumer financing.

12. As we discuss in Benigno and others (2009), a constraint expressed in terms of future income that could result from lender-borrower interaction in a limited commitment environment would introduce further computational difficulties that we need to avoid for tractability.

$$H_t : \left( C_{j,t} - \frac{H_{j,t}^{\delta}}{\delta} \right)^{-\rho} \left( H_{j,t}^{\delta-1} \right) = \mu_t W_t + \frac{1-\phi}{\phi} W_t \lambda_t. \tag{8}$$

When the credit constraint is binding ($\lambda_t > 0$), the Euler equation (7) incorporates an effect that can be interpreted as arising from a country-specific risk premium on external financing. In this framework, moreover, even if the constraint is not binding at time $t$, an intertemporal effect arises due to the possibility that the constraint might be binding in the future: this effect is embedded in the term $E_t(\mu_{t+1})$, which implies that current consumption of tradable goods would be lower than the unconstrained case, when the constraint is expected to bind in the future.

Based on the conditions above, we can combine equations (5) and (6) to obtain the intratemporal allocation of consumption, and equations (5) with (8) to obtain the labor supply schedule, respectively:

$$\frac{(1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = P_t^N, \tag{9}$$

$$H_{j,t}^{\delta-1} = \left( \frac{\omega C}{C^T} \right)^{\frac{1}{\kappa}} W_t \left( 1 + \frac{1-\phi}{\phi} \frac{\lambda_t}{\mu_t} \right). \tag{10}$$

Note here that

$$\left( \frac{\omega C}{C^T} \right)^{\frac{1}{\kappa}} = \left[ 1 + \left( \frac{1-\omega}{\omega} \right)^{\frac{1}{\kappa}} \left( \frac{C_t^N}{C_t^T} \right)^{\frac{\kappa-1}{\kappa}} \right]^{\frac{1}{\kappa-1}} \omega^{\frac{1}{\kappa-1}} = \left[ 1 + \left( \frac{1-\omega}{\omega} \right) (P_t^N)^{1-\kappa} \right]^{\frac{1}{\kappa-1}} \omega^{\frac{1}{\kappa-1}}.$$

So, if we were in a one-good economy, there would be no effect coming from the marginal utility of consumption for the labor supply choice, because of the GHH specification. For later use, it is also useful to note that an increase in $P^N$ would lower $(\omega C/C^T)^{1/\kappa}$, and the labor supply curve becomes flatter as $P^N$ increases.[13] When the constraint

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13. In what follows we refer to the labor supply curve in a diagram in which labor is on the vertical axis and the wage rate on the horizontal one.

is binding ($\lambda_t > 0$), the marginal utility of supplying one more unit of labor is higher and this helps to relax the constraint. In this case, the labor supply becomes steeper and agents substitute leisure with labor to increase the value of their collateral for given wages and prices.

Importantly, labor supply is also affected by the possibility that the constraint may be binding in the future. If in period $t$ the constraint is not binding but may bind in period $t + 1$, we have

$$\left( C_{j,t} - \frac{H_{j,t}^{\delta}}{\delta} \right)^{-\rho} \left( H_{j,t}^{\delta-1} \right) = \mu_t W_t$$

and

$$\mu_t = \beta(1+i)E_t \left[ \lambda_{t+1} + \beta(1+i)E_t(\mu_{t+2}) \right],$$

so that the marginal benefit of supplying one more unit of labor today rises in line with the probability of the constraint becoming binding in the future. This effect will induce agents to supply more labor for any given wage, and the labor supply curve will be steeper in this case than when there is no credit constraint. In equilibrium, this effect increases non-tradable production and consumption and affects tradable consumption, depending on the degree of substitutability between tradable and non-tradable goods. When goods are complements, any increase in non-tradable consumption is associated with an increase in tradable consumption that reduces the amount agents save in the competitive equilibrium. The opposite would occur if goods were substituted.

## 2.2 Firms

Firms are endowed with a stochastic stream of tradable goods, $\exp(\varepsilon_t) Y^T$, where $\varepsilon_t$ is a stochastic process, and produce non-tradable goods, $Y^N$. We assume that $\varepsilon$ follows an autoregressive process of the first order (AR(1)). For simplicity, we abstract from other sources of macroeconomic uncertainty, such as shocks to the technology for producing non-tradables and the world interest rate.

Firms produce non-tradable goods, $Y_t^N$, with a variable labor input and a Cobb-Douglas technology

$$Y_t^N = AH_t^{1-\alpha},$$

where $A$ is a scaling factor. The firm's problem is static and current-period profits, $\pi_t$, are

$$\pi_t = \exp(\varepsilon_t)Y^T + P_t^N AH_t^{1-\alpha} - W_t H_t.$$

The first-order condition for labor demand is

$$W_t = (1-\alpha)P_t^N AH_t^{-\alpha}, \tag{11}$$

so that the value of the marginal product of labor is set equal to the real wage ($W_t$). For the case in which we have constant returns to scale ($\alpha = 0$), we obtain

$$W_t = P_t^N A,$$

so that the real wage in terms of the relative price of non-tradables is constant (as long as we don't have any shock to productivity of non-tradables), and equilibrium labor is determined by the supply side while the wage rate is determined by the demand side of the labor market.

## 2.3 Aggregation and Equilibrium

To gain insight into the model, we focus on the labor market equilibrium condition when firms have constant returns to scale technology, such that $\alpha = 0$. Combining equations (11) and (10) we obtain

$$H_{j,t}^{\delta-1} = \left(\frac{\omega C}{C^T}\right)^{\frac{1}{\kappa}} P_t^N A\left(1 + \frac{1-\phi}{\phi}\frac{\lambda_t}{\mu_t}\right).$$

When the international borrowing constraint is not binding ($\lambda_t = 0$), a shock that triggers a decrease in $P_t^N$ will reduce the labor supply and production of non-tradable goods. Indeed, in this case, equilibrium in the labor market becomes

$$H_{j,t}^{\delta-1} = \left[1 + \left(\frac{1-\omega}{\omega}\right)(P_t^N)^{1-\kappa}\right]^{\frac{1}{\kappa-1}} P_t^N A.$$

To determine the goods market equilibrium, we combined the household budget constraint and company profits with the

equilibrium condition in the non-tradable goods market to obtain the current account equation for our small open economy:

$$C_t^T = Y_t^T - B_{t+1} + (1+i)B_t. \tag{12}$$

The non-tradable goods market equilibrium condition means that

$$C_t^N = Y_t^N = AH_t.$$

Finally, using the definitions of firm profits and wages, the credit constraint means that the amount that the country as a whole can borrow is constrained by a fraction of the value of its GDP:

$$B_{t+1} \geq -\frac{1-\phi}{\phi}\Big[\exp(\varepsilon_t)Y^T + P_t^N Y^N\Big], \tag{13}$$

Thus, together equations (12) and (13) determine the course of foreign borrowing.

## 2.4 Social Planner Problem

Let us now consider the social planner's problem. The planner maximizes equation (1) subject to resource constraints, the international borrowing constraint from an aggregate perspective, and the pricing rule for the competitive equilibrium allocation. In particular, noting that the competitive rule (9) determines the relative price, we can rewrite equation (13) as

$$B_{t+1} \geq -\frac{1-\phi}{\phi}\left\{\exp(\varepsilon_t)Y^T + \left[\frac{(1-\omega)(C_t^T)}{\omega}\right]^{\frac{1}{\kappa}}(AH_t)^{\frac{\kappa-1}{\kappa}}\right\}.$$

The planner chooses the optimal path $C_t^T$, $C_t^N$, $B_{t+1}$, and $H_t$, and the first-order conditions for this problem are given by

$$C^T: \left(C_{j,t} - \frac{H_{j,t}^{\delta}}{\delta}\right)^{-\rho} \omega^{\frac{1}{\kappa}}(C_t^T)^{-\frac{1}{\kappa}}C^{\frac{1}{\kappa}}$$

$$= \mu_{1,t} - \lambda_t \frac{1-\phi}{\phi}\frac{1}{\kappa}\frac{(1-\omega)}{\omega}\left[\frac{(1-\omega)(C_t^T)}{\omega}\right]^{\frac{1}{\kappa}-1}(AH_t)^{\frac{\kappa-1}{\kappa}}, \tag{14}$$

$$C^N : \left( C_{j,t} - \frac{H_{j,t}^{\delta}}{\delta} \right)^{-\rho} (1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}} C^{\frac{1}{\kappa}} = \mu_{2,t}, \tag{15}$$

$$B_{t+1} : \mu_{1,t} = \lambda_t + \beta(1+i)E_t(\mu_{1,t+1}), \tag{16}$$

and

$$H_t : \left( C_t - \frac{H_t^{\delta}}{\delta} \right)^{-\rho} \left( H_t^{\delta-1} \right)$$

$$= \mu_{2,t} A + \frac{1-\phi}{\phi} \lambda_t \left[ \frac{(1-\omega)(C_t^T)}{\omega} \right]^{\frac{1}{\kappa}} \frac{\kappa-1}{\kappa} A(AH_t)^{-\frac{1}{\kappa}}. \tag{17}$$

There are two main differences between the competitive equilibrium first-order conditions and those associated with the planner's problem, arising from occasionally binding financial friction. First, equation (14) shows that, in choosing tradable consumption, the planner takes into account how a change in tradable consumption affects the value of collateral (see also Korinek, 2010 and Bianchi, 2009). This is usually called the price externality in the related literature and occurs when the constraint is binding (that is, $\lambda_t > 0$). As noted above, however, even if the constraint is not binding today, the possibility that it might bind in the future can affect the marginal value of tradable consumption today (that is, the marginal value of saving). Indeed, as Bianchi (2009) notes, the Euler equation from the planner's perspective becomes

$$\mu_{1,t} = \beta(1+i)E_t \left[ \lambda_{t+1} + \beta(1+i)E_t(\mu_{1,t+2}) \right],$$

where $E_t(\mu_{1,t+2})$ is given by equation (14) and takes into account the future effect of the pecuniary externality. Crucially, this implies that through this effect and at the same allocation, the marginal social value of saving (the marginal value in the SP allocation) will be higher than the private value (in the CE allocation). Thus, the decentralized equilibrium might display overborrowing.

In the production economy under study, the presence of occasionally binding financial friction has an additional effect. In particular, we

can rewrite the labor supply equation by using equation (15) and the equilibrium condition in the non-tradable good market as follows:

$$H_t^{\delta-1} = \left[\frac{(1-\omega)C_t}{AH_t}\right]^{\frac{1}{\kappa}} A \left\{1 + \frac{1-\phi}{\phi}\frac{\lambda_t}{\mu_{2,t}}\left[\frac{(1-\omega)(C_t^T)}{\omega}\right]^{\frac{1}{\kappa}}\frac{\kappa-1}{\kappa}(AH_t)^{-\frac{1}{\kappa}}\right\}.$$

This expression shows that, when the constraint is binding, the marginal utility of supplying one extra unit of labor is affected by the degree of substitutability between tradables and non-tradables. If goods are substitutes then, when the borrowing constraint is binding, it is worth supplying one more unit of labor, as that helps relax the constraint. If goods are complements, however, it is worth decreasing the amount of labor supplied. In both cases the planner tends to relax the international borrowing constraint by increasing the value in units of tradable or non-tradable production. In the case of complements, this is achieved by an increase in prices that dominates the negative effect of lower non-tradable production and consumption. In the case of substitutes, this is achieved by increasing non-tradable production and consumption, which overcomes the effect of lower prices.

    More importantly, changes in labor supply also occur when the constraint is expected to bind in the future. Indeed, in this case, taking the ratio of equations (15) to (14) we have

$$\frac{(1-\omega)^{\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}}\frac{(C_t^N)^{-\frac{1}{\kappa}}}{(C_t^T)^{-\frac{1}{\kappa}}}\mu_{1,t} = \mu_{2,t}. \tag{18}$$

This expression shows that a higher current marginal utility of tradable consumption in the SP (arising because the constraint might bind in the future) also suggests a higher marginal utility for non-tradable consumption, which in turn boosts the marginal utility of supplying one unit of labor today. As a result, in the SP allocation, labor supply and non-tradable production are relatively higher in the CE than in the SP, even when the constraint is not binding. When goods are complements, this increase in non-tradable consumption will be associated with a higher increase in tradable consumption (reducing the amount agents save) in the SP allocation

compared to the CE allocation. When goods are substitutes, however, the amount the planner saves will increase, as agents substitute tradable consumption with non-tradable consumption.

Thus, this mechanism could generate underborrowing in the CE compared to the SP allocation. Underborrowing could occur both when goods are complements or substitutes. This depends on the strength of the labor supply effect and the relative adjustment to tradable consumption in the CE versus the SP allocation. For example, even when goods are substituted and tradable consumption falls (following the labor supply mechanism just mentioned), the decline in tradable consumption could end up larger in the CE than in the SP, suggesting that agents would underborrow.

## 3. PARAMETER VALUES AND SOLUTION METHOD

In this section we discuss the parameter values chosen and briefly describe the global solution method that we use in the numerical computations.

### 3.1 Parameter Values

The model is calibrated using a quarterly frequency and the parameter values we use are reported in table 1.[14] As in Benigno and others (forthcoming), these values are set according to work by Mendoza (2010) and Kehoe and Ruhl (2008) to the extent possible, but also to facilitate the convergence of the numerical solution procedure.

We set the world interest rate to $i = 0.0159$, which yields an annual real rate of interest of 6.5 percent; a value between 5 percent (Kehoe and Ruhl, 2008) and 8.6 percent (Mendoza, 2010). The elasticity of intratemporal substitution between tradables and non-tradables follows Ostry and Reinhart (1992) who estimate a value of $\kappa = 0.760$ for developing countries.[15] The value of $\delta$ is 2, reflecting a Frisch elasticity of labor of 2. For simplicity, the elasticity of intertemporal substitution is unitary ($\rho = 1$).

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14. When we calibrate the model at annual frequency, for robustness, the results are qualitatively the same. Some quantitative differences emerge due to the fact that the annual calibration allows for more foreign borrowing as a share of GDP in the stochastic steady state of the model for the same parameter values.

15. There is considerable debate about the value of this parameter. The estimate we use is consistent with Kehoe and Ruhl (2008) who set this parameter to 0.5.

**Table 1. Model Parameters**

| Parameter | Value |
|---|---|
| Structural parameter | |
|   Elasticity of substitution between tradable and non-tradable goods | $\kappa = 0.760$ |
|   Intertemporal substitution and risk aversion | $\rho = 1$ |
|   Labor supply elasticity | $\delta = 2$ |
|   Credit constraint parameter | $\phi = 0.7$ |
|   Labor share in production | $\alpha = 0$ |
|   Relative weight of tradable and non-tradable goods | $\omega = 0.48568$ |
|   Discount factor | $\beta = 0.98$ |
| Exogenous variable | |
|   World real interest rate | $i = 0.0159$ |
|   Steady state relative price of non-tradables | $P^N = 1$ |
| Productivity process | |
|   Persistence | $\rho_\varepsilon = 0.86$ |
|   Volatility | $\sigma_\varepsilon = 0.015$ |

For simplicity also, the labor share of production in the non-tradable sector is assumed to be unitary ($\alpha = 0$). We then normalize steady-state tradable output to one (that is, $Y^T = 1$) and set $\omega$ and $A$ to obtain a steady-state ratio of tradable to non-tradable output of 0.75 (slightly higher than Mendoza, 2002) and a unitary relative price of non-tradables in steady state (that is, $P^N = 1$).

We set $\beta = 0.98$ (implying an annual value of 0.92237) to obtain a ratio for foreign borrowing to annualized GDP of about 25 percent in the deterministic steady state.[16] The value of the credit constraint parameter ($\phi$) determines the probability of a sudden stop. We set this parameter to 0.7, which makes the constraint binding in the deterministic steady state and yields a realistic probability of a sudden stop, as typically defined in the empirical literature. In the competitive equilibrium, the unconditional probability of a sudden stop is about 2 percent per quarter (or 8.2 percent annually). For

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16. For this calculation we added an elastic discount factor to the model to pin down foreign debt in steady state.

this calculation, a sudden stop is defined as an event in which the constraint is strictly binding.

Finally, in our analysis, we focus on the behavior of the economy subject to only one stochastic shock to the endowed tradable output, which we model as an AR(1) process. Specifically, the shock process for tradable GDP is

$$\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + v_t, \tag{19}$$

where $v_t$ is an independent and identically distributed innovation, such that $v_t \sim N(0, \sigma_\varepsilon^2)$. The parameters of this process are set to $\rho_\varepsilon = 0.86$ and $\sigma_\varepsilon = 0.015$, which are the first autocorrelation and the standard deviation of total GDP reported by Mendoza (2010).

With these parameters, as Benigno and others (forthcoming) show, the model produces the sharp reversal in capital flows, the plunging output and consumption, and substantial real exchange rate depreciation (proxied by the fall in the relative price of non-tradable goods), typical of a sudden stop. In this sense, our model is quantitatively capturing the sudden stop phenomena we observe in the data.

## 3.2 Solution Method

To solve the competitive equilibrium, we use the algorithm proposed by Benigno and others (forthcoming). Here we summarize their solution procedure and explain how we apply this solution to the social planner's problem. A key step involves transforming the system of Kuhn-Tucker conditions into a standard system of nonlinear equations, as per Garcia and Zangwill (1981). The transformed system can then be solved using standard nonlinear equation solution methods.

We can then represent model equilibrium as a recursive dynamic programming problem, summarized by the following Bellman equation:

$$V(b, B, \varepsilon) = \max_{B'} \left\{ u[C - z(H)] + \beta E\left[ V(b', B', \varepsilon') \mid \varepsilon \right] \right\}, \tag{20}$$

where

$$u[C - z(H)] = \frac{1}{1 - \rho} \left( C_{j,t} - \frac{H_{j,t}^\delta}{\delta} \right)^{1 - \rho}.$$

The value function, $V(b,B,\varepsilon)$, depends on three state variables: individual borrowing, $b$, aggregate borrowing, $B$, and the stochastic shock to the tradable endowment, $\varepsilon$. In equilibrium, individual and aggregate borrowing must coincide, but from the perspective of the representative agent in our model, the borrowing constraint is imposed at the individual level, taking relative prices as given. Our solution explicitly accounts for this feature of the model specification by treating aggregate and individual debt separately in the value function.

A solution for the decentralized equilibrium defined above will be given by (i) a value function $V(B,\varepsilon)$ and (ii) a set of laws of motion (hereafter, also called decision rules or policy functions) for aggregate borrowing ($B = G_B^n(B,\varepsilon)$), aggregate employment ($H = G_H^n(B,\varepsilon)$), and the relative price of the non-tradable good basket ($P^N = G_{P^N}^n(B,\varepsilon)$) that satisfy the Bellman equation (20). Note that while the value function depends on both individual and aggregate borrowing, the decision rules for all other endogenous variables only depend on aggregate borrowing.

To solve for the social planning equilibrium we set up a dynamic programming problem. The programming problem is written as an optimization of the value function, subject only to resource constraints and the borrowing constraint. Thus, the planner chooses all quantities directly. Specifically, the problem can be written as

$$v(B,\varepsilon) = \max_{c^T,c^N,H,B'} \left\{ u\big[C - z(H)\big] + \beta E_\varepsilon \left[ v(B',\varepsilon') \mid \varepsilon \right] \right\},$$

subject to

$$C^T = (1+r)B + \varepsilon - B',$$

$$C^N = AH,$$

$$B' \geq -\frac{1-\varphi}{\varphi}(\varepsilon + P^N AH),$$

$$P^N = \frac{(1-\omega)^{\frac{1}{\kappa}}(C^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}}(C^T)^{-\frac{1}{\kappa}}}.$$

We compute a solution to this problem numerically. The shock is discretized into a Markov chain with 11 states, as in Floden (2008). Methods to solve the programming problem are standard (for example, Johnson and others, 1993). In particular, we use cubic splines to approximate the value function and we then solve the maximization problem using a feasible sequential quadratic programming routine.

## 4. QUANTIFYING OVERBORROWING

In this section we discuss the basic properties of the competitive equilibrium allocation, comparing it to the social planner version, to quantify overborrowing. We conduct this comparison using alternative model specifications and assumptions for key parameter values.

## 4.1 Competitive Equilibrium

The properties of the competitive equilibrium are more fully explained in Benigno and others (2009). Here we review them briefly. The policy function for $B_t$ is plotted in figure 1. In this figure, each solid line depicts the policy function for $B_t$ conditional on a particular state of the tradable shock. This line is drawn assuming the same shock occurs in each period. For illustrative purposes, we report the decision rule for the worst state (state 1), and progressively better ones, together with the 45-degree (dashed line) trajectory. If the first state occurs perpetually, then the policy function will meet the 45-degree line at exactly the point where the constraint binds. The economy remains from this point on and at this point, and the multiplier is still zero. If the economy is currently at the intersection between the decision rule for one of the better states and the 45-degree line and receives a worse shock, the constraint can bind strictly on impact, as the economy jumps to the corresponding new decision rule. For example, if we are at the point where state 3 intersects the 45-degree line and we receive a worse shock, we move up directly to a point where the constraint binds strictly (with positive multiplier). So the point on the decision rule where the constraint starts to bind strictly depends on the particular exogenous state at which we evaluate the rule and the value of endogenous state variable $B_t$.

Figure 2 reports the policy functions for other variables of the model as a function of the endogenous state, $B_t$. Policy functions are drawn assuming the continuation of the worst shock. All variables

**Figure 1. Decision Rule for Foreign Borrowing in the Competitive Equilibrium**
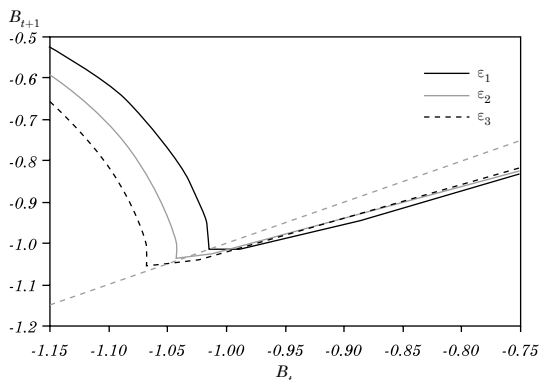


Source: Authors' calculations.

($C_t^T$, $P_t^N$, $C_t^N$, and $H_t$) follow a similar pattern. Before the constraint binds (that is, before the kink in these rules), the economy behaves in a seemingly linear manner as this shock continues to materialize. Far from the constraint, the ongoing realization of the shock reduces both tradable and non-tradable consumption and increases debt (not reported in figure 2), as agents smooth the impact of the shock by borrowing more from abroad. Once the constraint is reached, however, decision rules are driven by the need to respond to it. Agents can no longer borrow their desired amount: consumption of tradable goods decreases, lowering the relative price of non-tradable goods. A falling relative price of non-tradable goods has two effects. The first is to reduce borrowing capacity by lowering the collateral value of non-tradable income and hence generating an amplification mechanism similar to Irving Fisher's debt deflation, discussed by Mendoza (2010). This effect amplifies the fall in tradable consumption. The second effect occurs on the production side of the economy. As the price of non-tradable goods falls, the wage in units of tradables declines, thus reducing labor supply despite the fact that, as the constraint binds, the marginal utility of supplying one more unit of labor is higher. This second channel, combined with the amplified response of tradable consumption and the relative price of non-tradables, produces a fall in employment and non-tradable production and consumption.

## Figure 2. Decision Rules in the Competitive Equilibrium
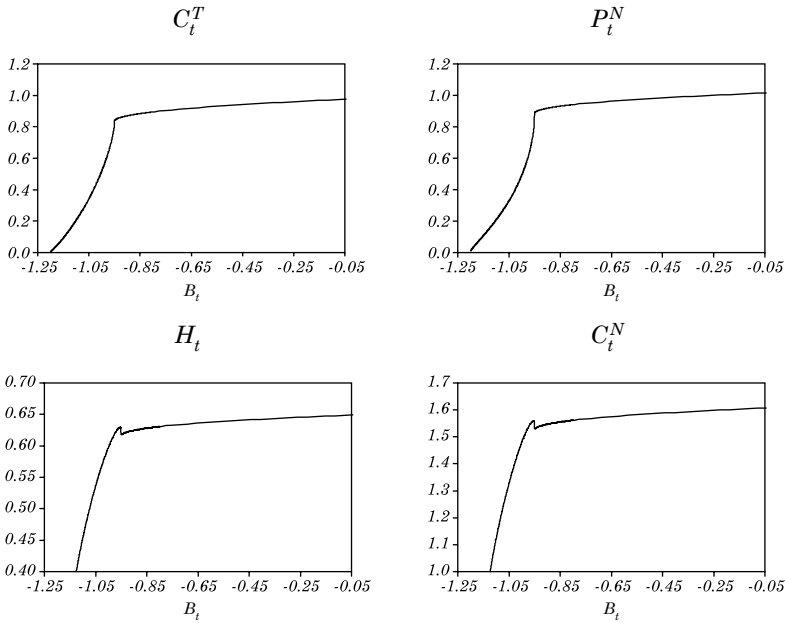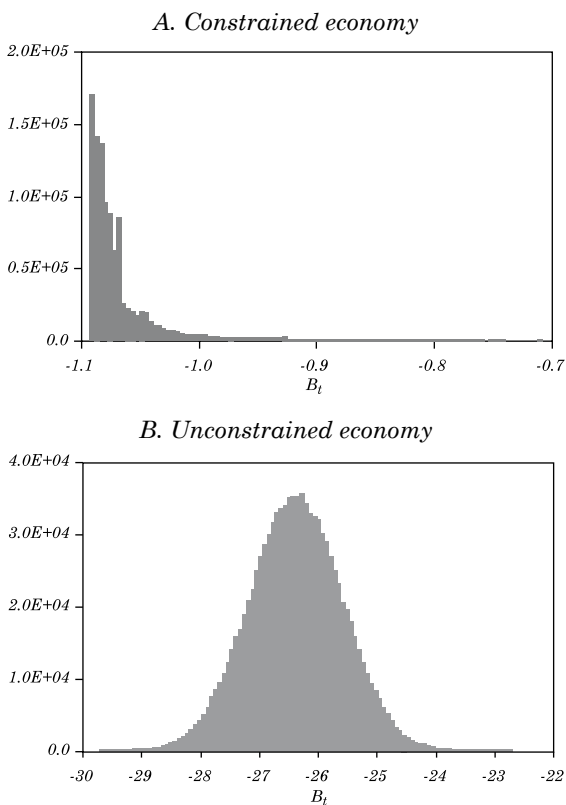


Source: Authors' calculations.

The foreign debt distribution in the stochastic steady state of the model illustrates a more intuitive working of the borrowing constraint. In figure 3, we compare the ergodic distribution of foreign debt for two economies, one with and one without the occasionally binding borrowing constraint.[17] As we can see, the foreign debt distribution of the economy with the constraint is shifted to the far right of the unconstrained economy and is truncated. That is, agents would like to borrow much more than they can in the constrained economy, and are aware of the state-contingent borrowing limit and the possibility of running into a sudden stop because of it. Private agents' precautionary saving motive, then, means that the average amount borrowed is lower than in the unconstrained economy. In the stochastic steady state

17. To compute the ergodic distribution of the unconstrained economy we need a stationary model. To achieve stationarity we use an elastic discount factor in both the constrained and the unconstrained economy. However, the elastic discount factor is not present in the model with the constraint that we use to produce all other results.

of the economy, which averages all possible equilibrium outcomes, there is therefore an endogenous debt limit beyond which agents do not want to go. The ergodic distribution of borrowing will be truncated at that point. Note however that this is not necessarily the point at which the borrowing constraint binds strictly at any particular time or state of the economy.

**Figure 3. Ergodic Distribution for Foreign Borrowing**

*A. Constrained economy*



*B. Unconstrained economy*
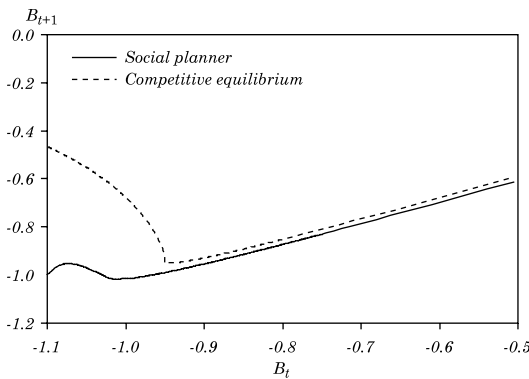


Source: Authors' calculations.

## 4.2 Comparing with the Social Planner Equilibrium

We now compare the allocations in the competitive equilibrium with those chosen by the social planner, under alternative model specifications and parameter assumptions.

### 4.2.1 Production economies

Figure 4 plots the decision rule for $B_t$ for the worst possible state of the exogenous state, $\varepsilon_t$, in our baseline model with endogenous labor supply. It shows that there is slight underborrowing when the constraint is not binding and much more underborrowing when the constraint is binding.[18] This shows that, in the benchmark economy, there is theoretical scope for both ex ante and ex post policy interventions, geared toward inducing more borrowing than private agents choose to take on, both before and after a sudden stop.

**Figure 4. Decision Rule for Foreign Borrowing**
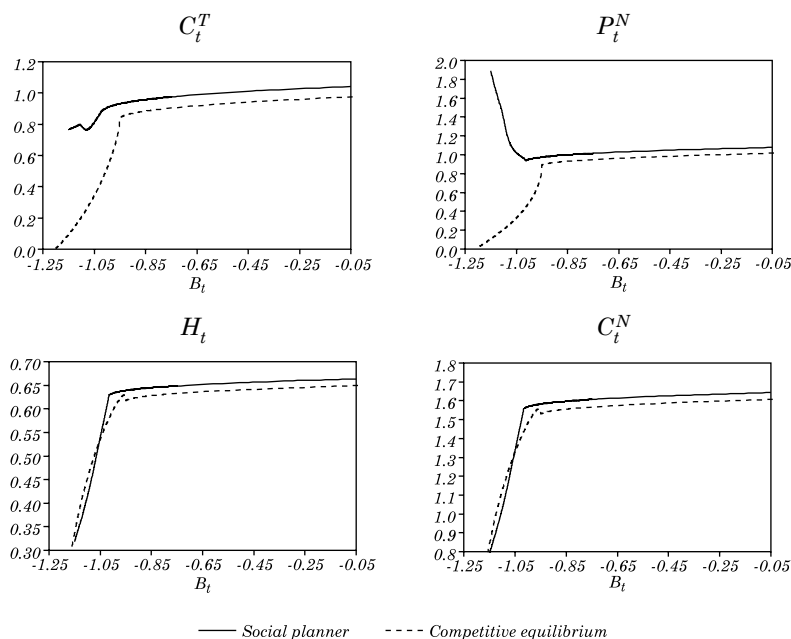


Source: Authors' calculations.

Figure 5 compares the behavior of the other endogenous variables for the worst value of the exogenous state $\varepsilon_t$, as in figure 2. Consistent with the underborrowing presented in figure 4, there is a wedge between the policy functions of the CE allocation and the SP, which is larger when the constraint binds. As we noticed earlier, when the constraint does not bind, two opposite forces are at work in our production economy. On the one hand, the social planner would like to reduce current consumption of tradables, thereby taking into account the amplification effects caused by any price externality that might arise in the future, when the constraint binds. On the other, the increase in the marginal utility of tradables causes an increase

18. That is, for each value of the endogenous state $B_t$, $B_{t+1}$ is smaller in the CE than in the SP throughout the support of the decision rule.

in the marginal utility of non-tradables and in labor supply, with higher non-tradable production and consumption. Under our baseline calibration, this second effect dominates the first one, causing tradable consumption to be higher and saving lower than in the CE allocation. The equilibrium relative price of non-tradables is also higher in the SP than in the CE. A policy intervention geared at moving the CE closer to the SP would therefore have to induce more borrowing in normal times and a more appreciated relative price for non-tradable goods.

When the constraint binds, the differences between the CE and the SP become even more marked. There are two key differences: first, the relative price of non-tradables increases in the SP, collapsing in the CE as the economy goes deeper into debt (see figure 5). Second, in the SP allocation, we see lower labor and non-tradables consumption than in the CE. These differences reflect how agents and the planner react to the constraint in the two equilibria. The planner limits the deflationary impact of meeting

## Figure 5. Decision Rules for Relative Prices, Consumption, and Labor
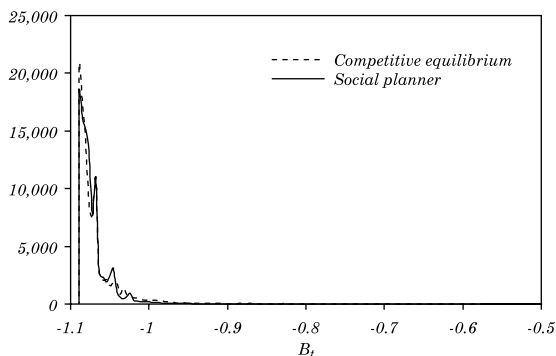


Source: Authors' calculations.

the borrowing constraint by increasing the value of collateral through prices (that is, by increasing $P^N$) rather than quantities (that is, it reduces $Y^N$). As we discussed in section 2, when goods are complements, supplying one less unit of labor generates a relative marginal benefit in the SP and not the CE. The value of collateral is higher in the SP than in the CE because, when goods are complements, the relative price of non-tradables increases and offsets the negative impact of lower non-tradables production and consumption. The overall implications of the planner's allocation is to allow for higher borrowing capacity and, as a consequence, higher tradable consumption, even when the constraint binds. In contrast, in the CE, when the constraint is binding, all else being equal, agents supply more labor to relax the constraint by increasing their non-tradable labor income. However, they don't internalize the effect that higher labor supply has, all else being equal, on the equilibrium relative price. Indeed a lower relative price will tighten the constraint even more and reduce tradable consumption. As a result, tradable consumption falls more and faster than in the SP.

Figure 6 compares the ergodic distributions of borrowing in the CE and the SP allocations. The two post a similar ergodic distribution of debt, despite differences in the decision rules conditional on the worst possible state.[19] Nonetheless, the mean debt-to-GDP ratio of this distribution is slightly lower in the CE than in the SP, as one would expect based on the discussion above. As table 2 reports, the average debt-to-annual-GDP ratio is –10.20 percent in the CE and –10.22 percent in the SP. This difference is very small, but statistically very significant (standard errors not reported).

The probability of having the constraint bind strictly is higher in the SP than in the CE (table 2). It amounts to 2.3 percent per quarters simulated in the SP (9.2 percent per year) and only 2.06 percent in the CE (8.2 percent per year). This difference can be interpreted in terms of precautionary saving behavior, and the decision rules we discussed above illustrate how the latter comes about in our benchmark production economy. The sudden stop is less costly in the SP than in the CE equilibrium, in terms of total consumption in units of tradable goods, with a welfare gain from

19. This is because the decision rules for better states are much closer to each other when the constraint does not bind and the economy spends little time in the worst state.

**Figure 6. Ergodic Distribution for Foreign Borrowing**



Source: Authors' calculations.

**Table 2. Average Foreign Borrowing and the Probability of a Sudden Stop**

| *Variable* | *CE* | *SP* |
|---|---|---|
| Annual average debt in the ergodic distribution (% of annual GDP) | | |
| Production, benchmark parameters | −10.20 | −10.22 |
| Production, alternative parameters | −7.31 | −6.90 |
| Endowment, benchmark parameters | −10.25 | −10.14 |
| Endowment, alternative parameters | −7.40 | −7.10 |
| Quarterly unconditional sudden stop probabilities (% per quarter) | | |
| Production, benchmark parameters | 2.06 | 2.30 |
| Production, alternative parameters | 1.53 | 2.20 |
| Endowment, benchmark parameters | 13.66 | 1.70 |
| Endowment, alternative parameters | 2.36 | 0.23 |

Source: Authors' calculations.

removing the constrained-inefficiency imposed, 0.03 percent of consumption at each state and date (table 3). Agents therefore try to borrow less and to face a sudden stop less frequently in the CE than the SP. Consistent with the small differences in average debt and the probability of sudden stop we reported, the overall welfare

**Table 3. Welfare Gain of Moving from the CE to the SP[a]**

Percent of tradable consumption

| Variable | Overall | At the sudden stop |
|---|---|---|
| Production, benchmark parameters | 0.01 | 0.03 |
| Production, alternative parameters | 0.30 | 0.90 |
| Endowment, benchmark parameters | 0.001 | 0.003 |
| Endowment, alternative parameters | 0.04 | 0.12 |

Source: Authors' calculations.

a. The welfare gains of moving from the competitive equilibrium (CE) to welfare-maximizing social planner (SP) are calculated as the percent of total consumption that agents are willing to forego, at every date and state, to move from one allocation to the other. That is the percentage reduction in consumption at all future dates and states in the SP that equates expected utility in the CE with expected utility in the SP. This cost is calculated at each point on the state space. The "overall" welfare cost is calculated by weighting the cost in each state by the unconditional probability of being in that state. We also construct the welfare gain when near a sudden stop. This calculation is complicated by the fact that the sudden stop does not always occur in the same state. Our solution is to simulate the model for 100,000 periods and keep track of the state(s) in which the economy is in before entering a sudden stop. We then average the gains over these states right before a sudden stop occurs.

gain of moving from the CE to the SP equilibrium is a mere 0.01 percent of consumption at each date and state.[20]

Consider now the same economy under an alternative calibration, in which agents are more impatient (that is, the discount factor is lower, at 0.91) and shocks are less persistent but four times more volatile than in the baseline (that is, $\rho_\varepsilon = 0.54$ and $\sigma_\varepsilon = 0.059$, as for instance in Bianchi, 2009). Figure 7 reports the same decision rules as figure 5, while figure 8 compares the ergodic distributions of $B_t$ in the CE and the SP allocations. As we can see from figure 7, with more impatient agents and more volatile shocks, we now generate overborrowing in the CE equilibrium compared to the SP equilibrium, when the constraint does not bind. Being more impatient, agents' current consumption of tradable goods is higher. Since the marginal utility of current consumption is now smaller than in the previous case, the increase in current consumption (away from the constraint) dominates the negative effect of lower current consumption of tradables induced by the labor margin, so that tradable consumption

20. The intuition for this result is that welfare is state dependent in our economy. The largest differences in the behavior of these economies arise at the sudden stop, which in turn occurs only infrequently. Given that the economy spends most of its time outside the sudden stop state, the overall welfare difference between the two allocations is very small. Indeed, as shown by Mendoza (2002), the second moments of an economy with or without such constraints are quite similar.
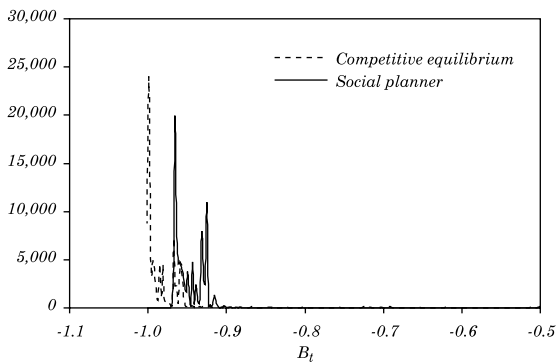
# Figure 7. Decision Rules under the Alternative Calibration[a]
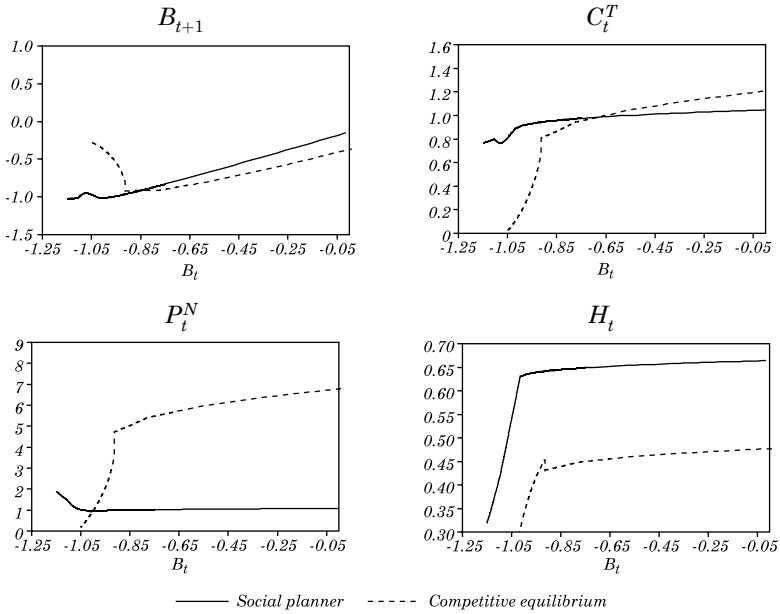


$B_{t+1}$

$C_t^T$

$P_t^N$

$H_t$

——— Social planner    - - - - - Competitive equilibrium

Source: Authors' calculations.
a. The alternative calibration considers more impatient agents and larger shocks.

# Figure 8. Ergodic Distribution for Foreign Borrowing under Alternative Calibration[a]



- - - - - Competitive equilibrium
——— Social planner

Source: Authors' calculations.
a. The alternative calibration considers more impatient agents and larger shocks.

is higher in the CE than the SP allocation. In equilibrium, as goods are complements, we see higher consumption of tradables, higher consumption of non-tradables, and a higher relative price of non-tradables in the SP allocation. In contrast, when the constraint is binding, the decision rules of the CE behave similarly to the benchmark economy, relative to those of the SP.

This economy's behavior thus differs not only quantitatively but also qualitatively with respect to the benchmark economy. The important policy implication is that this alternative economy would require an ex ante policy intervention of opposite sign to that in the benchmark model to close the gap between the CE and the SP. However, when the constraint binds (after the kink in the decision rules), the difference compared to the benchmark calibration is only quantitative. This suggests that the sign of an ex post policy intervention would be the same in the two economies, although the intensity of that intervention might vary because of different parameter values.

As table 2 reports, average debt in the stochastic steady state of the economy with the alternative calibration is smaller than in the benchmark model (despite the higher degree of impatience assumed), and larger in the CE than in the SP (at $-7.31$ and $-6.9$ percent of annual GDP, respectively) because there is overborrowing. Average debt is smaller in both the CE and the SP than in the benchmark economy, because the sudden stop is much more costly (about 30 times more costly in both allocations), with a welfare gain of moving from the CE to the SP at the sudden stop of 0.9 percent of consumption at every date and state (and an overall welfare gain of 0.3 percent). As a result, private agents self-insure more, as compared to the benchmark economy. This also leads to a significantly smaller probability of sudden stop in the CE in this case (1.53 percent per quarter). In contrast, the likelihood of the SP facing sudden stops is about the same as for the benchmark economy (2.2 percent of quarters).
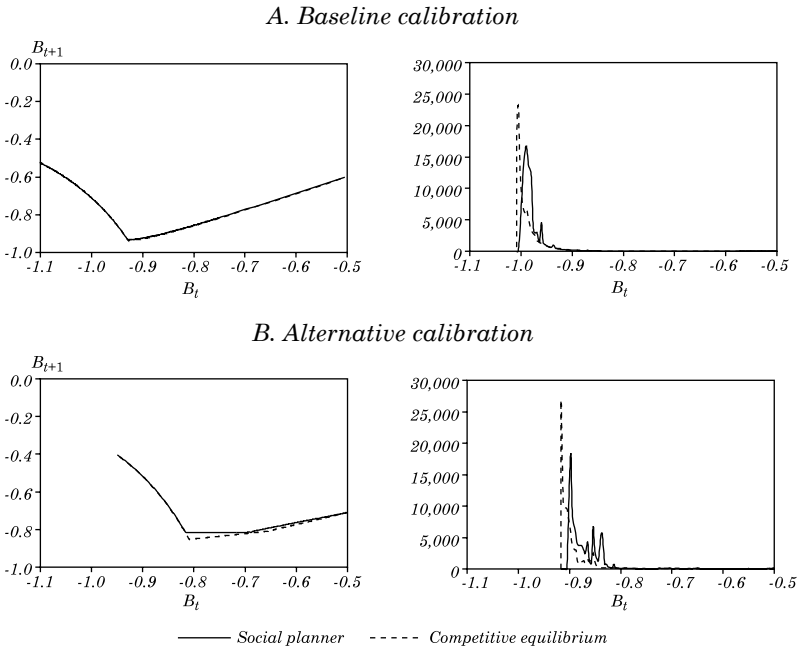
### 4.2.2 Endowment economies

Consider now an endowment economy under the baseline and alternative calibrations for the same two sets of parameter values used for the production economy. The only change compared to the benchmark economy presented in section 3 is that labor supply in the non-tradables sector is now exogenous. Figure 9 compares the

decision rule and the ergodic distribution for foreign borrowing in the CE and the SP for both calibrations. Figure 10 compares the decision rule for borrowing, tradable consumption, and the relative price of non-tradables. As we can see from panel A of figure 9, for the baseline parameter values and the worst realization of the shock, once we shut off the endogenous labor supply, there is essentially no difference in the decision rule for foreign borrowing between the CE and the SP allocations, either before or after the constraint binds. Nonetheless, we can see that in the ergodic distribution of foreign borrowing (which averages over all possible realizations of the shock and points on the support of the decision rules) there is slight overborrowing of about 0.10 percent of annual GDP (with average foreign borrowing reported in table 2 at –10.25 and –10.14 percent of annual GDP in the CE and the SP, respectively). This shows that, in this case, as discussed above, the distortion introduced by the credit constraint in the intertemporal margin leads households to undervalue the current marginal utility of tradable consumption for more favorable realizations of the exogenous state. The distortion, however, leads to a very small difference between the private and socially efficient level of foreign borrowing for the baseline parameter values.

Interestingly, the probabilities of sudden stops are 13.0 percent in the CE and 1.7 percent per quarter in the SP. In the CE, the probability of sudden stop is much higher in the endowment economy than in the production economy. This is because households cannot rely on the labor margin to supply more collateral when the constraint binds or is expected to bind in the future, despite facing the same incentive to borrow. As a result average borrowing is slightly higher as a share of total income and the probability of a sudden stop is much higher in the endowment than in the production economy. In contrast, in the social planner allocation for an endowment economy, in which there is no margin on which to act once the sudden stop is reached, there is less borrowing than in the production economy and a significantly lower probability of reaching the sudden stop, both with respect to the CE equilibrium of the endowment economy and the SP equilibrium of the production economy. Note here that the sudden stop is more costly for the SP of the endowment economy than the SP of the production economy, as tradable consumption falls by about 40 percent and 25 percent respectively (figure 5 and figure 10, panel A). However, the sudden stop cost is about the same in the CE and the SP equilibrium of the endowment economy, because the SP cannot improve on the CE when the constraint binds in the endowment economy. Consistent
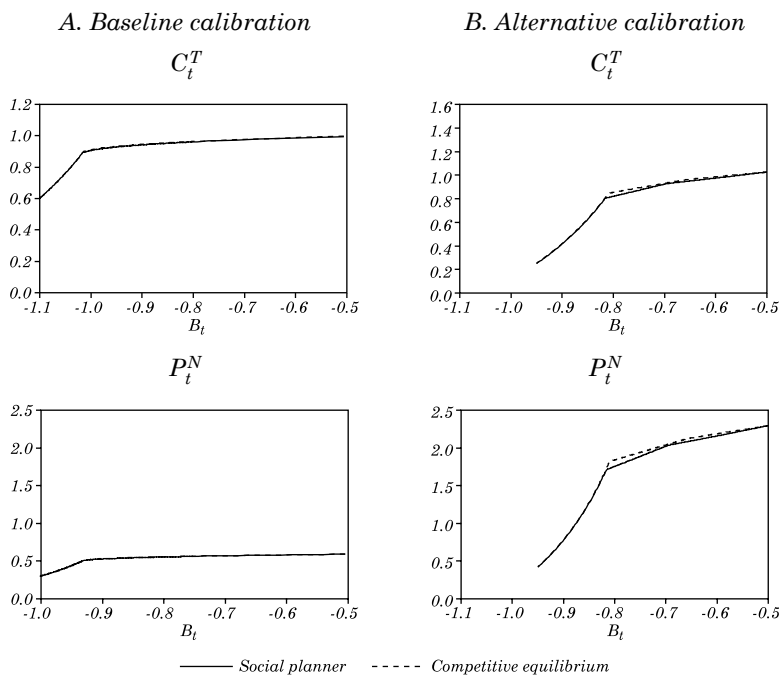
**Figure 9. Endowment Economies under Different Calibrations**

*A. Baseline calibration*



*B. Alternative calibration*



———— *Social planner*    - - - - - *Competitive equilibrium*

Source: Authors' calculations.

with this observation, the welfare gains of moving from the CE to the SP in this endowment economy, either overall or at the sudden stop, are one order of magnitude smaller than in the production economies above, at only 0.001 percent and 0.003 percent of consumption at each date and state, respectively (see table 3).

In an endowment economy with more impatient agents and larger shocks, there is more overborrowing than in the endowment economy with the baseline calibration, but precautionary saving is higher in both the CE and the SP equilibrium. Overborrowing, as measured by the difference in the average ergodic distribution of foreign borrowing, is about 0.30 percent of annual GDP, with average foreign borrowing of –7.40 and –7.10 in the CE and the SP, respectively (table 2). This is also evident from panel B of figure 9, which shows that the decision rule for $B_t$, conditional on the worst possible state, displays clearer evidence of overborrowing in the intermediate region of the state space.

**Figure 10. Endowment Economies under Different Calibrations**



*A. Baseline calibration*

$C_t^T$

*B. Alternative calibration*

$C_t^T$

$P_t^N$

$P_t^N$

—— *Social planner*    - - - - - *Competitive equilibrium*

Source: Authors' calculations.

Because of higher precautionary saving, the probabilities of sudden stops are also much smaller than in the endowment economy with base calibration (at 2.36 and 0.23 percent per quarter in the CE and the SP, compared to 13.66 and 1.70, respectively). The differences in the probability of a sudden stop across calibrations and the higher precautionary saving in this economy are associated with much more costly sudden stop dynamics in the alternative calibration than in the baseline. As we can see from figure 10, panel B, in fact, tradable consumption falls by about 75 percent with the alternative calibration compared to about 25 percent in the baseline one. A much higher cost of sudden stop leads to a large (overall and at the sudden stop) welfare gain of moving from the CE to the SP equilibrium in this economy, despite the fact that the planner cannot ameliorate the CE allocation at the sudden stop,

at 0.04 and 0.12 percent of consumption at all dates and states, respectively (table 3). The planner's incentive to curtail borrowing is particularly strong in this case.

## 5. POLICY IMPLICATIONS

The quantitative analysis in the previous section has important policy implications. The recent literature, reviewed in section 1, has focused on the theoretical and quantitative possibility of overborrowing, unambiguously recommending ex ante interventions to curtail it, such as a Tobin tax or other economy-wide prudential controls on international capital inflows.

While consistent with a theoretically second-best view of the world, in practice this clear-cut policy prescription warrants several qualifications. First, it is not possible to analyze the relative merits of both ex ante and ex post intervention strategies in models in which the planner can only intervene ex ante. In an endowment economy, by construction there is no scope for ex post policy interventions. As tradable consumption is pinned down by the constraint when this binds in an endowment economy, neither private agents nor the planner can manipulate the collateral value of non-tradable income to relax the borrowing constraint, and thus seek a better allocation.

Second, overborrowing is clearly a quantitative matter, and there is no solid basis to conclude that it is a key and general feature of emerging economies. As we saw in the previous section's quantitative analysis, simply by introducing small changes in key parameter values that are not easily anchored to the data in simple models, we find slight underborrowing instead of overborrowing in production economies. It follows that both sets of policy instruments should be implemented to "hedge" the model and parameter uncertainty that policy makers face.

By the well established standards of the dynamic stochastic general equilibrium (DSGE) methodology, such lack of robustness is sufficient to require a more cautious approach to economy-wide prudential controls on capital inflows, especially in light of the (at best mixed) historical experience with such policy tools.[21] DSGE standards indicate that the pros and cons of alternative policy

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21. See Ostry and others (2010) for a thorough review of the existing literature, as well as new empirical evidence on the effectiveness of economy-wide capital controls.

regimes should be evaluated quantitatively in models that fit the data well, as is now the case for traditional monetary and fiscal stabilization policy issues. But rich models with occasionally binding financial frictions are not as amenable to quantitative analysis as the canonical New Keynesian model that has been investigated in the monetary policy literature.

We must, therefore, recognize that these models are in their infancy and do not yet provide clear-cut policy recommendations. The important implication is that economy-wide capital controls alone, as recommended in the literature (and as recently implemented by Brazil), may not achieve constrained efficiency in more richly specified and parameterized economies.

Third, such interventions are distortionary and may hamper economic efficiency if imposed inappropriately. As Nikolov (2009) has pointed out, for instance, in this kind of model environment there is a trade-off between the higher volatility associated with mandating looser prudential controls (that is, a higher leverage ratio in his model) and the lower average growth associated with imposing tighter prudential controls (that is, lower leverage ratios in his model). So mandating lower, economy-wide regulatory leverage ratios on prudential grounds may impose significant efficiency costs in terms of lower average growth.[22] This point is largely absent from the current debate, in part because it is difficult to evaluate such a trade-off quantitatively in the models available. Nonetheless, Nikolov's (2009) analysis clearly highlights the risk involved, consistent with the traditional debate in the literature on capital controls reviewed by Ostry and others (2010).[23]

Fourth, even when ex ante economy-wide interventions reflect the appropriate economy-wide policy regime from a second-best welfare perspective, they do not eliminate sudden stops and financial crises completely; they only mitigate their severity and may reduce their likelihood, as our analysis highlights. Thus, even with prudential policies in place, we still need to design policies that can respond to

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22. Note however that this does not mean that specific sectors of the economy, such as the domestic financial system, would not benefit from such policy interventions.

23. As we noted already, this limitation does not apply to the policy analysis by Benigno and others (2009), in which the Ramsey planner explicitly trades off the benefits of intervening either ex ante or ex post with the efficiency costs of doing so using a distortionary tax on non-tradable consumption. In contrast, the existing literature only discusses implementation issues without accounting for implementation costs, when comparing competitive allocations with socially planned ones.

sudden stops in financial flows, as Caballero (2010) stresses. Our analysis of the two production economies, in which there is a wedge between the CE and the SP allocations both before and after the constraint binds, brings this out clearly.

Nonetheless, there are no moral hazard or time-consistency concerns in our setup. For instance, moral hazard considerations might surface in a microfounded specification of our constraint. Once moral hazard of ex post policies is considered, ex ante policies may become more desirable. Similarly, time-inconsistency problems are absent from these models. As Chari and Kehoe (2009) illustrate, the time-inconsistency of optimal ex post interventions may also call for ex ante interventions. The rationales for ex ante intervention policies would be different, however, addressing the need to avoid moral hazard and the time-inconsistency of ex post intervention policies, as opposed to correcting inefficient borrowing, as discussed in this paper.

## 6. CONCLUSIONS

The recent theoretical literature suggests that an economy-wide, macroprudential tax on leveraged borrowing might reduce the probability of a financial crisis and limit the ensuing adverse effects if one eventually occurs. These conclusions are based on the notion that agents do not save enough in tranquil times as a precaution against a possible crisis and hence overborrow. In our analysis in this paper we have shown that these policy conclusions are not robust. We examine production and endowment economies in which the pecuniary externality on which the literature has focused is present and creates the scope for policy intervention. While in endowment economies there is always overborrowing and there is no scope for policy intervention in crisis times, our baseline production economy displays underborrowing and a much larger welfare gain from ex post rather than ex ante policy intervention.

There are two important caveats to our analysis. First, the comparisons between the social planner and competitive equilibriums do not take into account the efficiency costs associated with any potentially distortionary policy tools needed to implement the social planner allocation. This suggests that the Ramsey allocation (which takes these costs into account) could differ from the social planner version. Second, the analysis in this paper and the relevant literature has neglected an important aspect of policy

design: the fact that there is an important link between ex ante and ex post policies. Full knowledge of ex post policies may influence agents' behavior in normal times, and hence modify the ex ante policy design as well. In a companion paper (Benigno and others, 2009) we look at both these important aspects using a framework similar to the one in this paper.

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