Modeling a Housing and Mortgage Crisis

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The current crisis has centered on borrower defaults on mortgages and the associated effects on banks' own credit standing (and in several cases their own default), which in turn led to tightened conditions for lending to new (mortgage) borrowers. Any model that does not incorporate all or most of these key elements cannot possibly hope to capture the defining features of the current crisis. This is particularly true of standard dynamic stochastic general equilibrium (DSGE) models, which (mostly) assume away the possibility of default altogether!

This paper builds on our previous model of a system in which default plays a central role for both borrowers and banks and in which financial intermediation and money thus have a necessary real function. Specifically, we include both an additional good, housing, in the prior composite basket of goods and services and an additional agent, a new entrant to the housing market. Our previous papers on this include Goodhart, Sunirand, and Tsomocos (2004, 2005, 2006).

Dealing with a model with default and heterogeneous agents is not straightforward, which is why standard DSGE models abstract from such concerns despite their resulting lack of realistic micro-foundations. We therefore regard this paper as a preliminary step in a longer exercise. In particular, the shocks that we model in the second period of our two-period model (the first being an initial predetermined

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set-up period) can be categorized as supply shocks, in which the agents' endowment declines greatly in the case of adverse shocks. Nevertheless, our model is general enough to allow for the examination of a wide variety of shocks that can lead to financial instability.

In practice, the main adverse shock in 2007–08 was a sharp decline in housing prices in the United States, whereas previously they had been expected to continue rising or, at worst, to hold steady. In a future version of this paper, we will experiment with this and other financial shocks. The main reason for proceeding with the current model is simply shortage of time. This kind of simulation model necessarily involves learning by doing, so we started with the shocks that we had used in our prior work. But Western economies were, in fact, facing adverse supply shocks in 2007–08, in the guise of rising energy and commodity prices, and these played a role in worsening the current downturn. Furthermore, our simulations include examples of changes in financial conditions, such as changes in the money stock (interest rates) and in bank capital endowments, so we can potentially explore how financial policy measures (including government recapitalization of banks) may affect the outcome. Nevertheless, this should be treated as a preliminary exercise.

The plan of the paper is as follows. In section 1, we briefly reprise the basic structure of the Goodhart-Sunirand-Tsomocos (GST) model and detail the innovations that we have made here. Section 2 sets out how the model works and its clearing conditions. In section 3, we report our choice of exogenous parameters for our numerical simulation and describe the resulting equilibrium values. In section 4, we report on the comparative statics of changes in the parameters chosen, and section 5 concludes.

With such a large, and alas complicated, model, there are a vast number of exercises that could be run, each with an accompanying set of tables and diagrams. In one sense this is a strength of the approach, since it supports the examination of a huge variety of potential shocks and policy responses, both individually and in conjunction. At the same time, however, it can lead to a mind-boggling multiplication of detail. In pursuit of focus and comparative simplicity, we focus here on just four examples: a decrease in the money supply in the initial period; an increased desire to take on risk (as occurred in 2003–06), which leads to adverse shocks having a stronger effect on the system; a (foreseen) intervention by the authorities to provide liquidity assistance in very bad states; and a combination of the former two simultaneously, which allows us to examine the extent to which the resulting effects are nonlinear.

1. The Background Setup

Goodhart, Sunirand, and Tsomocos (2004, 2006) and Tsomocos (2003) develop models of financial stability that are rich enough to include defaultable consumer loan, deposit, and interbank markets. In the models, consumers maximize their expected utility from consumption of goods, and banks maximize their expected profits. The main financial imperfection in Goodhart, Sunirand, and Tsomocos (2004, 2006) is that they assume that individual bank borrowers are assigned, by history or by informational constraints, to borrow from a single bank. Money is introduced by a cash-in-advance constraint, whereby a private agent needs money to buy commodities from other agents; commodities cannot be used to buy commodities. Similarly, they assume that agents needing money can always borrow cheaper from their (assigned) bank than from other agents; banks have an informational (and perhaps scale) advantage that gives them a role as an intermediary. The amount of loans they repay is a choice variable for consumers, so default in these models is endogenous.

The general model (Goodhart, Sunirand, and Tsomocos, 2006) features a set of heterogeneous private sector agents with initial endowments of both money and commodities; it is an endowment model without production. There is also a set of heterogeneous banks, who similarly have differing initial allocations of capital (in the form of government bonds). There are two other players, a central bank that can inject extra money into the system through open market operations (OMOs), and a financial supervisory agency, which can set minimum liquidity and capital requirements and imposes penalties on failures to meet such requirements and on defaults.

The main purpose of this paper is to model the market for mortgages and examine the implications of default in bank lending and of a housing market crisis. To do so, we alter the above framework in the following ways.

—First, we introduce another good into the economy, which is durable and gives utility in every period. The utility of consuming this good resembles the utility from buying a house. For tractability, the durable good (house) is assumed to be infinitely divisible.

—Second, we explicitly model a market for mortgages. Consumers enter a mortgage contract to buy housing, which they pledge as collateral. They default on their mortgage when the endogenous value of collateral is less than the amount they have to repay (Geanakoplos, 2003; Geanakoplos and Zame, 1997). When they default, the bank seizes the amount of housing pledged as collateral and immediately offers it in the next period housing market. In this sense default is highly discontinuous, as consumers do not choose the exact amount they want to default (as in the model discussed above), but only decide on whether to default.[1]

—Third, we introduce a new agent, λ, who is only born in period two. The motivation behind this is that the healthy functioning of the housing market generally depends on the existence of first-time buyers.

—Fourth, we allow for short-term loan markets operating within each period. This was not necessary in the Goodhart, Sunirand, and Tsomocos models, but in our analysis it plays a crucial role in providing credit to first-time buyers, namely, Mr. λ. For consistency, all agents can borrow short-term. In this market, there is no uncertainty regarding repayment. The central bank intervenes in the short-term loan markets in the second period to keep the interest rates (in the good state) at reasonable levels.

—Finally, since we are not considering wider asset markets, we exclude capital requirements for banks from our analysis.

2. The Model

Given the limited participation in the loan markets in our model, we need at least four agent-households (α, β, ϕ, λ) and two commercial banks (γ, δ). There are two periods and S states of the world. All agents maximize their utility over the consumption of the good and of housing in every period $t \in T = (0, 1)$ and state $s \in S$. Banks maximize their expected profits in the second period. The set of all states is given by $s \in S^* = \{0\} \cup S$.

Agent $h \in (\alpha, \beta)$ is endowed with the good at every $s \in S^*$, whereas agent λ is endowed at every $s \in S$, since he enters the economy only in the second period. Agent ϕ is endowed with houses only at $t = 0$. Agents α and ϕ interact with bank γ, while agents β and λ are associated with bank δ. All households are also given government cash free and clear of any obligations ($m_{s^*} \geq 0$). Both endowments and cash are allowed to vary across states of nature.

The central bank acts in the interbank market at $t = 0$ by providing liquidity M^{CB} and in the short-term loan markets at $t = 1$

1. Shubik and Wilson (1977) and Dubey, Geanakoplos, and Shubik (2005) analyze continuous default.

by providing liquidity $M^{CB}_{\gamma s}$ and $M^{CB}_{\delta s}$ in the markets organized by banks γ and δ, respectively.

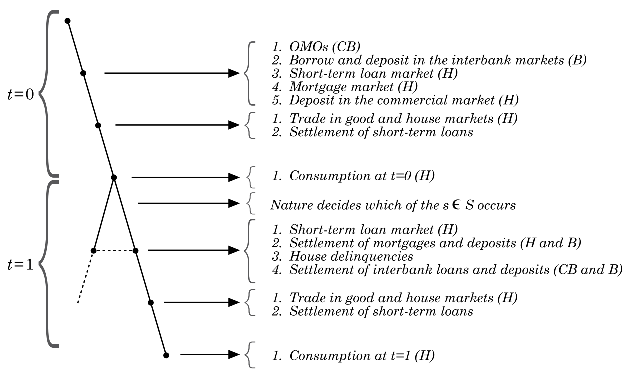
In the following subsections, we give the timeline of our model and specify the optimization problems for all the participants in the economy.

2.1 The Time Structure of Markets

In each period $t \in T$, six markets meet: the short-term (intraperiod) loan, mortgage, deposit, and interbank (intertemporal) markets meet simultaneously, and then the good and housing markets meet. Short-term loans come due at the end of the period. This setup maximizes the number of transactions possible and allows agents to borrow in the short-term money market in order to invest in the long-term bond or asset market. It also allows for an explicit speculative motive for holding money. Agents have the option of investing cash in the short loan market and then carrying it over to the next period. The only reason they may not do this is that they believe they will get a higher return from holding deposits. This not only preserves Keynesian motives on the uses of money, but also provides a rationale for an upward-sloping term structure.

Figure 1 indicates our time line, including the moments at which the various loans and assets come due. We make the sequence precise when we formally describe the budget set.

Figure 1. Time Line[a]



Source: Authors' drawing.
a. CB: central bank; B: commercial banks; H: households.

2.2 Household α's and β's Optimization Problem

Each consumer $h \in \{\alpha, \beta\}$ maximizes his payoff, which is his utility from consumption of the good and the house.[2] In order to acquire housing he enters a mortgage contract, which he has to repay in the last period. The amount of housing that he purchases is pledged as collateral. He honors his mortgage when the value of the housing that he has bought is greater than the amount he has to repay. If it is lower, then he defaults on his mortgage and the bank that extended the mortgage seizes the collateral. In essence, he repays the minimum between the two values, that is, min(value of collateral, mortgage amount). We denote by $S_1^h \subset S$ the set of states that agent h does not default on his mortgage, that is, S_1^h: value of collateral $\geq$ mortgage amount}. The maximization problem is as follows:

$$\max_{q_{s1}^h, b_{s2}^h, \mu_s^h, \bar{\mu}^h} \Pi^h = u(e_{01}^h - q_{01}^h) + u\left(\frac{b_{02}^h}{p_{02}}\right) + \sum_{s \in S} \theta_s u(e_{s1}^h - q_{s1}^h)$$
$$+ \sum_{s \in S^1} \theta_s u\left(\frac{b_{02}^h}{p_{02}} + \frac{b_{s2}^h}{p_{s2}}\right) + \sum_{s \notin S^1} \theta_s u\left(\frac{b_{s2}^h}{p_{s2}}\right)$$

subject to

$$b_{02}^h \leq \frac{\mu_0^h}{1 + r_0^k} + \frac{\bar{\mu}^h}{1 + \bar{r}^k} + m_0^h \tag{1}$$

(that is, the expenditure for housing at $t = 0 \leq$ amount borrowed short-term at $t = 0$ + mortgage amount + initial private monetary endowment);

$$\mu_0^h \leq p_{01} q_{01}^h \tag{2}$$

(that is, short-term loan repayment $\leq$ good sales at $t = 0$);

$$b_{s2}^h + \bar{\mu}^h \leq \frac{\mu_s^h}{1 + r_s^k} + m_s^h, \ \forall s \in S_1^h \tag{3}$$

(that is, expenditure for housing in the second period, state $s \in S_1^h$ + mortgage repayment $\leq$ amount borrowed short-term + private monetary endowment in $s \in S_1^h$);

2. In our simulations, we use a constant relative risk aversion (CRRA) utility function to account for wealth effects.

$$\mu_s^h \le p_{s1} q_{s1}^h, \ \forall\, s \in S_1^h \tag{4}$$

(that is, short-term loan repayment $\le$ good sales in $s \in S_1^h$);

$$b_{s2}^h \le \frac{\mu_s^h}{1 + r_s^k} + m_s^h, \ \forall\, s \notin S_1^h \tag{5}$$

(that is, expenditure for housing in the second period, state $s \notin S_1^h \le$ amount borrowed short term + private monetary endowment in $s \notin S_1^h$);

$$\mu_s^h \le p_{s1} q_{s1}^h, \ \forall\, s \notin S_1^h \tag{6}$$

(that is, short-term loan repayment $\le$ good sales in $s \notin S_1^h$); and

$$q_{s1}^h \le e_{s1}^h, \ \forall\, s \in S^* \tag{7}$$

(that is, quantity of goods sold in $s \le$ endowment of goods in s);

where

$k = \gamma$ for $h = \alpha$ and $k = \delta$ for $h = \beta$;
$b_{s2}^h \equiv$ amount of fiat money spent by $h \in H$ to trade in the housing market in $s \in S^*$;
$q_{s2}^h \equiv$ amount of goods offered for sale by $h \in H$ in $s \in S^*$;
$\bar{\mu}^h \equiv$ mortgage amount that $h \in H$ takes out;
$\mu_s^h \equiv$ short-term borrowing by $h \in H$ in $s \in S^*$;
$\bar{r}^k \equiv$ mortgage rate offered by bank k;
$r_s^k \equiv$ short-term rate offered by bank k in $s \in S^*$;
$p_{s1} \equiv$ price of the good in $s \in S^*$;
$p_{s2} \equiv$ price of housing in $s \in S^*$;
$e_{s1}^h \equiv$ endowment of goods of $h \in H$ in $s \in S^*$; and
$m_s^h \equiv$ monetary endowment of $h \in H$ in $s \in S^*$.

2.3 Household ϕ's Optimization Problem

Agent ϕ is endowed with housing at $t = 0$, some (much) of which he sells to buy goods for consumption. He then deposits interperiod a part of the sales receipts for use in the second period. His maximization problem is as follows:

$$\max_{q^{\phi}_{s2}, b^{\phi}_{s1}, \bar{d}^{\phi}, \mu^{\phi}_{s}} \Pi^{\phi} = u\left(\frac{b^{\phi}_{01}}{p_{01}}\right) + u\,(e^{\phi}_{02} - q^{\phi}_{02}) + \sum_{s\in S}\theta_s u\left(\frac{b^{\phi}_{s1}}{p_{s1}}\right) + \sum_{s}\theta_s u\,(e^{\phi}_{02} - q^{\phi}_{02} - q^{\phi}_{s2})$$

subject to

$$b^{\phi}_{01} + \bar{d}^{\phi} \le \frac{\mu^{\phi}_{0}}{1+r^{\gamma}_{0}} + m^{\phi}_{0} \tag{8}$$

(that is, expenditure for goods + interperiod deposits $\le$ amount borrowed short-term + private monetary endowment at $t = 0$);

$$\mu^{\phi}_{0} \le p_{02}q^{\phi}_{02} \tag{9}$$

(that is, short-term loan repayment $\le$ housing sales at $t = 0$);

$$q^{\phi}_{02} \le e^{\phi}_{02} \tag{10}$$

(that is, quantity of housing sold at $t = 0 \le$ endowment of housing at $t = 0$);

$$b^{\phi}_{s1} \le \frac{\mu^{\phi}_{s}}{1+r^{\gamma}_{s}} + \bar{d}^{\phi}(1+\bar{r}_d) + m^{\phi}_{s} \quad \forall\, s \in S \tag{11}$$

(that is, expenditure of goods $\le$ amount borrowed short term + deposits and interest payment + private monetary endowment in s);

$$\mu^{\phi}_{s} \le p_{s2}q^{\phi}_{s2} \quad \forall\, s \in S \tag{12}$$

(that is, short-term loan repayment $\le$ housing sales in s); and

$$q^{\phi}_{s2} \le e^{\phi}_{02} - q^{\phi}_{02} \quad \forall\, s \in S \tag{13}$$

(that is, quantity of housing sold in $s \le$ endowment of housing at $t = 0$ – quantity of housing sold $t = 0$);

where

$b^{\phi}_{s1} \equiv$ amount of fiat money spent by ϕ to trade in the goods market in $s \in S^{*}$;

q^{ϕ}_{s2} ≡ amount of housing offered for sale by ϕ in $s \in S^*$;
$\bar{d}^{\phi}$ ≡ deposit amount for ϕ;
μ^{ϕ}_{s} ≡ short-term borrowing by ϕ in $s \in S^*$;
$\bar{r}_d$ ≡ deposit rate;
r^{γ}_{s} ≡ short-term rate offered by bank γ in $s \in S^*$;
e^{ϕ}_{02} ≡ endowment of housing of ϕ at $t = 0$; and
m^{ϕ}_{s} ≡ monetary endowment of ϕ in $s \in S^*$.

2.4 Household λ's Optimization Problem

Agent λ enters the economy in the second period and is endowed with goods. His maximization problem is as follows:

$$\max_{q^{\lambda}_{s1}, b^{\lambda}_{s2}, \mu^{\lambda}_{s}} \Pi^{\lambda} = \sum_{s \in S} \theta_s u\,(e^{\lambda}_{s1} - q^{\lambda}_{s1}) + \sum_{s \in S} \theta_s u\left(\frac{b^{\lambda}_{s2}}{p_{s2}}\right)$$

subject to

$$b^{\lambda}_{s2} \leq \frac{\mu^{\lambda}_{s}}{1 + r^{\delta}_{s}} + m^{\lambda}_{s} \quad \forall\, s \in S \tag{14}$$

(that is, expenditure for housing ≤ amount borrowed short-term + private monetary endowment in s);

$$\mu^{\lambda}_{s} \leq p_{s1} q^{\lambda}_{s1} \quad \forall\, s \in S \tag{15}$$

(that is, short-term loan repayment ≤ good sales in s); and

$$q^{h}_{s1} \leq e^{h}_{s1} \quad \forall\, s \in S \tag{16}$$

(that is, quantity of goods sold in s ≤ endowment of goods in s);

where

b^{λ}_{s2} ≡ amount of fiat money spent by λ to trade in the housing market in $s \in S$;
q^{λ}_{s1} ≡ amount of goods offered for sale by λ in $s \in S$;
μ^{λ}_{s} ≡ short-term borrowing by λ in $s \in S$;
r^{δ}_{s} ≡ short-term rate offered by bank δ in $s \in S$;
e^{λ}_{s1} ≡ endowment of goods of λ in $s \in S$; and
m^{λ}_{s} ≡ monetary endowment of λ in $s \in S$;

2.5 Bank γ's Optimization Problem

Bank γ (as also bank δ) maximizes its expected profits in the second period. In the first period, it borrows from the interbank market, since it is relatively poor in initial capital, and extends credit in the short-term loan and mortgage markets. It also receives deposits from ϕ. In the second period, it receives the repayment on the mortgage it extended (full repayment for $s \in S_1^\alpha$ and partial repayment otherwise, since the value of the collateral is less than the amount of the mortgage), repays its interbank and deposit borrowing, and extends short-term credit. Its maximization problem is as follows:[3]

$$\max_{\pi_s^\gamma, m_s^\gamma, \bar{m}^\gamma, \mu_I^\gamma, \bar{\mu}_d^\gamma} \Pi^\gamma = \sum_{s \in S} \theta_s \left[\pi_s^\gamma - c^\gamma \left(\pi_s^\gamma\right)^2 \right]$$

subject to

$$m_0^\gamma + \bar{m}^\gamma \leq \frac{\mu_I^\gamma}{1+\rho} + \frac{\bar{\mu}_d^\gamma}{1+\bar{r}_\mathrm{d}} + e_0^\gamma \tag{17}$$

(that is, short-term lending + mortgage extension $\leq$ interbank loans + consumer deposits + initial capital endowment at $t = 0$);

$$m_s^\gamma + \bar{\mu}_d^\gamma + \mu_I^\gamma \leq \bar{m}^\gamma (1 + \bar{r}_s^\gamma) + m_0^\gamma (1 + r_0^\gamma) + e_s^\gamma \quad \forall\, s \in S \tag{18}$$

(that is, short-term lending + deposit repayment + interbank loan repayment $\leq$ effective mortgage repayment + first period short-term loan repayment + capital endowment in $s \in S$); and

$$\pi_s^\gamma = m_s^\gamma (1 + r_s^\gamma) \ \forall\, s \in S \tag{19}$$

(that is, profits = short-term loans repayment $s \in S$);

where
$\pi_s^\gamma \equiv$ bank γ's profits at state $s \in S$;

3. Banks' risk aversion is captured via a quadratic objective function, as in essence they are facing a portfolio problem and we want to capture diversification effects as closely as possible.

$\bar{m}^{\gamma} \equiv$ mortgage extension by bank γ;
$m_s^{\gamma} \equiv$ short-term loan extension by bank γ at state $s \in S^*$;
$\mu_I^{\gamma} \equiv$ interbank borrowing by bank γ;
$\bar{\mu}_d^{\gamma} \equiv$ amount borrowed from consumers in the form of deposits by bank γ;
$\bar{r}_s^{\gamma} \equiv$ effective repayment rate on the mortgage at state $s \in S$;
$r_s^{\gamma} \equiv$ short-term rate offered by bank γ in $s \in S^*$;
$\rho \equiv$ interbank rate; and
$e_s^{\gamma} \equiv$ capital endowment of bank γ at state $s \in S^*$.

2.6 Bank δ's Optimization Problem

Bank δ maximizes its expected profits in the second period. In the first period, it deposits in the interbank market, since it is relatively rich in initial capital, and extends credit in the short-term loan and mortgage markets. In the second period, it receives the repayment on the mortgage it extended (full repayment for $s \in S_1^{\beta}$ and partial repayment otherwise, since the value of the collateral is less than the amount of the mortgage), receives payment from depositing in the interbank market, and extends short-term credit. Its maximization problem is as follows:

$$\max_{\pi_s^{\delta}, m_s^{\delta}, \bar{m}^{\delta}, d_I^{\delta}} \Pi^{\delta} = \sum_{s \in S} \theta_s \left[\pi_s^{\delta} - c^{\delta} \left(\pi_s^{\delta} \right)^2 \right]$$

subject to

$$m_0^{\delta} + \bar{m}^{\delta} + d_I^{\delta} \leq e_0^{\delta} \tag{20}$$

(that is, short-term lending + mortgage extension + interbank deposits $\leq$ initial capital endowment at $t = 0$);

$$m_s^{\delta} \leq \bar{m}^{\delta}(1 + \bar{r}_s^{\delta}) + m_0^{\delta}(1 + r_0^{\delta}) + d_I^{\delta}(1 + \rho) + e_s^{\delta} \quad \forall\, s \in S \tag{21}$$

(that is, short-term lending $\leq$ effective mortgage repayment + first period short-term loan repayment + interbank deposits and interest payment + capital endowment in $s \in S$); and

$$\pi_s^{\delta} = m_s^{\delta}(1 + r_s^{\delta}) \quad \forall\, s \in S \tag{22}$$

(that is, profits = short-term loans repayment $s \in S$);

where

$\pi_s^\delta \equiv$ bank δ's profits at state $s \in S$;
$\bar{m}^\delta \equiv$ mortgage extension by bank δ;
$m_s^\delta \equiv$ short-term loan extension by bank δ at state $s \in S^*$;
$d_I^\delta \equiv$ interbank deposits by bank δ;
$\bar{r}_s^\delta \equiv$ effective repayment rate on the mortgage at state $s \in S$;
$r_s^\delta \equiv$ short-term rate offered by bank δ in $s \in S^*$; and
$e_s^\delta \equiv$ capital endowment of bank δ at state $s \in S^*$.

2.7 Market Clearing Conditions

There are six market categories in our model (namely, goods, housing, mortgage, short-term loan, consumer deposit, and interbank markets). Each of these markets determines a price that equilibrates demand and supply in equilibrium.

2.7.1 The goods market

The goods market clears when the amount of money offered for goods is exchanged for the quantity of goods offered for sale:

$$p_{01} = \frac{b_{01}^\phi}{q_{01}^\alpha + q_{01}^\beta}; \tag{23}$$

$$p_{s1} = \frac{b_{s1}^\phi}{q_{s1}^\alpha + q_{s1}^\beta + q_{s1}^\lambda} \quad \forall\, s \in S. \tag{24}$$

2.7.2 The housing market

The housing market clears when the amount of money offered for housing is exchanged for the quantity of housing offered for sale:

$$p_{02} = \frac{b_{02}^\alpha + b_{02}^\beta}{q_{02}^\phi}; \tag{25}$$

$$p_{s2} = \frac{b_{s2}^\alpha + b_{s2}^\beta + b_{s2}^\lambda}{q_{s2}^\phi} \quad \text{for } s \in S_1^\alpha \cap S_1^\beta; \tag{26}$$

$$p_{s2} = \frac{b_{s2}^{\alpha} + b_{s2}^{\beta} + b_{s2}^{\lambda}}{q_{s2}^{\phi} + (b_{02}^{\alpha} / p_{02})} \quad \text{for } s \in S_1^{\beta} \setminus S_1^{\alpha} \cap S_1^{\beta}; \tag{27}$$

$$p_{s2} = \frac{b_{s2}^{\alpha} + b_{s2}^{\beta} + b_{s2}^{\lambda}}{q_{s2}^{\phi} + (b_{02}^{\beta} / p_{02})} \quad \text{for } s \in S_1^{\alpha} \setminus S_1^{\alpha} \cap S_1^{\beta}; \tag{28}$$

$$p_{s2} = \frac{b_{s2}^{\alpha} + b_{s2}^{\beta} + b_{s2}^{\lambda}}{q_{s2}^{\phi} + (b_{02}^{\alpha} / p_{02}) + (b_{02}^{\beta} / p_{02})} \quad \text{for } s \notin S_1^{\alpha} \cup S_1^{\beta}. \tag{29}$$

When agent $h \in \{\alpha, \beta\}$ defaults on his mortgage, the amount of housing he has pledged as collateral will be offered by his bank for sale in the market. This amount is equal to the amount of housing he purchased in the initial period, that is, b_{02}^{h} / p_{02}. For example, in state $s \in S_1^{\beta} \setminus S_1^{\alpha} \cap S_1^{\beta}$, agent α (but not β) defaults, so the amount of housing he purchased in the initial period and pledged as collateral will be offered for sale by bank γ.

2.7.3 The mortgage market

Given that

$$1 + \bar{r}^{k} = \frac{\bar{\mu}^{h}}{\bar{m}^{k}}, \tag{30}$$

the effective return on the mortgage is min(value of collateral, mortgage amount) / initial credit extension, or

$$1 + \bar{r}_s^{k} = \frac{\min\left[(b_{02}^{h} / p_{02}) p_{s2}, \bar{\mu}^{h}\right]}{\bar{m}^{k}},$$

where $k = \gamma$ for $h = \alpha$ and $k = \delta$ for $h = \beta$. We thus get the following clearing conditions for effective returns on mortgages:

$$1 + \bar{r}_s^{k} = 1 + \bar{r}^{k} \quad \text{for } s \in S_1^{h} = \left\{ s \in S : \bar{\mu}^{h} \le \frac{b_{02}^{h}}{p_{02}} p_{s2} \right\}; \tag{31}$$

$$1 + \bar{r}_s^{k} = (1 + \bar{r}^{k}) \frac{b_{02}^{h}}{\bar{\mu}^{h}} \frac{p_{s2}}{p_{02}} \quad \text{for } s \notin S_1^{h}. \tag{32}$$

2.7.4 The short-term loan market

$$1+r_0^{\gamma}=\frac{\mu_0^{\alpha}+\mu_0^{\phi}}{m_0^{\gamma}}; \tag{33}$$

$$1+r_s^{\gamma}=\frac{\mu_s^{\alpha}+\mu_s^{\phi}}{m_s^{\gamma}+M_{\gamma s}^{CB}}; \tag{34}$$

$$1+r_0^{\delta}=\frac{\mu_0^{\beta}}{m_0^{\delta}}; \tag{35}$$

$$1+r_s^{\delta}=\frac{\mu_s^{\beta}+\mu_s^{\lambda}}{m_s^{\delta}+M_{\delta s}^{CB}}. \tag{36}$$

2.7.5 The consumer deposit market

$$1+\bar{r}_d=\frac{\bar{\mu}_d^{\gamma}}{\bar{d}^{\phi}}. \tag{37}$$

2.7.6 The interbank market

$$1+\rho=\frac{\mu_I^{\gamma}}{d_I^{\delta}+M^{CB}}. \tag{38}$$

2.8 Definition of Equilibrium

Let $\sigma^h=(q_{s1}^h,b_{s2}^h,\mu_s^h,\bar{\mu}^h)\in\mathbb{R}^{s+1}\times\mathbb{R}^{s+1}\times\mathbb{R}^{s+1}\times\mathbb{R}$ for $h\in\{\alpha,\beta\}$;

$$\sigma^{\phi}=(q_{s2}^{\phi},b_{s1}^{\phi},\bar{d}^{\phi},\mu_s^{\phi})\in\mathbb{R}^{s+1}\times\mathbb{R}^{s+1}\times\mathbb{R}\times\mathbb{R}^{s+1};$$

$$\sigma^{\lambda}=(q_{s1}^{\lambda},b_{s2}^{\lambda},\mu_s^{\lambda})\in\mathbb{R}^{s}\times\mathbb{R}^{s}\times\mathbb{R}^{s};$$

$$\sigma^{\gamma}=(\pi_s^{\gamma},m_s^{\gamma},\bar{m}^{\gamma},\mu_I^{\gamma},\bar{\mu}_d^{\gamma})\in\mathbb{R}^{s}\times\mathbb{R}^{s+1}\times\mathbb{R}\times\mathbb{R};$$

$$\sigma^{\delta}=(\pi_s^{\delta},m_s^{\delta},\bar{m}^{\delta},d_I^{\delta})\in\mathbb{R}^{s}\times\mathbb{R}^{s+1}\times\mathbb{R}\times\mathbb{R}.$$

Also, let $\eta = (p_{s1}, p_{s2}, r_s^{\gamma}, r_s^{\delta}, \overline{r}^{\gamma}, \overline{r}_s^{\gamma}, \overline{r}^{\delta}, \overline{r}_s^{\delta}, \rho)$
$\in \mathbb{R}^{s+1} \times \mathbb{R}^{s+1} \times \mathbb{R}^{s+1} \times \mathbb{R}^{s+1} \times \mathbb{R} \times \mathbb{R}^{s} \times \mathbb{R} \times \mathbb{R}^{s} \times \mathbb{R}$;

$$B^h(\eta) = \left\{ \sigma^h : \text{eqs. (1)} - \text{(7) hold} \right\} \quad \text{for } h \in \left\{ \alpha, \beta \right\};$$

$$\mathrm{B}^{\phi}(\eta) = \left\{ \sigma^{\phi} : \text{eqs. (8)} - \text{(13) hold} \right\};$$

$$B^{\lambda}(\eta) = \left\{ \sigma^{\lambda} : \text{eqs.(14)} - \text{(16) hold} \right\};$$

$$B^{\gamma}(\eta) = \left\{ \sigma^{\gamma} : \text{eqs.(17)} - \text{(19) hold} \right\}; \quad \text{and}$$

$$B^{\delta}(\eta) = \left\{ \sigma^{\delta} : \text{eqs.(20)} - \text{(22) hold} \right\}.$$

We say that $(\sigma^{\alpha}, \sigma^{\beta}, \sigma^{\phi}, \sigma^{\lambda}; p_{s1}, p_{s2}, r_s^{\gamma}, r_s^{\delta}, \overline{r}^{\gamma}, \overline{r}_s^{\gamma}, \overline{r}^{\delta}, \overline{r}_s^{\delta}, \rho)$ is a monetary equilibrium with commercial banks, collateral, and default if

(a) $\sigma^{\mathrm{n}} \in \mathrm{Argmax}_{\sigma^{\mathrm{n}} \in \mathrm{B}^{\mathrm{n}}(\eta)} \Pi^{\mathrm{n}}$, $n \in \{\alpha, \beta, \phi, \lambda\}$

and

(b) $\sigma^{\mathrm{k}} \in \mathrm{Argmax}_{\sigma^{\mathrm{k}} \in \mathrm{B}^{\mathrm{k}}(\eta)} \Pi^{\mathrm{k}}$, $k \in \{\gamma, \delta\}$

and if all markets in equations (23) through (38) clear.

3. Discussion of Equilibrium

In this section, we investigate a parametrized version of the model whereby only three states of nature are possible in the second period. We have chosen the exogenous parameters in our model so as to illustrate a housing and mortgage crisis. Their initial values are presented in table 1. The initial equilibrium yielded by the model is presented in table 2 and analyzed below.

Table 1. Exogenous Variables

Coefficient of risk aversion	*Endowment*	*Housing*	*Money*	*Capital*	*Other*
$c^{\alpha} = 1.3$	$e_{01}^{\alpha} = 11$	$e_{02}^{\phi} = 5.5$	$m_0^{\alpha} = 0.1$	$e_0^{\gamma} = 4$	$M^{CB} = 65$
$c^{\beta} = 1.3$	$e_{11}^{\alpha} = 10$		$m_1^{\alpha} = 0.1$	$e_1^{\gamma} = 0.7$	$M_{\gamma 1}^{CB} = 10.9$
$c^{\phi} = 1.3$	$e_{21}^{\alpha} = 10$		$m_2^{\alpha} = 4.4$	$e_2^{\gamma} = 0.7$	$M_{\gamma 2}^{CB} = 8$
$c^{\gamma} = 1.3$	$e_{31}^{\alpha} = 0.7$		$m_3^{\alpha} = 0.1$	$e_3^{\gamma} = 0.7$	$M_{\gamma 3}^{CB} = 0.5$
$c^{\gamma} = 0.005$	$e_{01}^{\beta} = 2$		$m_0^{\beta} = 5.8$	$e_0^{\delta} = 13$	$M_{\delta 1}^{CB} = 2.4$
$c^{\delta} = 0.005$	$e_{11}^{\beta} = 7$		$m_1^{\beta} = 0.1$	$e_1^{\delta} = 1$	$M_{\delta 2}^{CB} = 0.8$
	$e_{21}^{\beta} = 3$		$m_2^{\beta} = 0.1$	$e_2^{\delta} = 1$	$M_{\delta 3}^{CB} = 0.5$
	$e_{31}^{\beta} = 0.1$		$m_3^{\beta} = 0.1$	$e_3^{\delta} = 1$	$\theta_1 = 0.90$
	$e_{11}^{\lambda} = 4$		$m_0^{\phi} = 0.1$		$\theta_2 = 0.075$
	$e_{21}^{\lambda} = 4$		$m_1^{\phi} = 0.1$		$\theta_3 = 0.025$
	$e_{31}^{\lambda} = 3$		$m_2^{\phi} = 0.1$		
			$m_3^{\phi} = 0.1$		
			$m_1^{\lambda} = 0.1$		
			$m_2^{\lambda} = 0.1$		
			$m_3^{\lambda} = 0.1$		

e_{s1}: Endowment of goods in state $s \in S^*$.
e_{02}: Endowment of houses at $t = 0$.
m_s: Private money held by households in state $s \in S^*$.
e_0: Initial capital of banks.
e_s: Additional capital of banks in state $s \in S$.
M^{CB}: Money supply at $t = 0$.
M_{ks}^{CB}: Money injection by the central bank in the short-term loan market organized by bank $k \in \{\gamma, \delta\}$ in state $s \in S$.
θ_s: Probability of state $s \in S$.

In the initial equilibrium, we examine three different scenarios that can occur in the second period. State 1 occurs with the highest probability, and state 2 is more probable than state 3. State 1 is the good period in which neither borrower defaults. In state 2, one of the two agents, Mr. β, defaults on his mortgage debt, but the other does not. In state 3, both default. Agent α is richer in endowments of the good in the first period, whereas agent β is relatively richer in state 2 in the second period. Bank γ has less initial capital than bank δ, while it has more capital in the second period. The capital of both banks in the second period can be interpreted as outside banking profits or capital injections obtained in the second period and will play a crucial role in the comparative statics we perform. We have chosen the parametrization to motivate lending in the interbank market and in particular to motivate bank δ to deposit in the interbank rate.

The level of default on the mortgages depends on the relative (second period) differential between the value of houses that each agent bought and the mortgage amount they have to repay. Agent α, who is richer in the first period, needs to take a comparatively lower loan-to-value mortgage for the amount of housing he wants to purchase than agent β, since he can finance the purchase through the sale of goods in the first period. As a result, the effective return to the lending bank on the mortgages in state 3, when both agents default, will be higher for α than β. In combination with the fact that α does not default in state 2, this results in a lower interest rate on the mortgage for α than for β, since rational expectations are assumed throughout.

In our simulation, the prices of the good and the house move in opposite directions in the second period. The good is relatively more expensive in state 2 than state 1 and in state 3 than state 2, whereas the opposite holds for the price of the house. The intuition behind the result is quite simple, since the model is driven by adverse supply shocks to goods endowments, worse in state 3 than in state 2. Agents default on their mortgages when the value of the house is low. This happens when the endowments of goods are low (that is, an adverse supply shock) since agents will not have enough income to allocate to the housing market. This, in turn, implies that the price of the good should rise.

In order to buy the house, agents α and β sell goods in the first period and also take out a mortgage. This creates income for ϕ, the initial owner of the housing stock, who uses a portion to purchase

Table 2. Initial Equilibrium

Prices	*Interest rates*	*Loans/deposits households*	*Loans/deposits households*	*Loans/deposits banks*	*Repayment rates on mortgages*	*Goods*	*Houses*
$p_{01} = 4.10$	$\bar{r}^{\gamma} = 0.079$	$\bar{\mu}^{\alpha} = 12.33$	$\mu_1^{\lambda} = 6.23$	$\bar{m}^{\gamma} = 11.43$	$v_1^{\alpha} = 100\%$	$q_{01}^{\alpha} = 7.20$	$b_{02}^{\alpha} = 49.78$
$p_{11} = 2.60$	$\bar{r}^{\delta} = 0.123$	$\mu_0^{\alpha} = 29.47$	$\mu_2^{\lambda} = 6.97$	$\bar{\mu}_d^{\gamma} = 25.50$	$v_2^{\alpha} = 100\%$	$q_{11}^{\alpha} = 4.42$	$q_{12}^{\alpha} = 0.09$
$p_{21} = 3.10$	$r_0^{\gamma} = 0.043$	$\mu_1^{\alpha} = 12.80$	$\mu_3^{\lambda} = 18.46$	$\bar{\mu}_I^{\gamma} = 69.07$	$v_3^{\alpha} = 62\%$	$q_{21}^{\alpha} = 4.48$	$b_{22}^{\alpha} = 5.12$
$p_{31} = 12.31$	$r_1^{\gamma} = 0.047$	$\mu_2^{\alpha} = 13.65$		$m_0^{\gamma} = 83.24$	$v_1^{\beta} = 100\%$	$q_{31}^{\alpha} = 0.35$	$b_{32}^{\alpha} = 1.24$
$p_{02} = 26.12$	$r_2^{\gamma} = 0.047$	$\mu_3^{\alpha} = 4.35$		$m_1^{\gamma} = 5.28$	$v_2^{\beta} = 62\%$	$q_{01}^{\beta} = 0.29$	$b_{02}^{\beta} = 17.58$
$p_{12} = 15.17$	$r_3^{\gamma} = 2.83$	$\bar{\mu}^{\beta} = 11.96$		$m_2^{\gamma} = 5.28$	$v_3^{\beta} = 28\%$	$q_{11}^{\beta} = 4.54$	$q_{12}^{\beta} = 0.04$
$p_{22} = 10.96$	$r_0^{\delta} = 0.043$	$\mu_0^{\beta} = 1.18$		$m_3^{\gamma} = 0.64$		$q_{21}^{\beta} = 1.71$	$b_{22}^{\beta} = 5.08$
$p_{32} = 5.04$	$r_1^{\delta} = 0.048$	$\mu_1^{\beta} = 12.43$		$\bar{m}^{\delta} = 10.65$		$q_{31}^{\beta} = 0.04$	$b_{32}^{\beta} = 0.29$
	$r_2^{\delta} = 0.049$	$\mu_2^{\beta} = 5.22$		$d_I^{\delta} = 1.22$		$b_{01}^{\phi} = 30.64$	$q_{02}^{\phi} = 2.20$
	$r_3^{\delta} = 1.58$	$\mu_3^{\beta} = 0.50$		$m_0^{\delta} = 1.13$		$b_{11}^{\phi} = 29.55$	$q_{12}^{\phi} = 0.27$
	$\bar{r}_d = 0.043$	$\bar{d}^{\phi} = 24.44$		$m_1^{\delta} = 15.41$		$b_{21}^{\phi} = 25.84$	$q_{22}^{\phi} = 0.03$
	$\rho = 0.043$	$\mu_0^{\phi} = 57.36$		$m_2^{\delta} = 10.82$		$b_{31}^{\phi} = 23.30$	$b_{32}^{\phi} = 2.30$

Table 2. (continued)

Prices	*Interest rates*	*Loans/deposits households*	*Loans/deposits households*	*Loans/deposits banks*	*Repayment rates on mortgages*	*Goods*	*Houses*
		$\mu_1^{\phi} = 4.14$		$m_3^{\delta} = 6.84$		$q_{11}^{\lambda} = 2.40$	$b_{12}^{\lambda} = 6.05$
		$\mu_2^{\phi} = 0.25$				$q_{21}^{\lambda} = 2.29$	$b_{22}^{\lambda} = 6.75$
						$q_{31}^{\lambda} = 1.50$	$b_{32}^{\lambda} = 7.25$

Source: Authors' calculations.

p_{s1}: Price of goods in state $s \in S^*$.
p_{s2}: Price of houses in state $s \in S^*$.
$\bar{r}_d$: Consumer deposit rate.
ρ: Interbank rate.
$\bar{\mu}$: Mortgage amount.
μ_s: Short-term borrowing by households in state $s \in S^*$.
$\bar{d}^{\phi}$: Deposit by agent ϕ at t = 0.
$\bar{m}^k$: Mortgage extension by bank $k \in \{\gamma, \delta\}$.
$\bar{\mu}_I^{\gamma}$: Interbank borrowing by bank γ.

d_I^{δ}: Interbank deposits by bank δ.
v_s^h: Effective repayment on mortgage by agent $h \in \{\alpha, \beta\}$ in state $s \in S$.
b_{s1}, b_{s2}: Money spent in the goods and housing markets in state $s \in S^*$.
$\bar{r}^k$: Mortgage rate offered by bank $k \in \{\gamma, \delta\}$.
r_s^k: Short-term loan rate offered by bank $k \in \{\gamma, \delta\}$ in state $s \in S^*$.
$\bar{\mu}_d^{\gamma}$:Amount borrowed from consumers in the form of deposits by bank γ.
m_s^k: Short-term loan extension by bank $k \in \{\gamma, \delta\}$ in state $s \in S^*$.
q_{s1}: Amount of goods offered for sale in state $s \in S^*$.
q_{s2}: Amount of houses offered for sale in state $s \in S^*$.

goods and deposits the rest in the interperiod deposit market. In state 1, when α and β do not default on their mortgages and then find themselves with more housing than they want, they sell some of the amount they bought in the previous period (house prices are high relative to goods prices, and utility maximization leads α and β to switch out of housing into goods).[4] This is possible because the economy is going well, endowments of goods are high, and there is a strong demand for housing from agent λ, a first-time buyer who enters the economy in the second period. Agent ϕ also finds it profitable to sell some of the housing he is left with at those prices.

In state 2, however, Mr. β defaults on his mortgage in period 2 and essentially loses his house, but he still wants to purchase some housing. Although the housing supply is high as a result of the delinquencies, demand from α, β, and λ prevents the price of houses from collapsing. This gives incentives to ϕ to sell some of the housing he owns, as in state 1.

One would expect the same scenario to occur in state 3. However, since both agents α and β become extremely poor in their endowments of goods in that state, their demand for housing drops precipitously. As a result, the housing market should collapse and agent λ, who is the only one endowed with a sufficient amount of the scarce good, should be able to purchase housing at a very low price. The reason that this cannot happen is twofold. First, agent ϕ finds it profitable to purchase back some of the houses he sold in the first period, so both the demand for housing and the price are maintained.[5] The second and most important factor lies in the liquidity constraints that all agents face. In state 3, banks are short of liquidity, so they are only willing to lend money short term at a very high interest rate. Although the good is very expensive, agent λ can only find credit at an extremely high interest rate, which prevents him from enjoying the full benefits of the falling housing market. This is not the case for ϕ as he has money at hand from depositing in the first period.

Finally, lower prices is the last period are outweighed by the high real interest rates. Thus, short-term interest rates rise through the Fisher effect.

4. The agents' cash-in-advance constraints have been adjusted to include housing sales as well as goods sales.

5. Since ϕ does not sell any houses in state 3, he does not demand a short-term loan.

4. Comparative Statics

This section shows the effects of changes in the exogenous variables and parameters of the model. Tables A1 and A2 in the appendix describe the directional effects on endogenous variables of changing various parameters. Although we have performed a number of comparative statics, we discuss in detail only those that we reckon are the most interesting. The analysis involves the following principles, derived from the model structure, on the determination of interest rates, the quantity theory of money, and the Fisher effect.[6]

To determine the interest rate structure, we start with the facts that base money is fiat and the horizon is finite, so in the end no household will be left with fiat money. All households therefore finance their loan repayments to commercial banks via their private cash endowment and the initial capital endowments of banks. However, since we allow for default, the total amount of interest rate repayments is adjusted by the corresponding anticipated default rates. In sum, aggregate ex post interest rate payments, adjusted for default to commercial banks, is equal to the total amount of outside money (that is, sum of cash monetary and initial commercial banks' endowments). In this way, the overall liquidity of the economy and endogenous default codetermine the structure of interest rates.

The model possesses a nonmechanical quantity theory of money. Velocity will always be less than or equal to one (one if all interest rates are positive). However, since the quantities supplied in the markets are chosen by agents (unlike the representative agent model's sell-all assumption), the real velocity of money (that is, how many real transactions can be undertaken by money per unit of time) is endogenous. The upshot of this analysis is that nominal changes (that is, changes in monetary policy) affect both prices and quantities.

Finally, the nominal interest rate is equal to the real interest rate plus the expected rate of inflation.

6. The qualitative structure of the initial equilibrium does not change. For example, an increase in the price of goods in state 3 does not mean that this price has become higher than the prices of goods in states 1 and 2.

4.1 Decrease in the Money Supply

Let the central bank engage in contractionary monetary policy by decreasing the money supply (M^{CB}) in the interbank market in the initial period (or equivalently increasing the interbank interest rate, ρ). The effects on the endogenous variables are summarized in table A1 in the appendix.

Increasing the interbank rate induces bank γ to borrow less from the interbank market and therefore to reduce its supply of short-term loans and mortgages to Mr. α and Mr. ϕ, pushing up the corresponding lending rates, r_0^{γ} and $\bar{r}^{\gamma}$. Consequently, Mr. α reduces his short-term and mortgage borrowing, while Mr. ϕ similarly reduces his short-term borrowing and subsequent deposits in bank γ. Since bank δ increases its deposits in the interbank market, Mr. β faces stricter credit conditions in the short-term. He will therefore switch toward mortgages, which will induce bank δ to reallocate its portfolio and supply slightly more mortgages to him. Finally, from the liquidity structure of interest rates, last period short-term interest rates decrease, except for bank γ in the second state.

Given a higher interest rate, trade becomes less efficient.[7] Quantities of goods and houses traded in the initial period fall, as do prices. We see the quantity theory of money in action in our model. The reduction of the money supply, given that the velocity of money is at most one, typically leads to lower prices and quantities traded. Agent heterogeneity and positive trade volumes are necessary for this result to hold. Given the low price of housing and the increased mortgage extension by bank δ, Mr. β is led to demand more mortgages, which results in a higher mortgage rate ($\bar{r}^{\delta}$) for him, as well. According to the quantity theory of money, since less money chases the same amount of goods, prices will also drop in the last period. Recall that agents default on their mortgages when the value of their housing is less than that of their mortgage. Thus, lower house prices in the second period will result in lower effective returns on the mortgages (which can be interpreted as higher defaults) and even higher initial mortgage rates, given rational expectations. An increase in the interbank rate results not only in increased mortgage extension by the rich bank, but also in lower effective returns (higher levels of effective

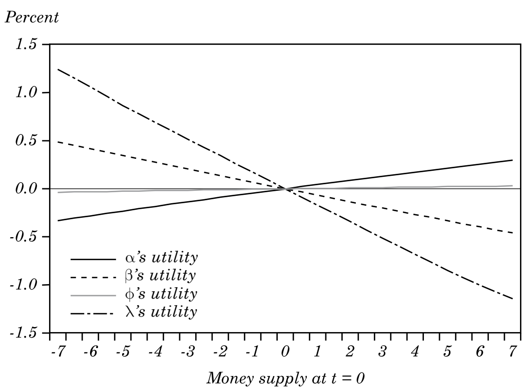
7. The reason is that agents encounter a higher transaction cost.

default) when the bad states materialize. Although the poor bank reduces its mortgage extension, it does not find itself in a better situation, since the effective return on its mortgages also falls when the very bad state obtains.

The higher mortgage extension and mortgage rates for bank δ do not outweigh the lower effective returns stemming from default on mortgages in the bad states of the world (that is, states 2 and 3). The impact on bank γ is the same. Contractionary monetary policy thus results in lower expected profits for the banking sector.

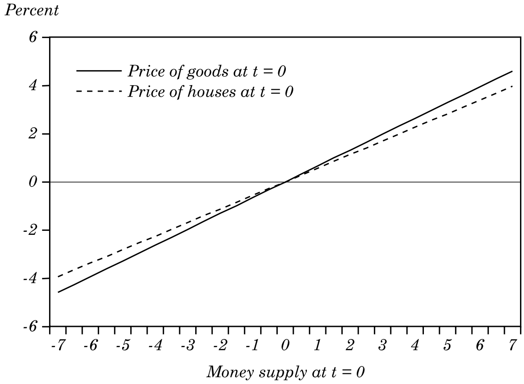
The effect on households differs. For Mr. α, an increase in the interbank rate has a negative effect on his welfare, whereas the opposite holds for Mr. β and Mr. λ. The welfare of Mr. ϕ remains almost unaffected (figure 2). The decrease in α's welfare is mainly due to the fact that he borrows less since he is affiliated with the poor bank. Although the price of housing drops at $t = 0$, the price of goods decreases even more (figure 3). Mr. β is affiliated with the rich bank and undertakes a bigger mortgage to take advantage of the falling housing prices in the initial period. In conjunction with the falling short-term rates in the last period (figure 4), Mr. β's welfare goes up. Mr. λ benefits as well from the lower short-term rates and enjoys an increase in his utility.

Figure 2. Household Welfare versus Money Supply



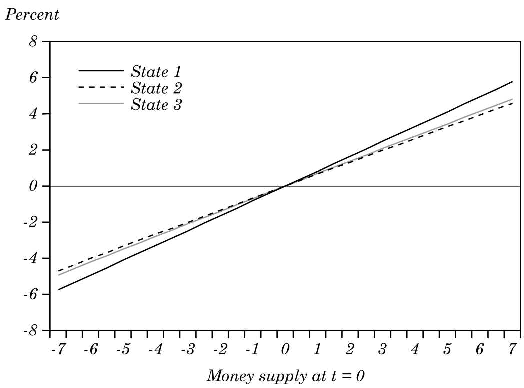
Source: Authors' calculations.

Figure 3. Housing and Goods Prices versus Money Supply



Source: Authors' calculations.

Figure 4. Short-Term Interest Rates by Bank δ versus Money Supply



Source: Authors' calculations.

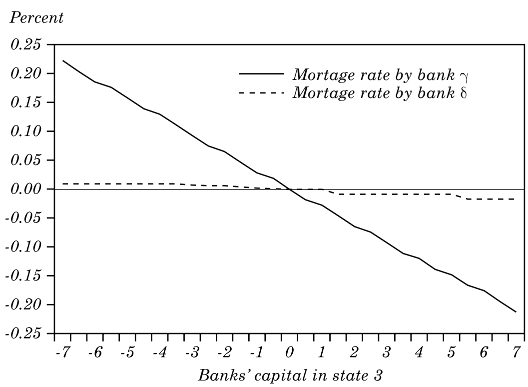
In sum, according to our financial stability measure, contractionary monetary policy results in higher financial instability since lower banking profits and higher default lead to welfare loses in the bad states of nature.

4.2 Liquidity Assistance to Banks in the Very Bad State of the World

Let there be an increase in both banks' capital in the third state of the world, which participants in the economy perfectly anticipate (table A1). This increase can be in the form of liquidity assistance by the government or new equity capital. An increase in the money endowments in the third state of the world will result in a price increase in goods and housing at that state, as expected from the quantity theory of money. A price increase in housing results in a higher effective return for both banks when both Mr. α and Mr. β default on their mortgages. Finding themselves with more money in the very bad state of the world, the banks will increase their extension of mortgages at the initial period. This will drive mortgage rates down and the demand for mortgages up. Bank δ will switch its portfolio from interbank deposits to mortgages, since the latter become less risky. Given the increased activity and higher prices overall, when government help is anticipated in (very) bad states of the world, interest rates in the short loan market rise in the initial period as a result of increased money demand by households.

Although the effective returns on mortgages rises and the overall default rate falls in absolute terms, both banks will sustain a drop in their expected profitability. The reason is that the rates on mortgages to which they switch their portfolios drop (figure 5). In addition,

Figure 5. Mortgage Rates versus Banks' Capital in State 3

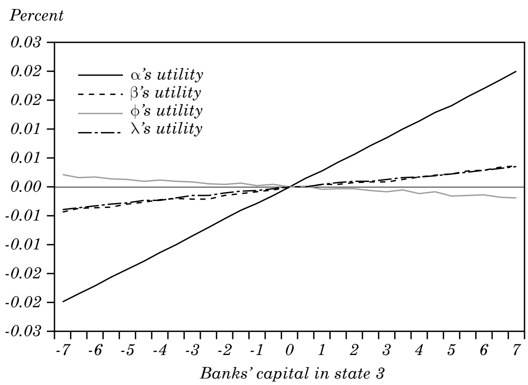


Source: Authors' calculations.

bank γ has to pay a higher interest rate for the money it borrows from depositors and the interbank market, and bank δ does not fully take advantage of the higher interbank rate, since it reallocates its portfolio toward mortgages that obtain higher effective returns.

The welfare of Mr. ϕ decreases because the liquidity injection occurs in state 3, when he is relatively rich, and he suffers a negative wealth effect due to higher prices in that state. Apart from Mr. ϕ, the effect on household welfare is positive (figure 6). Agents α, β, and γ are all better off since the first two benefit from the lower mortgage rates and all three take advantage of lower short-term rates in the last period, which translates into cheaper credit.

Figure 6. Household Welfare versus Banks' Capital in State 3



Source: Authors' calculations.

Liquidity assistance, unlike contractionary monetary policy, not only reduces aggregate default, but also improves the real sector of the economy since it eases credit conditions for households and first-time buyers.

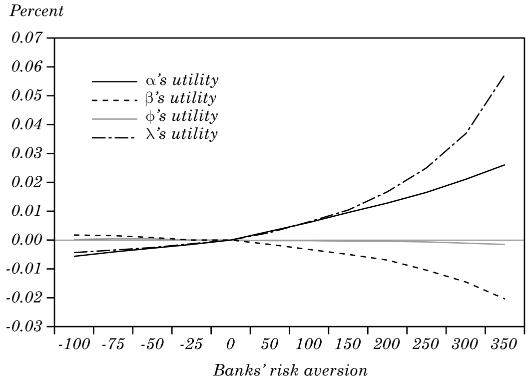
4.3 Banks Become Less Risk Averse

Assume that both banks become more risk loving (see table A2 in the appendix). The change in risk aversion is anticipated in the first period. Their first response will be to switch from safer to riskier investments. Consequently, the extension of mortgages increases and short-term lending decreases, which means lower mortgage rates and higher short-term rates in the initial period. Bank δ also reduces its interbank deposits, which results in bank γ having less funds for extending credit. Mr. α takes advantage of the lower mortgage rates and demands more mortgages. He also reduces his sale of goods in the initial period, since he can finance his housing purchases with more mortgages, and the transaction cost of selling his goods (that is, the short-term interest rate) has risen due to the banks' fund reallocation. Mr. β, who is poorly endowed in the initial period, will also reduce his sale of goods and his short-term borrowing. He will not demand more mortgages, however, as the drop in the mortgage rate allows him to maintain his housing purchases. The mortgage rate falls more for Mr. β than for Mr. γ because he is affiliated with bank δ, which has more funds to allocate to mortgages since it reduces its interbank and short-term lending. The demand for housing has increased, but its initial price will fall because Mr. α and Mr. β reduce their initial supply of goods to the market and Mr. ϕ has to sell more of his housing endowment to fund his purchase of goods. Thus, Mr. ϕ's disposable income falls, and he allocates less money to the goods markets, forcing their initial price to drop as well.

Lower housing prices and higher mortgage extension translate into lower effective returns on mortgages because of higher aggregate default in the economy in the bad states. Depending on the severity of the reduction in risk aversion and its initial level, aggregate default may increase a lot. In our exercise, we have chosen a relatively low initial risk aversion (to capture the banks' precrisis risk aversion in the initial equilibrium), so an even a relatively small increase in the appetite for risk results in a 0.5 percent increase in aggregate default. Of course, what matters is the directional effect and not the absolute number. Unlike our other comparative statics, a change in risk aversion, although exogenous in the model, is in reality a choice variable of the banks. The reason that they might adopt a more risk-loving behavior is that they expect higher profits. This is consistent with what our model yields.

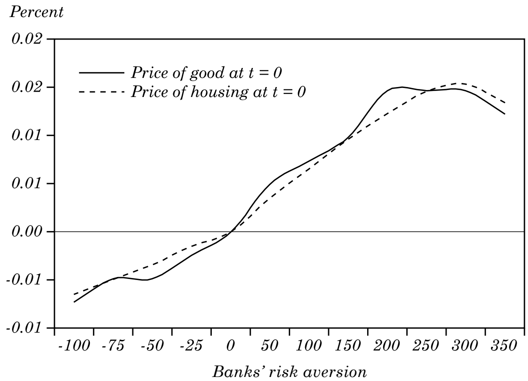
Households' welfare moves in different directions (figure 7). Mr. ϕ is better off because houses are a durable commodity and their price should be affected positively by a decrease in the overall risk aversion in the economy. As a result, the price of housing decreases less than the price of goods in the initial period, which generates a slightly positive effect on Mr. ϕ's welfare (see figure 8). Mr. β, who is poorly endowed in the initial period, is also better off, as he benefits from the lower mortgage rates and enjoys an increase in his utility. Mr. α, on the other hand, is worse off, since he faces a higher interest rate for short-term loans in the initial period, which is his main source for funding his housing purchases. Mr. λ also sees his welfare decrease because of the rise in short-term interest rates in the last period in response to higher aggregate default (figure 9) and higher real interest rates.

Figure 7. Household Welfare versus Banks' Risk Aversion



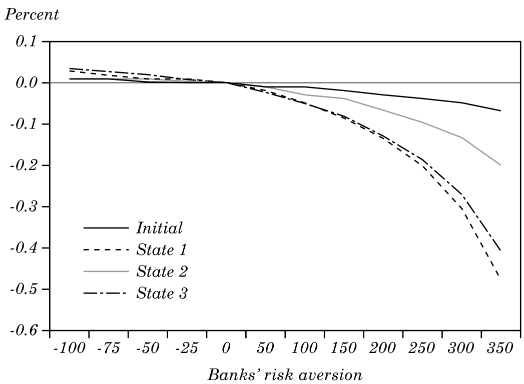
Source: Authors' calculations.

Figure 8. Housing and Goods Prices at *t* = 0 versus Banks' Risk Aversion



Source: Authors' calculations.

Figure 9. Short-Term Interest Rates by Bank δ versus Banks' Risk Aversion



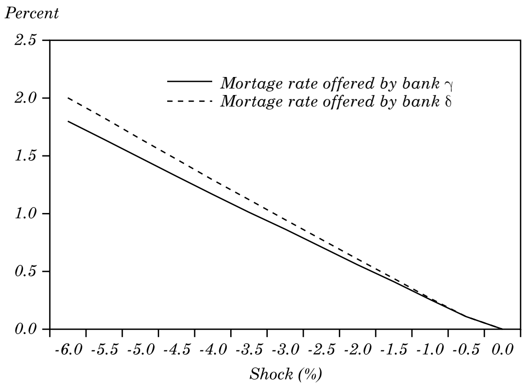
Source: Authors' calculations.

4.4 Compound Comparative Static

The comparative statics we examine above do not fully exhibit what we might expect to observe in a severe mortgage crisis. We therefore performed an exercise of letting more than one adverse shock occur at a time, in which we allow for contractionary monetary policy and a decrease in banks' risk aversion simultaneously. The results are summarize in table A2.

The reduction in the money supply yields a first-order effect that pushes up the interbank rate. Bank δ increases its interbank lending and reduces its mortgage extension. The reduction in risk aversion will moderate this pressure. The trade-off between these two effects will determine whether bank δ will extend more mortgages. In our simulation we find that mortgage extension by bank δ increases. The reduction is more severe for bank γ, since it is more dependent on monetary injections. Mortgage rates rise (figure 10), since demand does not decrease much due to the higher cost of short-term borrowing. Prices of goods and housing fall in all periods and states, as predicted by the quantity theory of money. The pressure is greater due to lower risk aversion (as discussed above). The result is lower expected returns on mortgages, which translate into higher defaults in conjunction with the fact that mortgage rates were higher to start with.

Figure 10. Mortgage Rates versus Compound Decrease in Money Supply and in Banks' Risk Aversion[a]



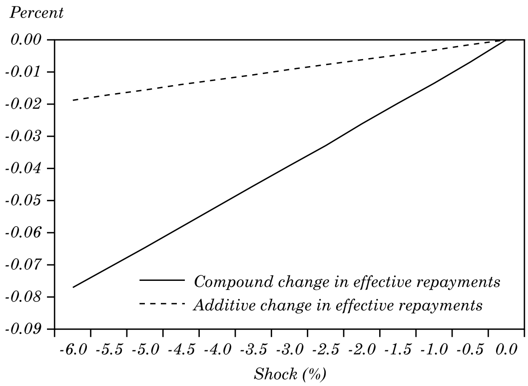
Source: Authors' calculations.
a. More severe shocks are to the left.

Higher default should mean higher mortgage rates, other things being equal, but a higher appetite for risk pushes mortgage rates down. Nevertheless, these second-order effects are outweighed by the increased default resulting from a lower money supply, as analyzed in the relevant section. An interesting result is that default increases disproportionally when contractionary monetary policy is combined with a higher appetite for risk by banks. When these adverse shocks occur at the same time, expected repayment on mortgages falls more than the aggregate change when they happen independently. In particular, nonlinear effects are not trivial, as shown in figure 11.

Expected banking profits go up. On the one hand, the lower money supply and increased default put downward pressure on expected profits, while on the other, lower risk aversion pushes them up. In our exercise, the latter forces prevail, but the effect of the former becomes more intense as the money supply continues to decrease.

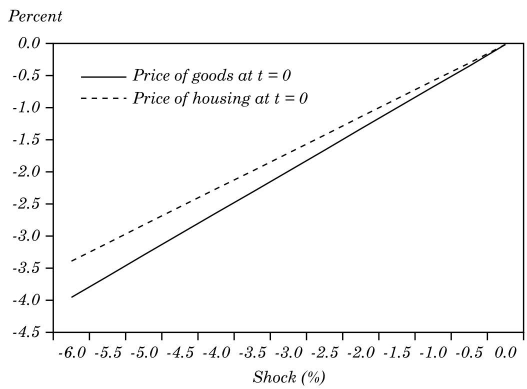
The effect on household welfare varies. Agents that are affiliated with the poor bank (that is, Mr. α and Mr. ϕ) observe a decrease in their expected welfare, because the stricter credit environment affects poorly capitalized banks more severely. In addition, the initial price of goods falls more than that of housing (figure 12),

Figure 11. Nonlinear Effects on Mortgage Repayment versus Compound Decrease in Money Supply and in Banks' Risk Aversion[a]



Source: Authors' calculations.
a. More severe shocks are to the left.

Figure 12. Housing and Goods Prices at $t = 0$ versus Compound Decrease in Money Supply and in Banks' Risk Aversion[a]



Source: Authors' calculations.
a. More severe shocks are to the left.

which negatively affects the purchasing power of Mr. α, who mainly finances his housing purchase though the sale of goods in the initial period. Mr. β is able to benefit from the falling housing prices in the initial period by entering a mortgage contract, since he is affiliated with a well capitalized—and more risk-loving—bank, and his welfare increases. Housing prices in state 1 fall more than goods prices (figure 13) because Mr. ϕ decreases his deposits in the initial period and increases his sales of housing in that state to finance the purchase of goods. The lower demand for money by Mr. β in the last period (partially reflecting lower prices and higher defaults) and the well-capitalized position of bank δ put downward pressure on short-term interest rates at the states in which agents default. Mr. λ benefits from the looser credit conditions and enjoys a higher utility.

Figure 13. Housing and Goods Prices in State 1 versus Compound Decrease in Money Supply and in Banks' Risk Aversion[a]

Source: Authors' calculations.
a. More severe shocks are to the left.

However, lower banking profits in the two bad states and the relatively higher aggregate default (relative to contractionary monetary policy only) result in higher welfare losses in these states. Hence, contractionary monetary policy coupled with an attempt to gamble on resurrecting the banks exacerbates the mortgage crisis and increases financial instability.

5. Conclusions

Central bank officials are prone to describe the months since August 2007 as being akin to wartime. In econometric exercises based on longer-run time-series stretching back, say, to 1900, the war years of 1914–18, and 1939–45 are frequently omitted (or dummied out) as involving regimes and structures too atypical for normal analysis. By analogy, the years 2007–08 may also become excluded from standard econometric analysis as too extraordinary to fit with our standard models. After all, such standard models abstract from counterparty risk, from default, from endogenous risk premiums, and even from financial intermediation.

If, however, we want to address and model current events, then we need a model that incorporates default as a central feature and treats credit risk as endogenous (rather than as an exogenous add-on). The model explored above is such a model, albeit an initial, preliminary attempt. Much more needs to be done.

For example, it is an endowment model, so the economy has a given time path of goods, houses, capital, and fiat money. With such predetermined endowments, the resulting time path of prices, interest rates, and quantities just redistributes goods and assets among agents. The welfare implications are never clear-cut since some gain and others lose. To explore the welfare implications of financial crises, they have to be incorporated into a production economy, wherein a credit crunch adversely affects output and employment, so that real incomes become generally reduced and not just redistributed. This can be done and should not be too difficult.

In general, the results of our simulations are more or less what most economists would have imagined. Tight money reduces prices and quantities traded. Government support to banks in crises stabilizes the economy. When banks become risk-loving, a subsequent crisis becomes even more extreme. We are encouraged that our model reproduces common-sense outcomes. The direction of effects seems correct.

This raises the question of whether such a model as this can be taken beyond numerical solution and simulation to the actual data. Could it be used to try to match and calibrate the actual time path of the major data series in existing countries and to explore alternative policy options in real time? We believe that it can, though it will not be straightforward to do so.

Running simulations often provides the analysts with more insight than the readers of the resulting paper. One of the lessons that this exercise has taught us concerns the limitations of a strict rational expectations model. In a rational expectations model, an event in some subsequent period, such as a change in risk aversion or a change in government policy, may be regarded at the outset as a low probability event, but in a fully rational world it cannot by definition have been entirely unexpected. One cannot run simulations, in a rational expectations world, in which the completely unexpected occurs. This makes it rather harder to simulate extraordinary time periods such as 2007–08. Thus, for example, the risk-seeking behavior of financial intermediaries in 2004–06 gave way to strong risk aversion in 2007–08 in a way that was entirely unexpected in 2004–06. Had it been anticipated, it would have been discounted in a rational expectations system. The solution is to assume that unexpected changes in behavior were actually previously expected, but with an extremely small probability—for example, that there would be a generalized fall in U.S. housing prices. When we started on this exercise, we had not appreciated this. It also raises the philosophical question of whether the subjective probability distribution of actual expectations, based on some combination of the accidents of human history and the limited stretch of our imaginations, can ever approximate the true underlying objective probability distribution. If that approximation is partial at best, in what sense can expectations be held to be rational? Keynes and Shackle would have appreciated that question.

Our purpose, however, is not so much to query the current rational expectations methodology as to demonstrate that within the format of existing best-practice, it should be possible to model a combined collapse of the housing and financial markets.

Appendix
Supplemental Tables

We illustrate the results from the comparative statics exercise in two tables. The directional effect of a change in the respective exogenous variables is presented. Other comparative statics we performed include a decrease in liquidity in the short-term loan markets in the last period, a decrease in banks' initial capital, a decrease in banks' capital in the last period, a change in agents' expectations regarding the occurrence of each state of the world, and a production shock in the goods market. The results can be found in our working paper (Goodhart, Tsomocos, and Vardoulakis, 2009).

Table A1. Comparative Statics A

Endogenous variable	*Decrease in money supply at t = 0*	*Liquidity assistance to banks in state 3*	*Endogenous variable*	*Decrease in money supply at t = 0*	*Liquidity assistance to banks in state 3*
p_{01}	–	–	$\bar{d}^{\phi}$	–	+
p_{11}	–	–	μ_0^{ϕ}	–	+
p_{21}	–	–	μ_1^{ϕ}	+	–
p_{31}	–	+	μ_2^{ϕ}	+	–
p_{02}	–	–	μ_1^{λ}	–	–
p_{12}	–	–	μ_2^{λ}	–	–
p_{22}	–	–	μ_3^{λ}	–	+
p_{32}	–	+	q_{01}^{α}	–	–
$\bar{r}^{\gamma}$	+	–	q_{11}^{α}	+	+
$\bar{r}_3^{\gamma}$	–	+	q_{21}^{α}	+	+
$\bar{r}^{\delta}$	+	–	q_{31}^{α}	–	–
$\bar{r}^{\delta}$	–	–	q_{01}^{β}	–	–
$\bar{r}_3^{\delta}$	–	+	q_{11}^{β}	+	+
$\rho, \bar{r}_d, r_0^{\gamma}, r_0^{\delta}$	+	+	q_{21}^{β}	–	–
r_1^{γ}	–	+	q_{31}^{β}	–	+
r_2^{γ}	+	–	b_{01}^{ϕ}	–	–
r_3^{γ}	–	–	b_{11}^{ϕ}	–	–

Table A1. (continued)

Endogenous variable	*Decrease in money supply at t = 0*	*Liquidity assistance to banks in state 3*	*Endogenous variable*	*Decrease in money supply at t = 0*	*Liquidity assistance to banks in state 3*
r_1^δ	–	–	b_{21}^ϕ	–	–
r_2^δ	–	–	b_{31}^ϕ	–	+
r_3^δ	–	–	q_{11}^λ	–	+
$\bar{m}^\gamma$	–	+	q_{21}^λ	–	–
$\bar{m}^\delta$	+	+	q_{31}^λ	–	–
m_0^γ	–	–	b_{02}^α	–	+
m_1^γ	+	–	q_{12}^α	–	+
m_2^γ	+	–	b_{22}^α	–	–
m_3^γ	+	+	b_{32}^α	+	+
m_0^δ	–	–	b_{02}^β	–	+
m_1^δ	+	–	q_{12}^β	+	+
m_2^δ	+	–	b_{22}^β	+	–
m_3^δ	–	+	b_{32}^β	–	+
$\bar{\mu}_d^\gamma$	–	+	q_{02}^ϕ	–	+
$\bar{\mu}_I^\gamma$	–	+	q_{12}^ϕ	+	–
d_I^δ	+	–	q_{22}^ϕ	+	–
$\bar{\mu}^\alpha$	–	+	b_{32}^ϕ	–	–
$\bar{\mu}^\beta$	+	+	b_{12}^λ	+	–
μ_0^α	–	–	b_{22}^λ	+	–
μ_1^α	–	+	b_{32}^λ	–	+
μ_2^α	–	+	U^α	–	+
μ_3^α	–	+	U^β	+	+
μ_0^β	–	–	U^ϕ	–	–
μ_1^β	–	–	U^λ	+	+
μ_2^β	–	–	$\gamma's$ profits	–	–
μ_3^β	–	+	$\delta's$ profits	–	–

Source: Authors' calculations.

Table A2. Comparative Statics B

Endogenous variable	*Decrease in money supply at t = 0*	*Liquidity assistance to banks in state 3*	*Endogenous variable*	*Decrease in money supply at t = 0*	*Liquidity assistance to banks in state 3*
p_{01}	–	–	$\bar{d}^{\phi}$	+	–
p_{11}	–	–	μ_0^{ϕ}	+	–
p_{21}	–	–	μ_1^{ϕ}	–	+
p_{31}	–	–	μ_2^{ϕ}	–	+
p_{02}	–	–	μ_1^{λ}	–	–
p_{12}	–	–	μ_2^{λ}	–	–
p_{22}	–	–	μ_3^{λ}	–	–
p_{32}	–	–	q_{01}^{α}	–	–
$\bar{r}^{\gamma}$	–	+	q_{11}^{α}	+	+
$\bar{r}_3^{\gamma}$	–	–	q_{21}^{α}	+	+
$\bar{r}^{\delta}$	–	+	q_{31}^{α}	+	+ *and* –
$\bar{r}_2^{\delta}$	–	–	q_{01}^{β}	–	–
$\bar{r}_3^{\delta}$	–	–	q_{11}^{β}	+	+
$\rho, \bar{r}_d, r_0^{\gamma}, r_0^{\delta}$	+	+	q_{21}^{β}	+	–
r_1^{γ}	–	–	q_{31}^{β}	–	–
r_2^{γ}	–	+	b_{01}^{ϕ}	–	–
r_3^{γ}	+	+ *and* –	b_{11}^{ϕ}	–	–
r_1^{δ}	+	+	b_{21}^{ϕ}	+	–
r_2^{δ}	+	–	b_{31}^{ϕ}	–	–
r_3^{δ}	+	–	q_{11}^{λ}	+	–
$\bar{m}^{\gamma}$	+	–	q_{21}^{λ}	+	–
$\bar{m}^{\delta}$	+	+	q_{31}^{λ}	–	–
m_0^{γ}	–	–	b_{02}^{α}	+	–
m_1^{γ}	–	+	q_{12}^{α}	+	+
m_2^{γ}	–	+	b_{22}^{α}	–	–
m_3^{γ}	–	+ *and* –	b_{32}^{α}	–	+ *and* –

Table A2. (continued)

Endogenous variable	*Decrease in money supply at t = 0*	*Liquidity assistance to banks in state 3*	*Endogenous variable*	*Decrease in money supply at t = 0*	*Liquidity assistance to banks in state 3*
m_0^δ	–	–	b_{02}^β	+	–
m_1^δ	–	+	q_{12}^β	+	–
m_2^δ	–	+	b_{22}^β	–	+
m_3^δ	–	–	b_{32}^β	–	–
$\bar{\mu}_d^\gamma$	+	–	q_{02}^ϕ	+	–
$\bar{\mu}_I^\gamma$	+	–	q_{12}^ϕ	–	+
d_I^δ	–	+	q_{22}^ϕ	–	+
$\bar{\mu}^\alpha$	+	–	b_{32}^ϕ	+	–
$\bar{\mu}^\beta$	–	+	b_{12}^λ	–	+
μ_0^α	–	–	b_{22}^λ	–	+
μ_1^α	+	–	b_{32}^λ	–	–
μ_2^α	+	–	U^α	–	–
μ_3^α	+	–	U^β	+	+
μ_0^β	–	–	U^ϕ	+	–
μ_1^β	–	–	U^λ	–	+
μ_2^β	–	–	$\gamma's$ profits	+	+
μ_3^β	–	–	$\delta's$ profits	+	+

Source: Authors' calculations.

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